Nonlinear Analysis: Real World Applications 11 (2010) 1918–1924

Contents lists available at ScienceDirect

Nonlinear Analysis: Real World Applications

journal homepage: www.elsevier.com/locate/nonrwa

Observer design for open and closed trophic chains

Z. Varga [a,*], M. Gámez [b], I. López [b]

[a] Institute of Mathematics and Informatics, Szent István University, Páter K. u. 1., H-2103 Godollo, Hungary

[b] Department of Statistics and Applied Mathematics, University of Almería, La Cañada de San Urbano, 04120 Almería, Espagne

ARTICLE INFO

Article history:
Received 18 March 2009
Accepted 29 April 2009

Keywords:
Ecological monitoring
Observer design
Trophic chain

ABSTRACT

Monitoring of ecological systems is one of the major issues in ecosystem research. The concepts and methodology of mathematical systems theory provide useful tools to face this problem. In many cases, state monitoring of a complex ecological system consists in observation (measurement) of certain state variables, and the whole state process has to be determined from the observed data. The solution proposed in the paper is the design of an observer system, which makes it possible to approximately recover the state process from its partial observation. Such systems-theoretical approach has been applied before by the authors to Lotka–Volterra type population systems. In the present paper this methodology is extended to a non-Lotka–Volterra type trophic chain of *resource–producer–primary consumer* type and numerical examples for different observation situations are also presented.

1. Introduction

The problem of sustainability of economic and social development in a broader sense also involves conservation aspects of ecology. The problem of state monitoring of population systems, even under natural conditions, is an important issue in conservation ecology. Nearly natural populations are often exposed to a strong human intervention, e.g. by wildlife management, fisheries or environmental pollution. This means that human activity may improve or break the equilibrium of the population system in question, it may also increase or decrease the genetic variability of the given populations. One of the main tasks of conservation biology is to preserve the diversity of population systems and genetic variability of certain populations. These problems make it necessary to extend the traditional approach of theoretical biology focusing only on a biological object, to the study of the system "biological object – man". This, in dynamic situation, i.e. in case of a long-term human intervention, typically requires the approach of mathematical systems theory (in frequently used terms, state-space modelling). On the state-space approach to modelling in population biology, [1] is an early reference, see also [2].

Mathematical systems theory offers a methodology to face the monitoring problem. This discipline had been developed by the 1960s to solve a variety of problems in engineering and industry. A basic reference is [3], see also [4]. A recent reference on linear systems theory is [5]. While by now, mathematical systems theory became quite familiar to system engineers, observability and controllability analysis of dynamic models in population biology is relatively new. In many cases, state monitoring of a complex ecological system consists in observation (measurement) of certain state variables, and the whole state process has to be determined from the observed data. In a more general setting, the state process is a system of differential equations, and instead of its concrete solution only a transform (in particular a subset of the components) of it is known (measured). The considered system is called (locally) observable, if from the observation, the underlying state process can be uniquely recovered (near an equilibrium state). Based on the sufficient condition for nonlinear observation

* Corresponding author. Tel.: +36 12752058; fax: +36 28410804.
E-mail addresses: Varga.Zoltan@gek.szie.hu (Z. Varga), mgamez@ual.es (M. Gámez), milopez@ual.es (I. López).

1468-1218/$ – see front matter
doi:10.1016/j.nonrwa.2009.04.015

systems published in [6], for different coexisting Lotka–Volterra type population systems, local observability results have been obtained in part by some of the coauthors of the present paper in [7,8]. Later on, in addition to these theoretical results, for *Lotka–Volterra systems*even a so-called observer systems has been constructed that made it possible to numerically recover the state process from the observation data, see [7–11]. We also mention that, based on an observability result of [12] for nonlinear observation systems with invariant manifold, in [13] an observer system was designed for the frequency-dependent model of phenotypic observation of genetic processes.

