Why tuning rules for feedforward control are required

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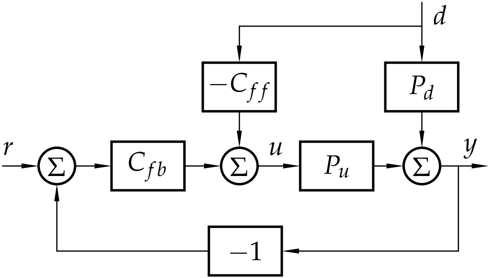
Outline

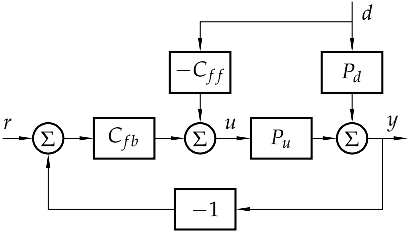
Outline

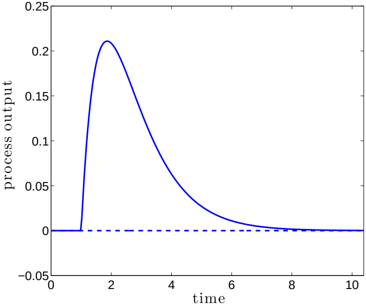
Motivation: feedforward compensator



$$Y = \frac{P_d - C_{ff}P_u}{1 + C_{fb}P_u}D, \qquad C_{ff} = \frac{P_d}{P_u}$$

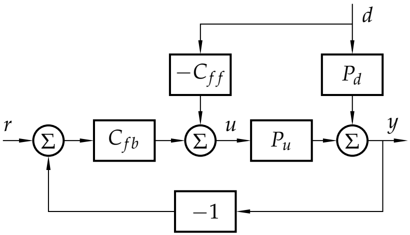
Motivation: feedforward compensator

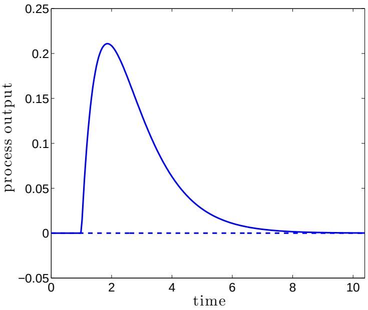




Ideal compensation: $C_{ff} = \dfrac{P_d}{P_u} = P_d\, P_u^{-1}$

Motivation: feedforward compensator





Ideal compensation: $C_{ff} = \frac{P_d}{P_u} = P_d P_u^{-1}$

Feedforward control problem

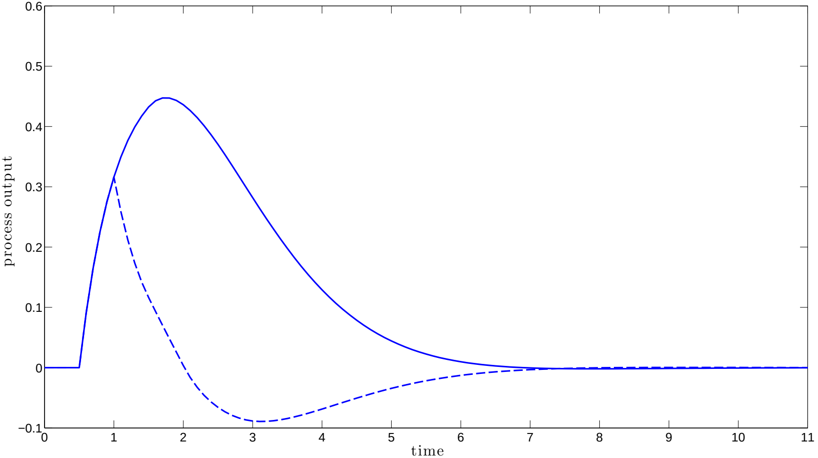
Perfect compensation is seldom realizable:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.
- Improper transfer functions.

