Game-theoretical model for marketing cooperative in fisheries

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Abstract

The classical game-theoretical models described the conflict in fisheries arising from harvesting a 'common pool resource' which without an efficient regulation leads to an overexploitation of a renewable but not unlimited resource, known as the 'tragedy of the commons'. Unlike these studies, the present paper deals with a marketing cooperative of micro or small enterprises in fishing industry, formed to negotiate a contracted price with large buyers, sharing risk among members of the cooperative. In the paper a game-theoretical

model for the behaviour in this cooperative is set up. By the time of the actual commercialization of the product, the market price may be higher than what the cooperative can guarantee for members, negotiated on beforehand. Therefore some “unfaithful” members may be interested in selling at least a part of their product on the free market, the cooperative, however, can punish them for this. This conflict is modelled with a multi-person normal form game. An evolutionary dynamics is proposed for the continuous change of the applied strategies, which in the long term leads to a particular Nash equilibrium, considered the solution of the game. This strategy dynamics is continuously influenced by an ”exosystem” describing the dynamics of fishing, based on a classical fishing effort model. This approach focuses only on the conflict within the marketing cooperative, since it is supposed that the single enterprises fish from independent resources.

Keywords: fishery management, marketing cooperative, oligopoly, evolutionary game dynamics.

1. Introduction and preliminaries

1.1. Introduction

A cooperative in a given region may perform several activities, ranging from product processing to complex marketing, see e.g. Cobia (1989). In particular, concerning fisheries, Freeman (2010) gives a quick checklist of benefits and drawbacks of fishing cooperatives. Micro and small enterprises often have difficulties in the commercialization of their product.

Varga *et al*. (2010) analyze a game-theoretical model for the behaviour in a marketing cooperative. The model studies a ’one-shot game’, where at the end of a given production cycle, each member of the cooperative may decide to sell a part of its production on the free market, if the market price is higher than the price set by the cooperative beforehand. In the present study we will deal with a similar conflict, but in the context of fisheries based on a dynamic fishing effort model.

An overview of different conflicts fishery management should face, is given e.g. in Caddy (1999), Cochrane *et al*. (1998), Castilla and Defeo (2001). From these papers and the references therein it is clear that the classical game-theoretical conflict of exploiting a common-pool resource has been widely studied over the last decades. Less attention has been payed to the marketing conflicts related to fisheries. As examples of dealing with conflicts in oligopoly market environment, we recall Szidarovszky and Okuguchi (1998) and Bischi *et al*. (2005).

In the present paper a model of marketing cooperative in fisheries is set up and studied, where in a given time period a continuous production (harvested biomass) is being sold, under the condition that the actual offer is determined by the dynamics of the harvested fish population. We emphasize that in the considered situation the game-theoretical conflict arises on the marketing side, while the production in unit time $L_i(t)$ of each cooperative member *i*, comes from the solution of the corresponding classical logistic fishing effort model (see e.g.Clark, 1990).

First we set up a normal form game to describe the considered conflict and apply a solution concept called attractive solution, which is a special type of Nash equilibrium, introduced by Larbani (1997), see also Larbani and Lebbah H. (1999). Then this solution concept is also studied in dynamic context, applying an evolutionary dynamics introduced by Garay (2002). The reason for the application of this solution concept is that it takes into consideration that in the definition of an equilibrium there is a distinguished player which in our case will be the cooperative, and the rest of the players will be its members.

The paper is organized as follows. In the rest of Section 1, a classical fishery model is recalled that will be a component of the model we will set up. In Section 2, following a general description of a marketing cooperative, the oligopoly market environment is formalized, where the price is determined by the total offer. In Section 3 a time-dependent game-theoretical model of the cooperative is introduced, and the existence of an attractive solution is proved. In Section 4 a model of dynamic strategy choice is introduced and sufficient conditions are given under which the strategy choice of the players leads to the desired attractive solution of the game. In Section 5 the strategy dynamics with discrete-time delivery of the catch is shortly touched on. A Discussion and outlook section closes the main body of the paper. In the Appendix, for the reader's convenience some further details of the applied classical fishing effort model are recalled.

Finally, we note that the simulations illustrating our theoretical study have been programmed in MatLab environment.