In the present paper ecological systems of *non-Lotka–Volterra* type will be considered. Up until now in [14], only observability results have been obtained for systems of type *resource–producer–primary consumer*. In Section 2, from [14], the model setup and basic conditions for the existence of an equilibrium of the system are shortly recalled. Section 3 is the main body of the paper. First the theoretical background of the observer design is set up. Then the construction of the observer and the asymptotic recovery of the state process is illustrated with numerical examples for different observation situations. Section 4 is devoted to the discussion of the results.

2. Description of the dynamic model

In order to illustrate the application of the methodology of mathematical systems theory, a relatively simple food web, a trophic chain has been chosen, that in addition to populations also involves a resource (energy or nutrient). In the following, the model setup is shortly recalled from [14], see also [15,16]. For further details on trophic chains (and general food webs) see e.g. [17,18].

The considered model describes how a resource moves through a trophic chain. A typical terrestrial trophic chain consists of the following components:
resource, the 0th *trophic level* (solar energy or inorganic nutrient),
which is incorporated by
a plant population, the 1st *trophic level* (*producer*),
which transfers it to
a herbivorous animal population, the 2nd *trophic level* (*primary consumer*).

Let us note that, in a longer trophic chain, the herbivores can be consumed by a predator population, the 3rd *trophic level* (*secondary consumer*), which can be followed by a top predator population (*tertiary consumers*). In the present paper, for technical simplicity only trophic chains of the type *resource– producer–primary consumer* will be studied. According to the possible types of 0th level (energy or nutrient), two types of trophic chains will be considered: *open chains* (without recycling) and *closed chains* (with recycling). At the 0th *trophic level, resource* is the common term for energy and nutrient.

Let x_0 denote the time-varying quantity of free resource, say nutrient present in the system, x_1 and x_2, in function of time, the biomass (or density) of the producer (species 1) and the primary consumer (species 2), respectively. Let Q be the resource supply considered constant in the model. Let $\alpha_0 x_0$ be the velocity at which a unit of biomass of species 1 consumes the resource, and assume that this consumption increases the biomass of species 1 at rate k_1. A unit of biomass of species 2 consumes the biomass of species 1 at velocity $\alpha_1 x_1$, converting it into biomass at rate k_2. Both the plant and the animal populations are supposed to decrease exponentially in the absence of the resource and the other species, with respective rates of decrease (Malthus parameters) m_1 and m_2.

Finally, in a *closed system* the dead individuals of species 1 and 2 are recycled into nutrient at respective rates $0 < \beta_1 < 1$ and $0 < \beta_2 < 1$, while for an *open system* (where there is no natural recycling) $\beta_1 = 0, \beta_2 = 0$ holds. Then with model parameters

$$Q, \alpha_0, \alpha_1, m_1, m_2 > 0; \quad k_1, k_2 \in]0, 1[; \beta_1, \beta_2 \in [0, 1[,$$

for the trophic chain the following dynamic model can be set up:

$$\dot{x}_0 = Q - \alpha_0 x_0 x_1 + \beta_1 m_1 x_1 + \beta_2 m_2 x_2 \tag{2.1}$$

$$\dot{x}_1 = x_1(-m_1 + k_1 \alpha_0 x_0 - \alpha_1 x_2) \tag{2.2}$$

$$\dot{x}_2 = x_2(-m_2 + k_2 \alpha_1 x_1). \tag{2.3}$$

Let function f be defined in terms of the right-hand side of this system:

$$f : R^3 \to R^3, \quad f(x) = f(x_0, x_1, x_2) := \begin{bmatrix} Q - \alpha_0 x_0 x_1 + \beta_1 m_1 x_1 + \beta_2 m_2 x_2 \\ x_1(-m_1 + k_1 \alpha_0 x_0 - \alpha_1 x_2) \\ x_2(-m_2 + k_2 \alpha_1 x_1) \end{bmatrix}.$$

