

Available online at www.sciencedirect.com



BioSystems 73 (2004) 99-109



www.elsevier.com/locate/biosystems

Observability in dynamic evolutionary models

I. López*, M. Gámez, R. Carreño*

Departamento de Estadística y Matemática Aplicada, Universidad de Almería La Cañada de San Urbano sn, 04120 Almeria, Spain

Received 9 June 2003; received in revised form 22 October 2003; accepted 22 October 2003

Abstract

In the paper observability problems are considered in basic dynamic evolutionary models for sexual and asexual populations. Observability means that from the (partial) knowledge of certain phenotypic characteristics the whole evolutionary process can be uniquely recovered. Sufficient conditions are given to guarantee observability for both sexual and asexual populations near an evolutionarily stable state.

© 2003 Elsevier Ireland Ltd. All rights reserved.

Keywords: Evolutionary game; Replicator dynamics; Strategic evolutionary model; Observability of non-linear systems

1. Introduction

The static concept of an evolutionarily stable state (ESS) was introduced by Maynard Smith and Price (1973) to describe the terminal state of phenotypic evolution of an asexual population. ESS is a state of the population in which no rare mutant phenotype can propagate via natural selection. (A formal definition of ESS in terms of the pay-off matrix of the evolutionary game is recalled in Section 3.)

Taylor and Jonker (1978) proposed a dynamic evolutionary model (evolutionary game dynamics, replicator dynamics) and proved (see also Zeeman, 1979; Hofbauer and Sigmund, 1988) that ESS is an asymptotically stable equilibrium for this dynamics.

Later, the notion of evolutionary stability was extended to sexual populations defining the corresponding dynamic model, the so-called strategic model of viability selection, where in a diploid model the state of the population is described in terms of allele frequencies, and at the phenotypic level an evolutionary game models the selection process. (See Cressman et al., 1996, 2003; Garay and Varga, 1998; Garay, 2003.)

In the above dynamic models, an important qualitative property, asymptotic stability (in particular the convergence to the evolutionarily stable state) was proved. Further important qualitative properties of these models can be explored applying concepts and methods of mathematical systems theory. This discipline was developed to deal with problems of system engineering (see Kalman et al., 1969). Its application to dynamic selection models was initiated by Varga (1989). In his paper controllability of Fisher's selection model was considered. In biological terms the main result was the following: If a population is subject to natural selection, certain perturbation changes the state of the population from a polymorphic equilibrium to a nearby state, then under generic conditions on the fitness matrix the population can be controlled by artificial selection into this equilibrium in given time. Recently, in Scarelli and Varga (2002) these results have been extended to selection-mutation processes.

^{*} Corresponding author. Fax: +34-950-015167.

E-mail address: milopez@ual.es (R. Carreño).

 $^{0303\}mathchar`2647/\$$ – see front matter @ 2003 Elsevier Ireland Ltd. All rights reserved. doi:10.1016/j.biosystems.2003.10.003