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PERSISTENCE AND GLOBAL STABILITY IN A PREDATOR-PREY SYSTEM WITH DELAY

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In this paper, several sufficient conditions are established for the persistence and extinction in a Lotka–Volterra system with time delay. Based on the use of Lyapunov functionals techniques, necessary and sufficient conditions are also given for global asymptotic stability of the positive equilibrium for autonomous systems.

Keywords: Delayed Lotka–Volterra systems; global asymptotic stability; bifurcation; persistence; extinction.

1. Introduction

Consider the system of differential equations with finite time delay,

$$\begin{aligned} x'(t) &= x(t)\left[a(t) - b(t)x(t) - c(t)\int_0^r k_1(s)y(t-s)ds\right] \\ y'(t) &= y(t)\left[-d(t) + e(t)\int_0^r k_2(s)x(t-s)ds - f(t)y(t)\right] \end{aligned} \tag{1}$$

where functions $a(t), b(t), \ldots, f(t)$ are assumed to be continuous and bounded above and below by positive constants, whereas kernels $k_1(s), k_2(s)$ defined in $[0, r]$ are nonnegative functions with $\int_0^r k_i(s)ds = 1$ for $i = 1, 2$.

Systems of type Eq. (1) are used often to model the dynamics of a two-species system consisting of a prey and predator populations, $x(t), y(t)$ denote the sizes (or densities) of the prey and predator populations at time t respectively. Roughly speaking, we say that system Eq. (1) is persistent if

$$\limsup_{t\to+\infty} x(t) > 0, \quad \limsup_{t\to+\infty} y(t) > 0.$$

System Eq. (1) is said to be uniformly persistent if both populations will eventually have densities larger than some positive constant; if, in addition, these populations are also bounded, then we say the system is permanent.

Recently, the study of persistence and extinction for nonautonomous Lotka–Volterra systems with delay has received much attention. Li and Teng [2003] studied the persistence of nonautonomous n-species food chain systems with finite delay. Teng and Chen [2003], Xu *et al.* [2004a] discussed uniform persistence for a periodic predator-prey model with dispersion and time delays.

Another important question for this type of systems is to analyze the effect of time delays on the

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