In [14], necessary and sufficient conditions were found for the existence of a non-trivial ecological equilibrium $x^*$of dynamic system (2.1)–(2.3), where all components are present: system (2.1)–(2.3) has a unique equilibrium $x^* = (x_0^*, x_1^*, x_2^*) > 0$ if and only if the resource supply is high enough, i.e.

$$Q > Q_1 := \frac{m_1 m_2}{\alpha_1 k_1 k_2} - \frac{\beta_1 m_1 m_2}{\alpha_1 k_2}. \tag{2.4}$$

Remark 2.1. For $\beta_1 > 0$ the threshold Q_1 is lower than that for $\beta_1 = 0$. Clearly, in the latter case the lack of recycling from species 1, a higher value of resource supply is necessary to produce the required positive equilibrium.

Remark 2.2. A linearization shows that under the same condition (2.4), this equilibrium x^* is also asymptotically stable, which means stable coexistence in ecological sense.

3. Construction of an observer system for a trophic chain

Concerning different observation situations, in [14] local observability of system (2.1)–(2.3) has been proved. (For the concept of local observability see Appendix). Now, following the Theorem of [19] (see Appendix), we shall construct, in an explicit way, the local exponential observer for the three cases considered by [14]. To this end, for the linearization of system (2.1)–(2.3), we calculate the corresponding Jacobian at equilibrium x^*,

$$A := f'(x^*) = \begin{bmatrix} -\alpha_0 x_1^* & -\alpha_0 x_0^* + \beta_1 m_1 & \beta_2 m_2 \\ k_1\alpha_0 x_1^* & 0 & -\alpha_1 x_1^* \\ 0 & k_2\alpha_1 x_2^* & 0 \end{bmatrix}.$$

Case 1. We consider the observation of the resources of system (2.1)–(2.3), where the observation function is

$$h(x) := x_0 - x_0^* \Rightarrow C := h'(x^*) = (1, 0, 0). \tag{3.1}$$

In order to construct the local observer for the considered observation system, we need to determine a matrix $H = col(h_1, h_2, h_3)$ such that matrix *A-HC* is Hurwitz, i.e. all its eigenvalues have negative real parts. According to the Hurwitz criterion (see e.g. [5]), in terms of the normed characteristic polynomial of *A-HC*, the following necessary and sufficient condition holds:

$$p(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0 \quad \text{is Hurwitz } \Leftrightarrow a_0, a_1, a_2 > 0 \quad \text{and} \quad a_2 \cdot a_1 > a_0. \tag{3.2}$$

This matrix H can be determined from the following theorem:

Theorem 3.1. *Let us suppose that the resource supply is high enough,* $Q > \frac{m_1 m_2}{\alpha_1 k_1 k_2}$ *and matrix*

$$H := \begin{pmatrix} h_1 \\ 0 \\ 1 \end{pmatrix}$$

is such that $h_1 > \max\left\{\frac{m_1 x_1^*}{m_2 x_2^*}, \frac{\alpha_0 x_0^*}{\beta_2 k_2}\right\}$. *Then the dynamic system defined by*

$$\dot{z} = f(z) + H[y - h(z)]$$

is a local exponential observer for system (2.1)–(2.3) with the observation of the resource defined by (3.1).

Proof. It is sufficient to show that under the conditions of the theorem, x^* is a Lyapunov stable equilibrium of system (2.1)–(2.3), and the matrix *A-HC* is Hurwitz. Then the proof can be concluded by applying the Theorem of [19] (see Appendix).

First, from $Q > \frac{m_1 m_2}{\alpha_1 k_1 k_2}$ inequality $Q > Q_1$ also follows, which on the one hand, as quoted at the end of Section 2, implies the existence of a unique positive equilibrium. On the other hand, in [14,15] it was proved, both in open systems (with $\beta_1 = 0$, $\beta_2 = 0$) and in partially or totally closed systems (i.e. at least one of inequalities $0 < \beta_1 < 1$ and $0 < \beta_2 < 1$ holds) condition $Q > Q_1$ also implies (asymptotic) stability of the equilibrium.

