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Observability in strategic models of viability selection

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Abstract

Strategic models of frequency-dependent viability selection, in terms of mathematical systems theory, are considered as a dynamic observation system. Using a general sufficient condition for observability of nonlinear systems with invariant manifold, it is studied whether, observing certain phenotypic characteristics of the population, the development of its genetic state can be recovered, at least near equilibrium.

Keywords: Observability; Frequency-dependent selection; Single-locus system

1. Introduction and preliminaries

The concept of an evolutionarily stable strategy (ESS) was introduced by Maynard Smith and Price (1973) to describe the terminal state of phenotypic evolution of an asexual population undergoing frequency-dependent selection. Later, a game dynamics was proposed to explain how an asexual population can arrive at an ESS, see Taylor and Jonker (1978), Zeeman (1979, 1980), and Hofbauer et al. (1979). For the study of the genetic background of phenotypic evolution, complex dynamic population genetic models were constructed, and the asymptotic behaviour of such systems near the ESS was widely investigated in the literature. For references on the subject, see, e.g. Cressman et al. (1996), also Garay and Varga (1998).

In Garay and Garay (1998), in a static situation, the case of a single locus model was considered where the number of alleles coincides with the number of pure phenotypes. In terms of the hereditary system, a necessary and sufficient condition was given for the allele frequency–phenotype frequency correspondence to be one-to-one. Their proof is based on topological degree arguments.

An important initiative to apply mathematical systems theory (state space models) in population biology was taken in Metz (1977), see also Metz and Dickman (1986). Taking advantage of the dynamics of the considered evolution process, mathematical systems theory offers an efficient technique to recover the underlying genetic process, out of the dynamic observation of certain phenotypic characteristics of the population. This technique has already provided observability results for the classical Fisher selection model (Varga, 1992), as well as for models of reaction kinetics (Farkas, 1998). For an application in population ecology see Varga et al. (2003).

Following Cressman et al. (1996), we shall consider a large panmictic diploid Mendelian population with alleles $A_1, \ldots, A_n$ at a single autosomal locus. By assumption, zygotes are produced according to the Hardy–Weinberg proportions. We consider N possible phenotypes or behavior strategies identified with

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