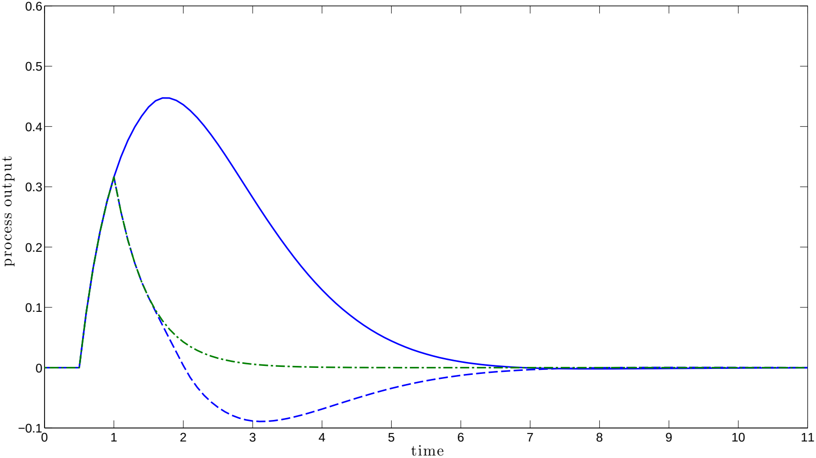
Classical solution

Ignore the non-realizable part of the compensator and implement the realizable one. In practice, static gain feedfoward compensators are quite common.

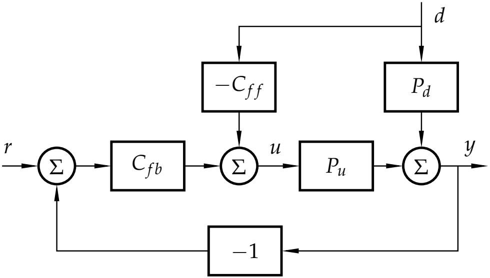
Motivation: non-ideal feedforward compensator



Motivation: non-ideal feedforward compensator



Motivation: residual term



$$Y = \frac{P_d - C_{ff}P_u}{1 + C_{fb}P_u}D, \qquad C_{ff} = \frac{P_d}{P_u}$$

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - C_{ff}P_u}{1+C_{fb}P_u}d$$

Other design strategies are required!

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - C_{ff}P_u}{1+C_{fb}P_u}d$$

Other design strategies are required!

Motivation

Surprisingly there are very few studies in literature (we starting the project in 2010):

- D. Seborg, T. Edgar, D. Mellichamp, Process Dynamics and Control, Wiley, New York, 1989.
- F. G. Shinskey, Process Control Systems. Application Design Adjustment, McGraw-Hill, New York, 1996.
- C. Brosilow, B. Joseph, Techniques of Model-Based Control, Prentice-Hall, New Jersey, 2002.
- A. Isaksson, M. Molander, P. Moden, T. Matsko, K. Starr, Low-Order Feedforward Design Optimizing the Closed-Loop Response, Preprints, Control Systems, 2008, Vancouver, Canada.

Outline

Feedforward control problem

PID control is used as feedback controller and process transfer functions are modeled as FOPDT, i.e.

$$C_{fb} = \kappa_{fb}\left(1 + \frac{1}{s\tau_i} + s\tau_d\right), \;\; P_u = \frac{\kappa_u}{1 + \tau_u} e^{-s\lambda_u}, \;\; P_d = \frac{\kappa_d}{1 + s\tau_d} e^{-s\lambda_d}$$

Two structures for the feedforwrad comensator:

Static with delay: $C_{ff} = \kappa_{ff} e^{-sL_{ff}}$

Lead-lag with delay: $C_{ff} = \kappa_{ff} \dfrac{1 + s\beta_{ff}}{1 + s\tau_{ff}} e^{-sL_{ff}}$