1.2. A classical fishery model

There are two basic types of classical fishery models. In both types the population dynamics of the considered fish population is described by the logistic model. In the *quota model* the per unit time catch is independent of the population biomass, with a constant $h > 0$ we have

$$\dot{z} = rz\left(1 - \frac{z}{K}\right) - h ,$$

where r and K are the Malthus parameter and the carrying capacity of the habitat, respectively.

We will use the more flexible *fishing effort model*, where the per unit time catch is a function of the actual biomass. From the different function types (see e.g. Marcos *et al.*, 2015; Kar, 2004), for our analytical study we will use the linear dependence (see Schaefer, 1954)

$$\dot{z} = rz\left(1 - \frac{z}{K}\right) - E\rho z , \tag{1.1}$$

where E is the fishing effort (number of vessels or gears applied) and ρ is the catchability constant (the biomass caught from unit stock by one vessel, in unit time). From Clark (1990), in the Appendix for the reader's convenience we recall some basic properties of this model, also giving the analytic solution to it. It can be shown that if $E < \frac{\rho}{r}$, dynamics (1.1) has an equilibrium $0 < z^* < K$, which is attractor in the sense that from both initial values $0 < z(0) < z^*$ and $z^* < z(0) < K$, the solution tends to z^*, monotonically increasing in the first case, monotonically decreasing in the second one (see Appendix, Figure A1.). In Figure 1, the numerical solution is illustrated with parameters: r=0.2, K=1000, E=1.1, $\rho = 0.01$. Now z^*=950, and with initial values $z(0)$=150$< z^*$, $z(0)$=1700$>$ z* we obtain the solutions shown in Figure 1.

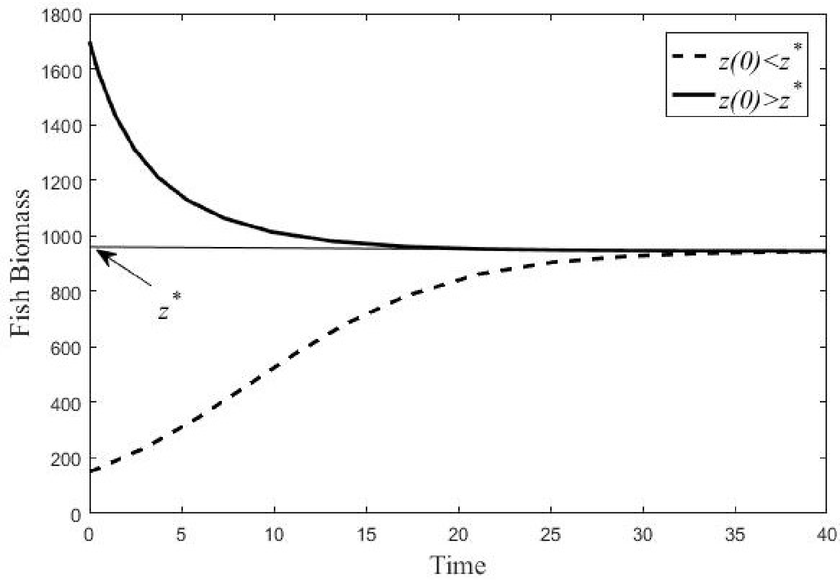

Figure 1. Solutions of fishing effort model (1.1)

2. Description of a marketing cooperative of fishing enterprises

2.1. General description of the considered marketing cooperative

Let us assume that there are n micro or small enterprises fishing in different lakes of the same geographic area, with somewhat similar ecologic conditions. (For an example, one can think of the volcanic lakes of the Latium region in Central Italy, or the glacial lakes of Northern Italy). In fact, our modelling conditions will primarily correspond to freshwater fisheries. For the sake of simplicity only one fish species will be included in our model. For such enterprises, because of the relatively small quantity they can offer for sale, it is reasonable to form a marketing cooperative and find a large buyer who would contract them, say for a time period ahead, at a negotiated price, for the total production of the cooperative. However, in the meanwhile on the free market there may appear a price higher than the contracted price. Then the member enterprises of the cooperative may be interested in selling a part of their production on the free market. If member i sells the x_i -part of its production to the cooperative, and $x_i < 1$, then the cooperative may punish it for "unfaithfulness". In the formalization of this conflict as a normal form game, $x_i \in [0,1]$ will be the strategy of member (player) i. With vector $x = (x_1, x_2, ..., x_n) \in [0,1]^n$, $(x, y) \in [0,1]^n \times [0,1]$ will be called a multi-strategy. The cooperative as player $n+1$, can threaten the unfaithful with penalty proportional to its extra revenue, with rate $y \in [0,1]$. The net revenue (payoff) of the players will depend on the actual free market price formed by the outputs on oligopoly basis, as described in the next subsection.