From (2.1)–(2.3) the coordinates of the positive equilibrium x^* are

$$x_0^* = \frac{-\alpha_1 Q - \frac{\beta_1 m_1 m_2}{k_2} + \beta_2 m_1 m_2}{-\frac{\alpha_0 m_2}{k_2} + \alpha_0\beta_2 k_1 m_2},$$

$$x_1^* = \frac{m_2}{k_2\alpha_1},$$

$$x_2^* = \frac{-\alpha_0 k_1 Q + -\frac{\alpha_0 m_1 m_2}{k_2\alpha_1} - \frac{\beta_1 k_1 \alpha_0 m_1 m_2}{k_2\alpha_1}}{-\frac{\alpha_0 m_2}{k_2} + \alpha_0\beta_2 k_1 m_2}.$$

Now it will be proved that for the coefficients of the normed characteristic polynomial of *A-HC* conditions (3.2) hold. To cut short the rather tedious calculations, the following statements can be checked: Hypotheses $Q > \frac{m_1 m_2}{\alpha_1 k_1 k_2}$ and $k_1, k_2 \in]0, 1[$; $\beta_1, \beta_2 \in [0, 1[$ imply $Q > \frac{m_1 m_2}{\alpha_1}$ and also $\alpha_0 x_0^* - \beta_1 m_1 > 0$, furthermore, the latter is sufficient for $a_1 > 0$ and also used in the proof of $a_2 \cdot a_1 - a_0 > 0$. On the other hand,

$$h_1 > \frac{m_1 x_1^*}{m_2 x_2^*} \Rightarrow \alpha_1 h_1 k_2 x_2^* - \beta_1 m_1 > 0, \quad \text{to be used in the proof of } a_0 > 0$$

$$h_1 > \frac{\alpha_0 x_0^*}{\beta_2 k_2} \Rightarrow \beta_2 m_2 h_1 - \alpha_0\alpha_1 x_0^* x_1^* > 0 \Rightarrow a_2 \cdot a_1 - a_0 > 0.$$

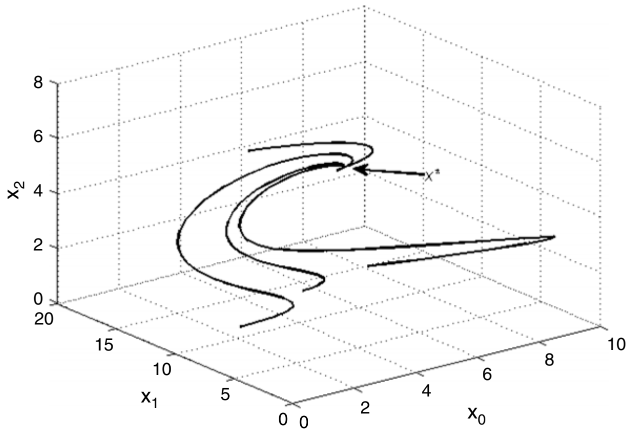

Fig. 1. Some solutions of systems (2.1)–(2.3).

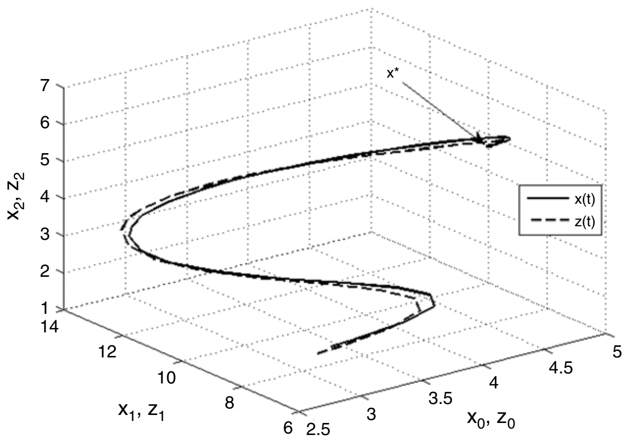

Fig. 2. Solutions of systems (2.1)–(2.3) and (3.3).