Motivation

Then, let's consider a delay inversion problem, i.e., $\lambda_d < \lambda_u$. Then, the resulting feedforward compensators are given by:

$$C_{ff} = K_{ff} = \frac{\kappa_d}{\kappa_u}$$

$$C_{ff} = \frac{\kappa_d}{\kappa_u} \frac{\tau_u s + 1}{\tau_d s + 1}$$

Motivation

Example:

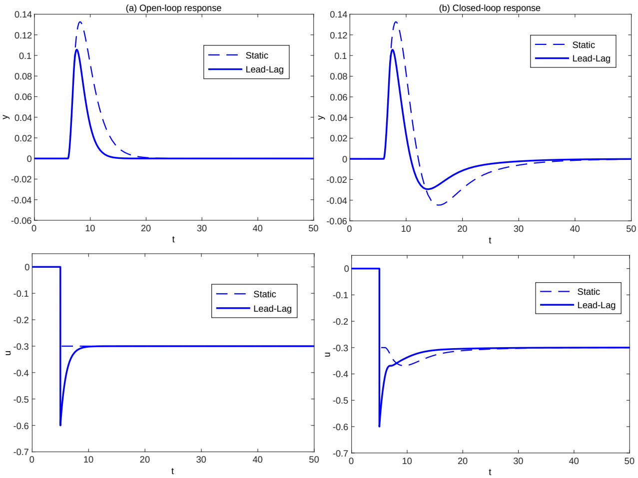
$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

$$C_{ff} = 1, \quad C_{ff} = \frac{2s+1}{s+1}$$

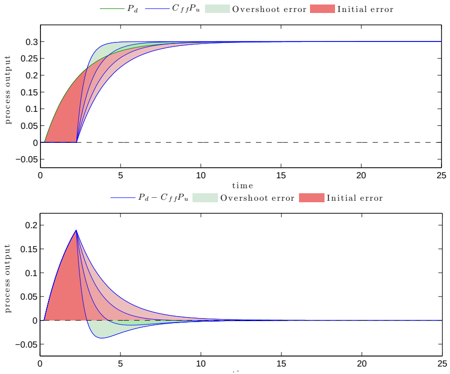
The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.32$ and $\tau_i = 2.85$.

Feedforward control problem

Motivation

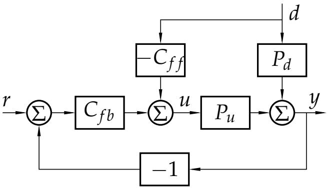


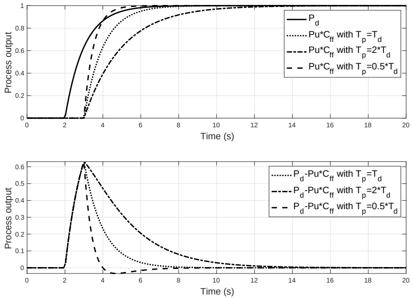
Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d + u_{fb}$$

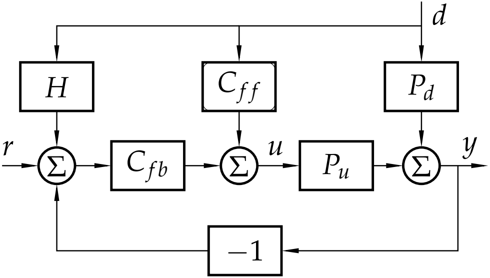
Delay inversion: open-loop and closed-loop interaction





$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d$$

Feedforward control problem



$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

Outline

Feedforward tuning rules

Since 2011, we have been working on this topic for 10 years.

Cases to be evaluated in this research:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.

Feedforward design: non-realizable delay

Objective

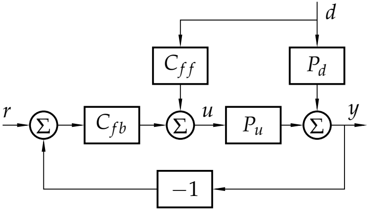
To improve the final disturbance response of the closed-loop system when delay inversion is not realizable ($\lambda_u > \lambda_d$)

Methodology

- Obtain new tuning rules to reduce overshoot or to minimize IAE or ISE criteria.
- Adapt the open-loop tuning rules to closed-loop design for Classical control scheme.
- Open-loop solutions for Non-interactive control scheme.

