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BioSystems 81 (2005) 65–75
BioSystems
www.elsevier.com/locate/biosystems

# Equilibrium, observability and controllability in selection-mutation models

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Received 4 November 2004; received in revised form 12 February 2005; accepted 21 February 2005

**Abstract**

In this paper we shortly discuss the problem of the equilibrium in the well-known Fisher type selection-mutation model, also providing a formula for particular three-allele models. The considered continuous-time dynamics is a known extension of the classical model of natural selection given by Fisher. We also extend the existing investigation of the observability of Fisher's model to the case when another evolutionary factor, mutation is also present. Moreover, we prove a result of technical character, which makes it possible to apply the methodology of nonlinear systems with invariant manifold, to models of artificial selection. For an illustration, a class of three-allele systems is presented in which the controllability into equilibrium is guaranteed without any condition on the biological parameters.


*Keywords:* Selection-mutation dynamics; Observability of nonlinear systems; Controllability of nonlinear systems

## 1. Introduction

Alleles can mutation during DNA replication in meiosis. Mutations that occur are a source of genetic variation, despite the action of selection. Mutant alleles are maintained by neutral drift or a lack of direct selection pressure.

Therefore, in a one-locus diploid model, we consider the simultaneous action of selection and mutation on the dynamics, where the effects of the latter, being small in general, change the allelic frequencies in a linear way. We study the well-known Fisher type selection-mutation model from the view point of mathematical systems theory. This model is an extension of the classical continuous-time model of natural selection given by Fisher (1930, 1958), and studied by many authors. In the following sections, we shall also discuss the model, according to the absence or presence of mutation.

Fundamental results concerning the basic selection-mutation model can be found in Crow and Kimura (1970), Moran (1976, 1977) and Hadeler (1981). A general survey of the most recent advances in the study

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doi:10.1016/j.biosystems.2005.02.006