Nonlinear Studies
Vol. 9, No. 3, pp. 273-282, 2002

Nonlinear Studies

Daytona Beach, FL 2002

Globally Stable Kolmogorov Systems

Antonio Tineo[1] and Manuel Gámez[2]

[1]Departamento de Matemáticas,
Facultad de Ciencias, Universidad de los Andes,
Merida.Venezuela

[2]Departamento de Estadística y Matemática
Aplicada.Universidad de Almería.España

AbstractIn this paper, we consider a biological community consisting of p competing subcommunities, such that each subcommunity, in isolation, behaves as a cooperative system. Assuming that the Jacobian matrix is "uniformly stable", we prove that our system is globally asymptotically stable. We also prove a result about persistence, if each subcommunity is globally asymptotically stable. Finally, for $p = 2$, we use a comparison result by H. L. Smith to prove the existence of a coexistence state.

1 Introduction

In this paper, we study the periodic Kolmogorov system

$$x_i' = x_i f_i(t, x_1, \cdots, x_n);\ 1 \le i \le n; \tag{1.1}$$

where $f = (f_1, \cdots, f_n) : \mathbb{R} \times \mathbb{R}_+^n \to \mathbb{R}^n$ is a continuous function such that $f(t, x)$ is T-periodic in t and has partial derivative $f_x(t, x)$, defined and continuous in $\mathbb{R} \times \mathbb{R}_+^n$.

We shall assume that there exists a "decomposition" $I_1, \cdots, I_p$ of $I := \{1, \cdots, n\}$ such that,

$$\frac{\partial f_i}{\partial x_j} \ge 0 \text{ if } i \ne j \text{ and } (i,j) \in \bigcup_{\alpha=1}^{p} (I_\alpha \times I_\alpha) \tag{1.2}$$

$$\frac{\partial f_i}{\partial x_j} \le 0 \text{ if } (i,j) \in \bigcup_{\alpha \ne \beta} (I_\alpha \times I_\beta).$$

The case $p = 2$, for autonomous systems, has been considered by several authors: [5], [6], [4], [10], [11], etc.

This paper was partially supported by CDCHT, Universidad de los Andes.
2000 *Mathematics Subject Classification: Primary* 34A37; 34B37; 34B18.; Secondary: 39A99
Keywords: Kolmogorov periodic systems, Jacobian matrix, Stability.