Feedforward design: non-realizable delay

First approach



$$P_k(s) = \frac{\kappa_k}{\tau_k s + 1} e^{-\lambda_k s} \qquad k \in [u, d] \qquad \lambda_u > \lambda_d$$

$$C_{fb}(s) = \kappa_{fb} \frac{\tau_i s + 1}{\tau_i s} \qquad C_{ff}(s) = \kappa_{ff} \frac{\beta_{ff} s + 1}{\tau_{ff} s + 1}$$

Nominal feedforward design: non-realizable delay

First approach

To deal with the non-realizable delay case, the first approach was to work with the following:

- Use the classical feedforward control scheme.
- Remove the overshoot observed in the response.
- Proposed a tuning rule to minimize Integral Absolute Error (IAE).
- The rules should be simple and based on the feedback and model parameters.

Nominal feedforward design: non-realizable delay

To remove the overshoot, the feedback control action is taken into account to calculate the feedforward gain, κ_{ff}.

$$\Delta u = \frac{\kappa_{fb}}{\tau_i} \int e dt = \frac{\kappa_{fb}}{\tau_i} IE \cdot d$$

So, in the new rule, the goal is to take the control signal to the correct stationary level $-\Delta u$ in order to take the feedback control signal into account and reduce the overshoot. The gain is therefore reduced to

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

Closed-loop design

Nominal feedforward design: non-realizable delay

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Closed-loop design

Nominal feedforward design: non-realizable delay

IE estimation:

$$IE = \begin{cases} k_d(\tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d + \tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d < \lambda_u \end{cases}$$

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

Nominal feedforward design: non-realizable delay

Once the overshoot is reduced, the second goal is to design β_{ff} and τ_{ff} to minimize the IAE value. In this way, we keep $\beta_{ff} = \tau_u$ to cancel the pole of P_u and fix the zero of the compensator:

$$IAE = \int_0^{\infty} |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^{\infty} y(t)dt$$

where t_0 is the time when y crosses the setpoint, with $y_{sp} = 0$ and $d = 1$.

$$\frac{d}{d\tau}IAE = -1 + 2e^{-\frac{\lambda_b}{\tau}} + 2\frac{\lambda_b}{\tau}e^{-\frac{\lambda_b}{\tau}} = -1 + 2(1+x)e^{-x} = 0$$

where $x = \lambda_b/\tau$. A numerical solution of this equation gives $x \approx 1.7$, which gives

$$\tau_{ff} = T_b - \tau_d + \tau_u = \tau_d - \tau \approx \tau_d - \frac{\lambda_b}{1.7}$$

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \end{cases}$$

First approach: Guideline summary

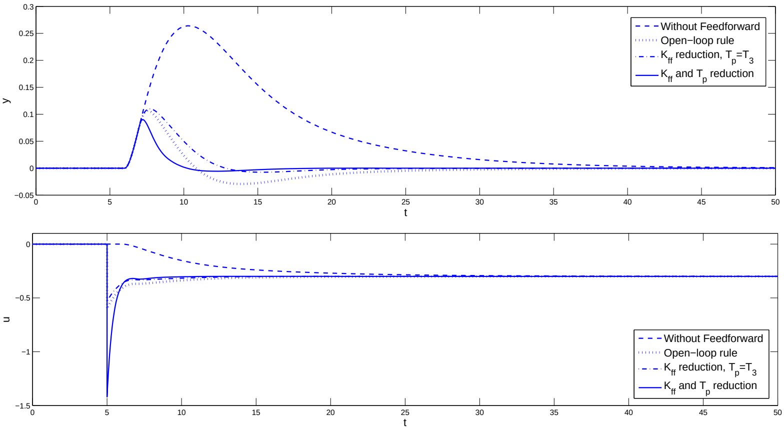
1. Set $\beta_{ff} = \tau_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \end{cases}$$