From $x_1^* = \frac{m_2}{k_2\alpha_1}$, $k_1, k_2 \in]0, 1[$ and $\beta_2 \in [0, 1[$ inequality $\alpha_1 k_2 x_1^* x_2^* - \beta_2 k_1 k_2 m_2 x_2^* > 0$ can be derived, which implies $a_0 > 0$. Finally, inequalities $h_1, a_0, x_1^* > 0$ directly imply $a_2 > 0$. As a conclusion, all inequalities in (3.2) hold for the characteristic polynomial of p. Therefore matrix A-HC is Hurwitz, which concludes the proof. □

Example 3.2. As a numerical example, we consider the following parameter values: $Q := 10$; $\alpha_0 := 0.3$; $\alpha_1 := 0.1$; $\beta_1 := 0.2$; $\beta_2 := 0.3$; $m_1 := 0.1$; $m_2 := 0.4$; $k_1 := 0.5$; $k_2 := 0.5$. In this case the considered system (2.1)–(2.3) has a positive equilibrium $x^* = (4.52, 8, 5.78)$, which is asymptotically stable (see Fig. 1).

Now, with matrix

$$H := \begin{pmatrix} 10 \\ 0 \\ 1 \end{pmatrix},$$

conditions of Theorem 3.1 are satisfied, therefore we can construct the following observer system

$$\begin{aligned} \dot{z}_0 &= 10 - 0.3 z_0 z_1 + 0.2 \cdot 0.1 z_1 + 0.3 \cdot 0.4 z_2 + 10\left[y - \left(z_0 - x_0^*\right)\right] \\ \dot{z}_1 &= z_1(-0.1 + 0.5 \cdot 0.3 z_0 - 0.1 z_2) \\ \dot{z}_2 &= z_2(-0.4 + 0.5 \cdot 0.1 z_1) + 1[y - (z_0 - x_0^*)]. \end{aligned} \tag{3.3}$$

If we set initial condition $x(0) := (3, 7, 2)$ near the equilibrium of system (2.1)–(2.3), and similarly, we consider another nearby initial condition, $z(0) := (2.9, 7.2, 1.8)$ for the observer system (3.3), Fig. 2 shows that the corresponding solution z tends to the solution x of the original system.

Case 2. Now we consider the case when the plant of system (2.1)–(2.3) is observed. The observation function then is

$$h(x) := x_1 - x_1^* \Rightarrow C := h'(x^*) = (0, 1, 0). \tag{3.4}$$

Similarly to Case 1, we can prove the following theorem providing an observer for the case (3.4).

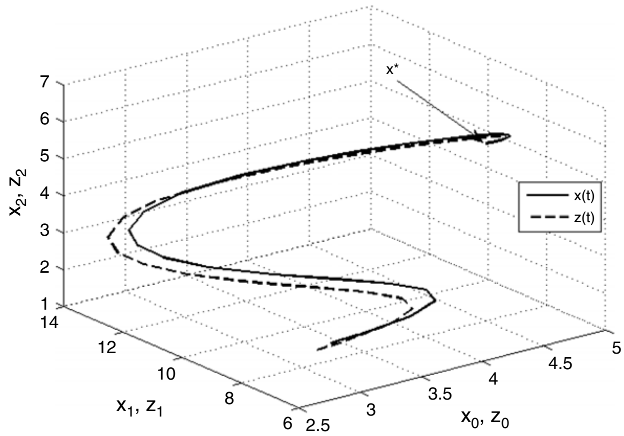

Fig. 3. Solutions of systems (2.1)–(2.3) and (3.5).