2.2. Cournot type oligopoly market with time-dependent outputs

Since the quantity offered on the market by the enterprises will be proportional to the time-dependent catch obtained from a dynamic fishery model (1.1), below we will consider a time-dependent market situation. A market where the price is defined by the total quantity of goods offered by several producers, is usually called *Cournot type oligopoly market.*

With $a, b > 0$, for all $t \in [0, \infty[$ and multi-strategy $(x, y) \in [0,1]^n \times [0,1]$ we define the time-dependent inverse demand function (or price function) as a decreasing linear function of the total output:

$$q(t, x) := a - b \sum_{i=1}^{n} L_i(t)(1 - x_i), \tag{2.1}$$

where $L_i(t)$ is the time-dependent total catch shared by enterprise *i*, between the cooperative and the local market. Since in lack of local product (when all $L_i(t)$-s are zero) the price is *a*, the latter can be considered as import price. Suppose that the "import price" a is greater than the contracted price p (i.e. $a > p$), functions $L_i(t)$ are bounded, and the oligopoly effect is weak enough (i.e. *b* is small enough). Then the oligopoly model is considered consistent, i.e. for all $(t, x) \in [0, \infty[\times[0,1]^n$ we have

$$q(t, x) := a - b\sum_{i=1}^{n} L_i(t)(1 - x_i) > p. \tag{2.2}$$

In particular, the inverse demand function is always positive, and the market price is attractive for the members of the cooperative.

3. Game-theoretical model of the conflict between the cooperative and its members

3.1. A solution concept for *N*-person games

Since in the considered conflict the cooperative has a distinguished role, we need a special solution concept for an *N*-player game, called *attractive solution* (see e.g. Larbani, 1997) of the game, in the sense of the following definition. Consider an *N*-player game where

X_i is the strategy set of player *i*,

$X := \prod_{i=1}^{N} X_i$ the set of multi-strategies,

$F_i : X \to \mathbf{R}$ the payoff of player *i*, $F := (F_1, ..., F_N)$.

Definition 1. Multi-strategy x^0 is said to be an *attractive solution* of the normal form game (X, F), if there exists a player $i \in \overline{1, N}$ such that the following conditions are satisfied:

A) $F_j(x_1, ..., x_i^0, ..., x_N) \leq F_j(x^0)$, ($j \in \overline{1, N} \setminus \{i\}$, $x_k \in X_k$, $k \in \overline{1, N} \setminus \{i\}$);

B) $F_i(x_1^0, ..., x_i, ..., x_N^0) \leq F_i(x^0)$, ($x_i \in X_i$).

Remark 1. If in condition A), for each $j \in \overline{1,N} \setminus \{i\}$ we choose $x_k := x_k^0 \in X_k$ for $k \in \overline{1,N} \setminus \{i,j\}$, and an arbitrary $x_j \in X_j$, we obtain that any attractive solution is a Nash equilibrium (NE), too. The interpretation of the distinguished player i is the following: if player i sticks to his equilibrium strategy, the rest of the players cannot increase their payoff even if they deviate together from their equilibrium strategies. In the context of our game, the distinguished player is the cooperative.

3.2. Game model for the marketing cooperative

Let c be the production cost per unit biomass, and $\beta \geq 1$ a penalty parameter. For any $t \in [0,\infty[$ and multi-strategy $(x,y) \in [0,1]^n \times [0,1]$, the payoff (profit = revenue – cost – penalty) of player i is

$$f_i(t,x,y) := [(p-c)x_i L_i(t) + (q(t,x)-c)(1-x_i)L_i(t) - \beta y(q(t,x)-p)(1-x_i)L_i(t)]$$

$$= L_i(t)\{(p-c)x_i + (1-x_i)[q(t,x)-c-\beta y(q(t,x)-p)]\}, \qquad (i \in \overline{1,n}) \qquad (3.1)$$

and for player $(n+1)$ the payoff is

$$f_{n+1}(t,x,y) := \beta y(q(t,x)-p)\sum_{j=1}^{n} L_j(t)(1-x_j). \qquad (3.2)$$