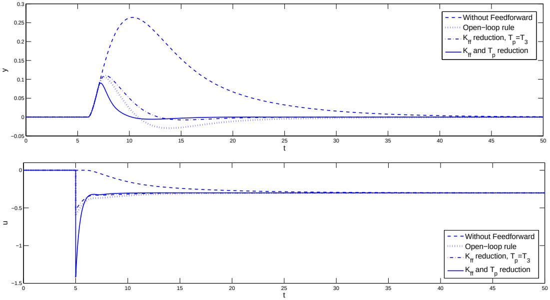
2. Calculate the compensator gain, κ_{ff}, as

$$\begin{aligned} \kappa_{ff} &= \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE \\ IE &= \begin{cases} k_d(\tau_{ff} - \tau_d) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d - \tau_d + \tau_{ff}) & \lambda_d < \lambda_u \end{cases} \end{aligned}$$

Gain and τ_{ff} reduction rule:



Gain and τ_{ff} reduction rule:



	No FF	Open-loop rule	κ_{ff} reduction	κ_{ff} &τ_{ff} reduction
IAE	9.03	1.76	1.37	0.59

Second approach: non-interacting structure

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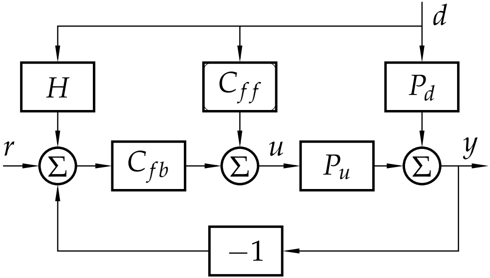


$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

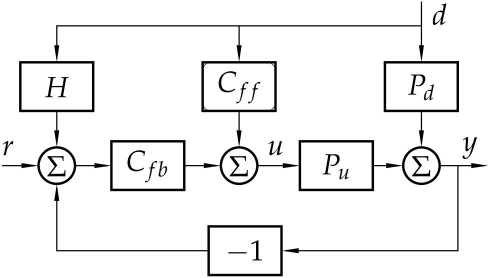
Nominal feedforward design: non-realizable delay

Second approach: non-interacting structure

To deal with the non-realizable delay case, the second approach was to work with the following:

- Use the non-interacting feedforward control scheme (feedback effect removed).
- Obtain a generalized tuning rule for τ_{ff} for moderate, aggressive and conservative responses.
- The rules should be simple and based on the feedback and model parameters.

Second approach: non-interacting structure



$$\frac{y}{d} = P_d - P_u C_{ff}$$

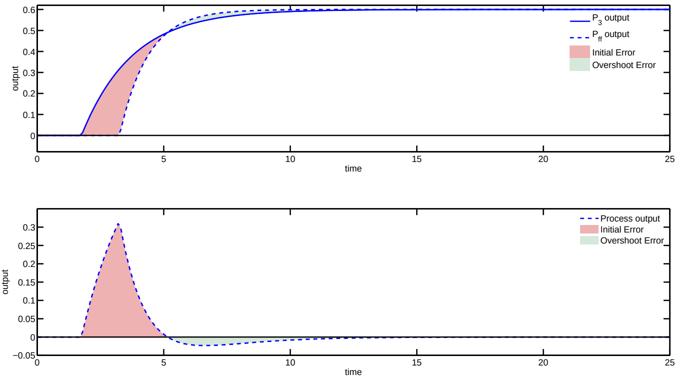
Second approach

The main idea of this second approach relies on analyzing the residual term appearing when perfect cancelation is not possible:

$$\frac{y}{d} = P_d - P_u C_{ff}$$

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

Nominal feedforward design: non-realizable delay



$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

Nominal feedforward design: non-realizable delay

From the previous analysis, it can be concluded that in order to totally remove the overshoot for the disturbance rejection problem by using a lead-lag filter, the settling times of both transfer functions must be the same:

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}, \quad \lambda_b = \lambda_d - \lambda_u$$

Nominal feedforward design: non-realizable delay

So, considering the IAE rule obtained for the first approach, two tuning rules are available:

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}$$

$$\tau_{ff} = \tau_d - \frac{\lambda_u - \lambda_d}{1.7} = \tau_d - \frac{\lambda_b}{1.7}$$

And a third one (a more agreessive rule) can be calculated to minimize Integral Squared Error (ISE) instead of IAE such as proposed in the first approach.

Nominal feedforward design: non-realizable delay

ISE minimization:

$$\frac{d\,\text{ISE}}{d\,\tau_{ff}} = \frac{1}{2} - 2\tau_d e^{-\frac{\lambda_b}{\tau_d}} \left(\frac{1}{\tau_d + \tau_{ff}} + \frac{-\tau_{ff}}{(\tau_d + \tau_{ff})^2} \right) = \frac{1}{2} - \frac{2\tau_d^2}{(\tau_d + \tau_{ff})^2} e^{-\frac{\lambda_b}{\tau_d}} = 0$$

$$\tau_{ff}^2 + 2\tau_d \tau_{ff} + \tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}}) = 0$$

$$\tau_{ff} = \frac{-2\tau_d + \sqrt{4\tau_d^2 - 4\tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}})}}{2} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1 \right)$$

Nominal feedforward design: non-realizable delay

Thus, three tuning rules are available:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{4}$$

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{1.7}$$

$$\tau_{ff} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1 \right)$$

which can be generalized as:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{\alpha}$$

Nominal feedforward design: non-realizable delay

Second approach: Guideline summary

1. Set $\beta_{ff} = \tau_u$, $\kappa_{ff} = k_d/k_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_b \leq 0 \\ \tau_d - \frac{\lambda_b}{\alpha} & 0 < \lambda_b < 4\tau_d \\ 0 & \lambda_b \geq 4\tau_d \end{cases}$$

2. Determine τ_{ff} with $\lambda_b/\tau_d < \alpha < \infty$ using:

$$\alpha = \begin{cases} \dfrac{\lambda_b}{2\tau_d\left(1-\sqrt{e^{-\lambda_b/\tau_d}}\right)} & \text{aggressive (ISE minimization)} \\ 1.7 & \text{moderate (IAE minimization)} \\ 4 & \text{conservative (Overshoot removal)} \end{cases}$$

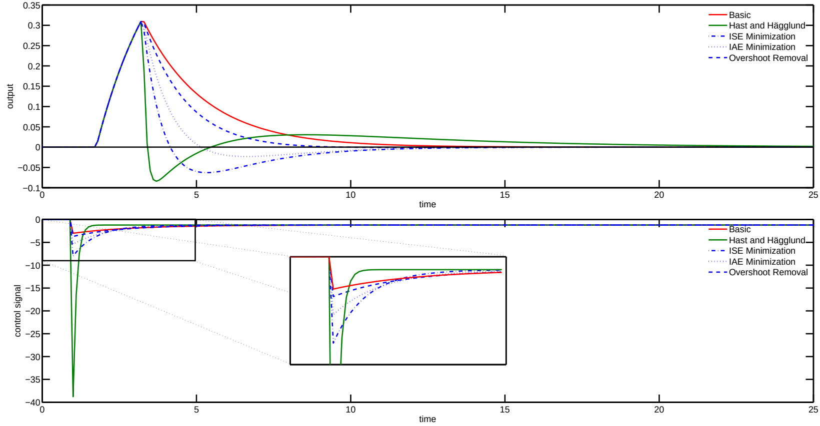
Nominal feedforward design: non-realizable delay

Example:

$$P_u(s) = \frac{0.5}{5s+1}e^{-2.25s}, \quad P_d(s) = \frac{1}{2s+1}e^{-0.75s}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.9$ and $\tau_i = 4.53$.