Theorem 3.3. *Given a matrix*

$$H := \begin{pmatrix} h_1 \\ h_2 \\ 0 \end{pmatrix}$$

with $h_1 > m_1$ and $h_2 > 0$, dynamic system defined by

$$\dot{z} = f(z) + H[y - h(z)]$$

is a local exponential observer for system (2.1)–(2.3) with the observation of the plant, as given in (3.4).

Proof. The scheme of the proof is similar to that of the previous one. We only have to prove that matrix *A-HC* is Hurwitz and the application Theorem of [19] will conclude the proof. Since from [14] we have $x_1^* = \frac{m_2}{k_2\alpha_1}$, and $k_1, k_2 \in]0, 1[$; $\beta_2 \in [0, 1[$, therefore we obtain that $\beta_2 k_1 m_2 - \alpha_1 x_1^* < 0$. Moreover, since $\beta_1 \in [0, 1[$ and $h_1 > m_1$, we have that $h_1 - \beta_1 m_1 > 0$. Applying these inequalities and taking into account that the case of a positive equilibrium $x^* > 0$ is considered and $h_2 > 0$, it is easy to check that conditions (3.2) hold, therefore matrix *A-HC* is Hurwitz, and the proof is complete. □

Example 3.4. With the same model parameters as in Example 3.2, we consider

$$H = \begin{pmatrix} 0.5 \\ 0.1 \\ 0 \end{pmatrix}.$$

Then conditions of Theorem 3.3 are verified and therefore we can construct the following observer system

$$\begin{aligned} \dot{z}_0 &= 10 - 0.3z_0z_1 + 0.2 \cdot 0.1z_1 + 0.3 \cdot 0.4z_2 + 0.5\,[y - (z_1 - x_1^*)] \\ \dot{z}_1 &= z_1(-0.1 + 0.5 \cdot 0.3z_0 - 0.1z_2) + 0.1\,[y - (z_1 - x_1^*)] \\ \dot{z}_2 &= z_2(-0.4 + 0.5 \cdot 0.1z_1). \end{aligned} \tag{3.5}$$

If we set again initial condition $x(0) := (3, 7, 2)$, near the equilibrium of system (2.1)–(2.3), and similarly, we consider another nearby initial condition, $z(0) := (2.9, 7.2, 1.8)$ for observer system (3.5), Fig. 3 shows that the corresponding solution z tends to the solution x of the original system.

Case 3. Let us finally consider the observation of the herbivorous species of system (2.1)–(2.3), where the observation function is

$$h(x) := x_2 - x_2^* \Rightarrow C := h'(x^*) = (0, 0, 1). \tag{3.6}$$

Similarly to Theorems 3.1 and 3.3, it is not hard to prove the following theorem providing an observer for the case (3.6).

Theorem 3.5. *Let $Q > \frac{m_1 m_2}{\alpha_1 k_1 k_2}$ be satisfied, and define matrix*

$$H := \begin{pmatrix} h_1 \\ h_2 \\ 0 \end{pmatrix},$$

where $h_1 > m_1$ and $h_2 > m_2$. Then the dynamic system defined by

$$\dot{z} = f(z) + H[y - h(z)]$$

is a local exponential observer for system (2.1)–(2.3) with the observation of the plant, with h defined in (3.6).

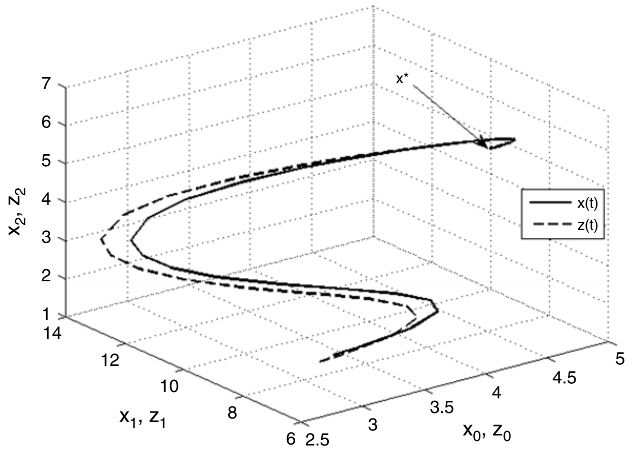

Fig. 4. Solutions of systems (2.1)–(2.3) and (3.7).