With notation $f := (f_1, f_2, ..., f_{n+1})$, for the description of the cooperative we have a time-dependent normal form game

$$([0,1]^n \times [0,1], f). \qquad (3.3)$$

Solution of the game

Let us fix a time moment $t \in [0,\infty[$, and denote $\mathbf{1} := (1,1,...,1) \in \mathbf{R}^n$. Then for any multi-strategy $(x,y) \in [0,1]^n \times [0,1]$ we easily obtain that

$$f_i(t,x,1) - f_i(t,\mathbf{1},1) = L_i(t)(1-\beta)(1-x_i)(q(t,x)-p) \leq 0, \qquad (3.4)$$

and

$$f_{n+1}(t,\mathbf{1},y) - f_{n+1}(t,\mathbf{1},1) = \beta y(q(t,x)-p)\sum_{j=1}^{n} L_j(t)(1-1) - \beta(q(t,x)-p)\sum_{j=1}^{n} L_j(t)(1-1) = 0. \quad (3.5)$$

Furthermore, it is easy to see that under the conditions of subsection 2.2, in case $\beta > 1$, $x \in [0,1]^n$ with $x_i < 1$ for some $i \in \overline{1,n}$, inequality (3.4) is strict.

Hence we obtain the following theorem.

Theorem 1. Multi-strategy $(\mathbf{1},1)$ is an attractive solution of game (3.3).

4. Strategy dynamics and stabilization of the cooperative

Different evolutionary models are often used to describe economic behaviour, see e.g. Cressman *et al.* (2004). For the time-invariant case, where the production of the members is constant and the game is played continuously with improving strategies, we have already applied the so-called *partial adaptive dynamics* of Garay (2002). Below we will develop this dynamics for our model of marketing cooperative in fisheries.

4.1. Evolutionary strategy dynamics for the game of the cooperative

The idea of the partial adaptive dynamics is that a player should continuously change his strategy in order to improve his payoff, provided the other players maintain their strategies. To this end the time derivative of the strategy should be proportional to the partial derivative of the player's payoff with respect to his own strategy.

With payoff functions of (3.1)-(3.2) the strategy dynamics for the time-dependent strategies $x_i(t)$ and $y(t)$ of member *i* and the cooperative, respectively, will be

$$\dot{x}_i = x_i(1-x_i)\frac{\partial}{\partial x_i} f_i(t,x,y) \quad (i \in \overline{1,n}), \tag{4.1}$$

$$\dot{y} = y(1-y)\frac{\partial}{\partial y} f_{n+1}(t,x,y). \tag{4.2}$$

From (3.1) and (3.2) we obtain

$$\frac{\partial}{\partial x_i} f_i(t,x,y) = L_i(t)(\beta y - 1)[q(t,x) - p - (1-x_i)bL_i(t)] \quad (i \in \overline{1,n}),$$

$$\frac{\partial}{\partial y} f_{n+1}(t,x,y) = \beta(q(t,x)-p)\sum_{j=1}^{n} L_j(t)(1-x_j)\,.$$

For all $(t,x,y) \in [0,\infty[\times[0,1]^n \times [0,1]$, we get the strategy dynamics

$$\dot{x}_i = x_i(1-x_i)L_i(t)(\beta y-1)[q(t,x)-p-(1-x_i)bL_i(t)] \qquad (i \in \overline{1,n}), \tag{4.3}$$

$$\dot{y} = y(1-y)\beta(q(t,x)-p)\sum_{j=1}^{n} L_j(t)(1-x_j)\,. \tag{4.4}$$

Remark 2. The attractive solution $(\mathbf{1},1)$ is obviously an equilibrium of dynamics (4.3)-(4.4), and the first two factors in the right-hand sides ensure that dynamics (4.3)-(4.4) leaves the multi-strategy set $[0,1]^n \times [0,1]$ positively invariant, which is necessary for the consistency of the model.

4.2. Asymptotic properties of the strategy dynamics

For the following two theorems we suppose that the production in unit time $L_i(t)$ of each cooperative member *i*, comes from the solution of the corresponding fishing effort model,

$$L_i(t) = E_i\rho_i z_i(t) \;\; (t \geq 0) \tag{4.5}$$

Case of limited penalty ($\beta = 1$)

Now we will see that by limited penalty for unfaithfulness of the members, the cooperative cannot be stabilized.