Nominal feedforward design: non-realizable delay



Nominal feedforward design: non-realizable delay

	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	**0.0739**	0.6423	38.7800	2.5710	0.8979
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	**0.5641**	5.3680	**0.9113**	**0.8315**
Overshoot Removal	0.1277	0.6833	**3.6920**	0.9323	0.8870

$$J_1(F,B) = \frac{1}{2}\left(\frac{\text{ISE}(F)}{\text{ISE}(B)} + \frac{\text{ISC}(F)}{\text{ISC}(B)}\right), \quad \text{ISC} = \int_0^\infty u(t)^2\,\mathrm{d}t$$

$$J_2(F,B) = \frac{1}{2}\left(\frac{\text{IAE}(F)}{\text{IAE}(B)} + \frac{\text{IAC}(F)}{\text{IAC}(B)}\right), \quad \text{IAC} = \int_0^\infty |u(t)|\,\mathrm{d}t$$

Outline

Experimental evaluation

Diurnal greenhouse temperature control

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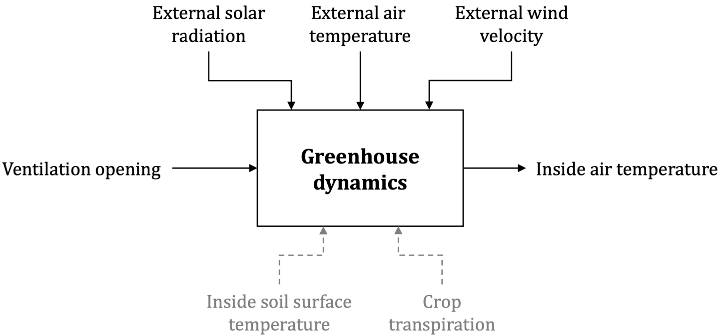
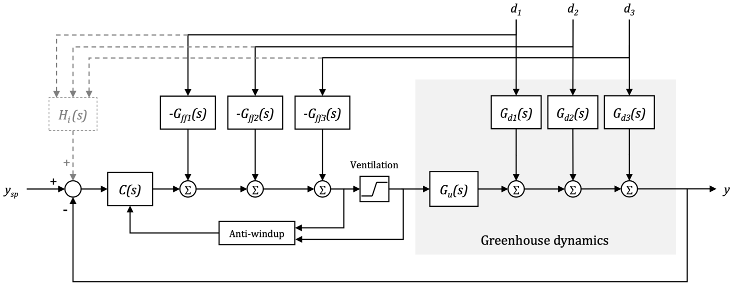
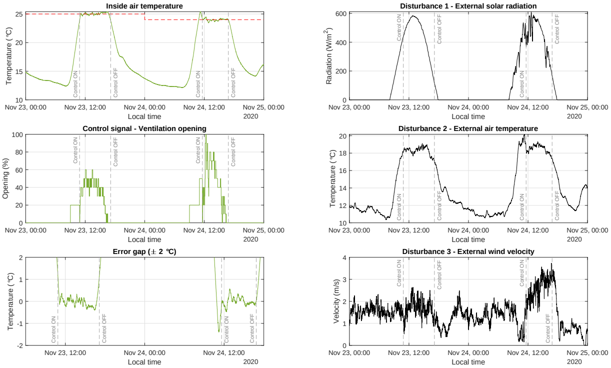


Diurnal greenhouse temperature control



Experimental evaluation

Diurnal greenhouse temperature control



Outline

Conclusions

- The motivation for feedforward tuning rules was introduced.
- The feedback effect on the feedforward design was analyzed.
- The delay inversion problem was studied.
- Simple tuning rules based on the process and feedback controllers parameters were derived.
- An example of experimental evaluation was presented.

End of the presentation

Thank you for your attention