Example 3.6. For the model parameters of the previous examples, with

$$H := \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \end{pmatrix},$$

conditions of Theorem 3.5 hold, and hence we obtain the following observer system

$$\begin{aligned} \dot{z}_0 &= 10 - 0.3z_0z_1 + 0.2 \cdot 0.1z_1 + 0.3 \cdot 0.4z_2 + 0.5\,[y - (z_2 - x_2^*)] \\ \dot{z}_1 &= z_1(-0.1 + 0.5 \cdot 0.3z_0 - 0.1z_2) + 0.5\,[y - (z_2 - x_2^*)] \\ \dot{z}_2 &= z_2(-0.4 + 0.5 \cdot 0.1z_1). \end{aligned} \tag{3.7}$$

Set again initial condition $x(0) := (3, 7, 2)$, close to the equilibrium of system (2.1)–(2.3), and as a nearby initial condition for the observer system (3.7) also choose $z(0) := (2.9, 7.2, 1.8)$. Now Fig. 4 shows that the corresponding solution z tends again to the solution x of the original system.

4. Discussion

In the paper the construction of an observer system was applied for the state monitoring of a simple trophic chain of the type *resource–producer–primary consumer*, recovering the whole state process from the only observation of different components of the systems, such as the resource, the plant (producer) and a herbivorous animal. The applied methodology can also be extended to more complex models of food webs, involving the observation of certain abiotic environmental components and/or certain indicator species. A similar approach may be also useful for the monitoring of population systems in changing environment, where the change of certain abiotic parameters of the ecosystem is governed by an “external” dynamic system (describing an industrial pollution or climatic changes).

Acknowledgements

The authors wish to thank the Ministry of Education and Science of Spain for the financial support of the project TIN2007-67418-C03-02, which has partially supported this work. The research was also supported by the Hungarian National Scientific Research Fund (OTKA 62000 and 68187), and a bilateral project funded by the Scientific and Technological Innovation Fund (of Hungary) and the Ministry of Education and Sciences (of Spain, grant No. HH2008-0023).

Appendix

Given positive integers m, n, let

$$f : R^n \to R^n, \quad h : R^n \to R^m$$

be continuously differentiable functions and suppose for some $x^* \in R^n$ we have that $f(x^*) = 0$ and $h(x^*) = 0$.

We consider the following observation system

$$\dot{x} = f(x) \tag{A.1}$$

$$y = h(x), \tag{A.2}$$

where y is called the *observed function*.

Definition A.1. Observation system (A.1) and (A.2) is called *locally observable* near equilibrium x^*, over a given time interval $[0, T]$, if there exists $\varepsilon > 0$, such that for any two different solutions x and $\bar{x}$ of system (2.1) with $|x(t) - x^*| < \varepsilon$ and $|\bar{x}(t) - x^*| < \varepsilon (t \in [0, T])$, the observed functions $h \circ x$ and $h \circ \bar{x}$ are different. ($\circ$ denotes the composition of functions. For brevity, the reference to $[0, T]$ is suppressed.)

For the formulation of a sufficient condition for local observability consider the linearization of the observation system (A.1) and (A.2), consisting in the calculation of the Jacobians

$$A := f'(x^*) \quad \text{and} \quad C := h'(x^*).$$

Theorem A.2 (*[6]*). *Suppose that*

$$rank[C|CA|CA^2|\cdots|CA^{n-1}]^{\mathrm{T}} = n. \tag{A.3}$$

Then observation system (A.1) *and* (A.2) *is locally observable near equilibrium x^*.*

Now, the construction of an observer system will be based on [19]. Let us consider observation system (A.1) and (A.2).