Theorem 2. Suppose that $\beta = 1$, and the oligopoly effect is weak enough (parameter *b* in the inverse demand function is small enough). Then attractive solution $(\mathbf{1},1)$ of the game described above is an *unstable* equilibrium of dynamics (4.3)-(4.4).

Proof. Since now for $(t,x,y) \in [0,\infty[\times]0,1[^n\times]0,1[$ in equation (4.3) obviously $\beta y - 1 < 0$, from (4.5) we have $L_i(t) > 0$, and we need to check the sign of the term [...] in (4.3). In addition, solutions $z_i(t)$ of the fishing effort equations are bounded from above (see Appendix, (A.4)). Therefore, $L_i(t) = E_i\rho_i z_i(t)$ ($t \geq 0$) is also bounded from above. Now since $a > p$ is supposed, similarly to the reasoning of subsection 2.2 for the consistence of the oligopoly market, we obtain that for b sufficiently small we have

$$q(t,x)-p-(1-x_i)bL_i(t)=a-b[\sum_{j=1}^{n}L_j(t)(1-x_j)-L_i(t)(1-x_i)]-p>0$$

for $(t,x)\in[0,\infty[\times]0,1]^n$.

Since $\beta y-1<0$, the right-hand side of equation (4.3) is negative, $(x(t),y(t))$ will diverge from dynamic equilibrium (and also NE) $(\mathbf{1},1)$.

Example 1. For an illustration let us consider a marketing cooperative of three small enterprises, fishing the same species in three different lakes, according to the fishing effort model (1.1),

$$\dot{z}_i=r_iz_i\left(1-\frac{z_i}{K_i}\right)-E_i\rho_iz_i \quad (i\in\overline{1,3}), \tag{4.6}$$

where the parameters are K_1=1100; K_2=1000; K_3=1050; r_1=0.25; r_2=0.21; r_2=0.23; $\rho_1=0.01$; $\rho_2=0.015$; $\rho_3=0.013$; E_1=2; E_2=1; E_3=3. Furthermore, the parameters of the game model are $\beta=1$; p=1.5; a=2.5; b=0.0087. In Figures 2 and 3 we illustrate Theorem 2, with z^*=(969, 952.38, 871.96). Set $z(0)$=(450, 450 , 450) for Figure 2, and $x(0)$=(0.5, 0.7, 0.8); y(0)=0.9 for Figure 3. Then we obviously have $z_i(0)<z_i^*$.

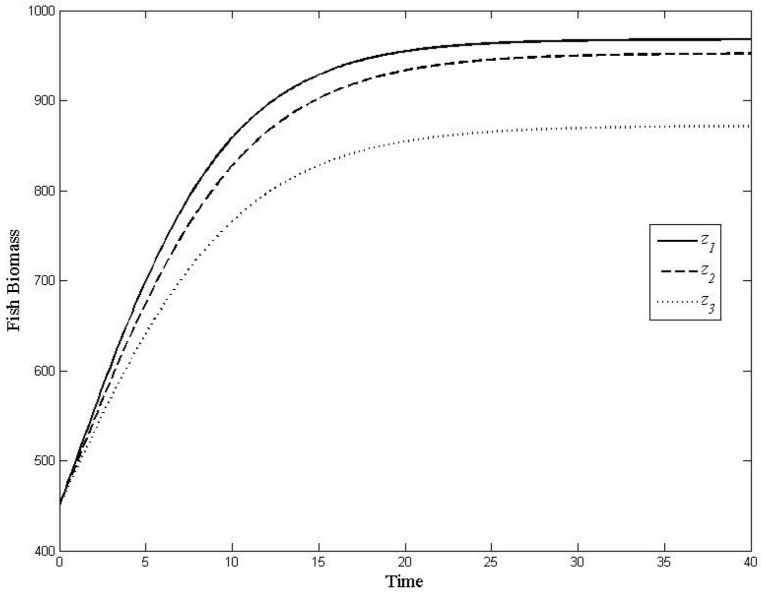

Figure 2. Solutions of fish dynamics (4.6) for cooperative members, with $\beta = 1$ and $z(0) < z^*$

The corresponding development of strategy dynamics is shown in Figure 3.