Definition A.3. Given a continuously differentiable function $G : R^n \times R^m \to R^n$, system

$$\dot{z} = G(z, y) \tag{A.4}$$

is called a *local asymptotic (respectively, exponential) observer for observation system* (A.1) and (A.2), if the composite system (A.1) and (A.2), (A.4) satisfies the following two requirements:

(i) If $x(0) = z(0)$, then $x(t) = z(t)$, for all $t \geq 0$.
(ii) There exists a neighbourhood V of the equilibrium x^* of R^n such that for all $x(0), z(0) \in V$, the estimation error $z(t) - x(t)$ decays asymptotically (respectively, exponentially) to zero.

Theorem A.4 (*[19]*). *Suppose that equilibrium x^* of system* (A.1) *and* (A.2) *is Lyapunov stable, and that there exists a matrix H such that matrix A-HC is Hurwitz (i.e. its eigenvalues have negative real parts), where $A := f'(x^*)$ and $C := h'(x^*)$. Then dynamic system defined by*

$$\dot{z} = f(z) + H[y - h(z)] \tag{A.5}$$

is a local exponential observer for observation system (A.1) *and* (A.2).

References

[1] J.A.J. Metz, State space model for animal behaviour, Annals of Systems Research 6 (1977) 65–109.
[2] J.A.J. Metz, O. Dickmann (Eds.), The Dynamics of Physiologically structured Populations, in: Lecture Notes in Biomathematics, vol. 68, Springer, 1986, pp. 1–511.
[3] R.E. Kalman, P.L. Falb, M.A. Arbib, Topics in Mathematical System Theory, McGraw-Hill, New York, 1969.
[4] L.A. Zadeh, C.A. Desoer, Linear System Theory-The State Space Approach, McGraw-Hill Book Co, New York, 1963.
[5] Ben M. Chen, Zongli Lin, Yacov A. Shamesh, Linear Systems Theory. A Structural Decomposition Approach, Birkhauser, Boston, 2004.
[6] E.B. Lee, L. Markus, Foundations of Optimal Control Theory, Wiley, New York-London-Sydney, 1971.
[7] I. López, M. Gámez, S. Molnár, Observability and observers in a food web, Applied Mathematics Letters 20 (8) (2007) 951–957.
[8] I. López, M. Gámez, J. Garay, Z. Varga, Monitoring in a Lotka–Volterra model, Biosystems 83 (2007) 68–74.
[9] M. Gámez, I. López, Z. Varga, Iterative scheme for the observation of a competitive Lotka–Volterra system, Applied Mathematics and Computation 201 (2008) 811–818.
[10] M. Gámez, I. López, S. Molnár, Monitoring environmental change in an ecosystem, Biosystems 93 (2008) 211–217.
[11] Z. Varga, Applications of mathematical systems theory in population biology, Periodica Mathematica Hungarica 51 (1) (2008) 157–168.
[12] Z. Varga, On observability of Fisher's model of selection, Pure Mathematics and Applications, Series B 3 (1) (1992) 15–25.
[13] I. López, M. Gámez, Z. Varga, Observer design for phenotypic observation of genetic processes, Nonlinear Analysis: Real World Applications 9 (2008) 290–302.
[14] A. Shamandy, Monitoring of trophic chains, Biosystems 81 (1) (2005) 43–48.
[15] Yu.M. Svirezhev, D.O. Logofet, Stability of Biological Communities, Mir Publishers, Moscow, 1983.
[16] S. Jorgensen, Y. Svirezhev (Eds.), Towards a Thermodynamic Theory for Ecological Systems, Pergamon, 2004.
[17] E.P. Odum, Fundamentals of Ecology, 3rd ed., Saunders, Philadelphia, 1971, pp. 1–574.
[18] P. Yodzis, Introduction to Theoretical Ecology, Harper & Row, New York, 1989.
[19] V. Sundarapandian, Local observer design for nonlinear systems, Mathematical and computer modelling 35 (2002) 25–36.