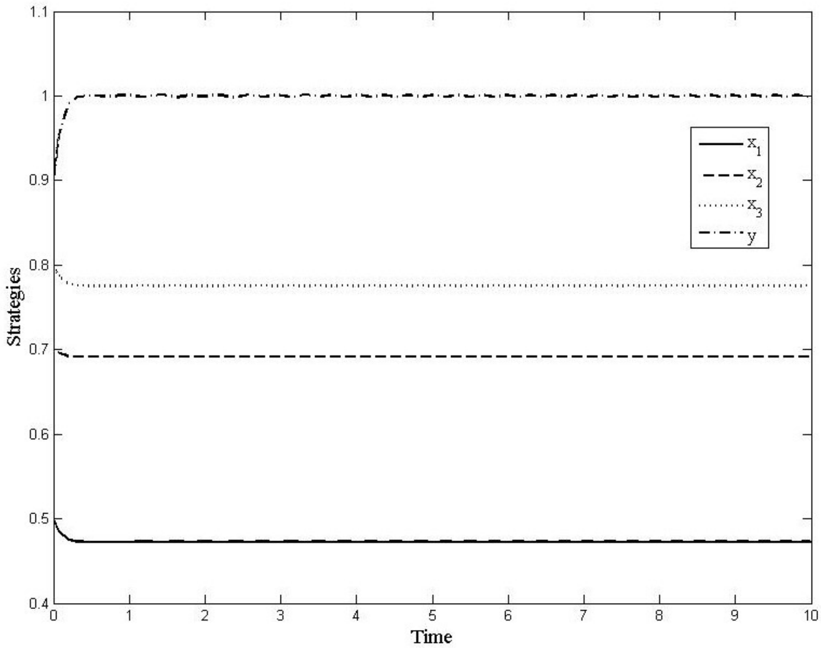

Figure 3. Strategies dynamics, corresponding to fish dynamics of Figure 2. Members "escape" from the Nash strategies **1**.

Next, with z(0)= (970, 970 , 970), we have $z(0) > z^*$. Theorem 2 is illustrated in Figures 4 and 5, with results analogous to Figures 2 and 3.

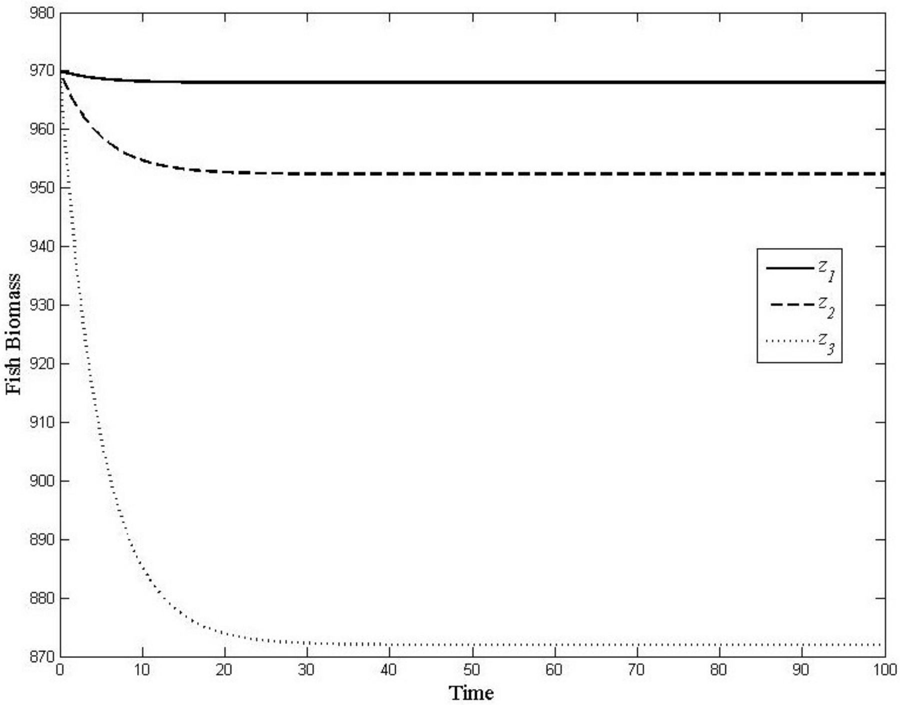

Figure 4. Solutions of fish dynamics (4.6), for $\beta = 1$ and $z(0) > z^*$

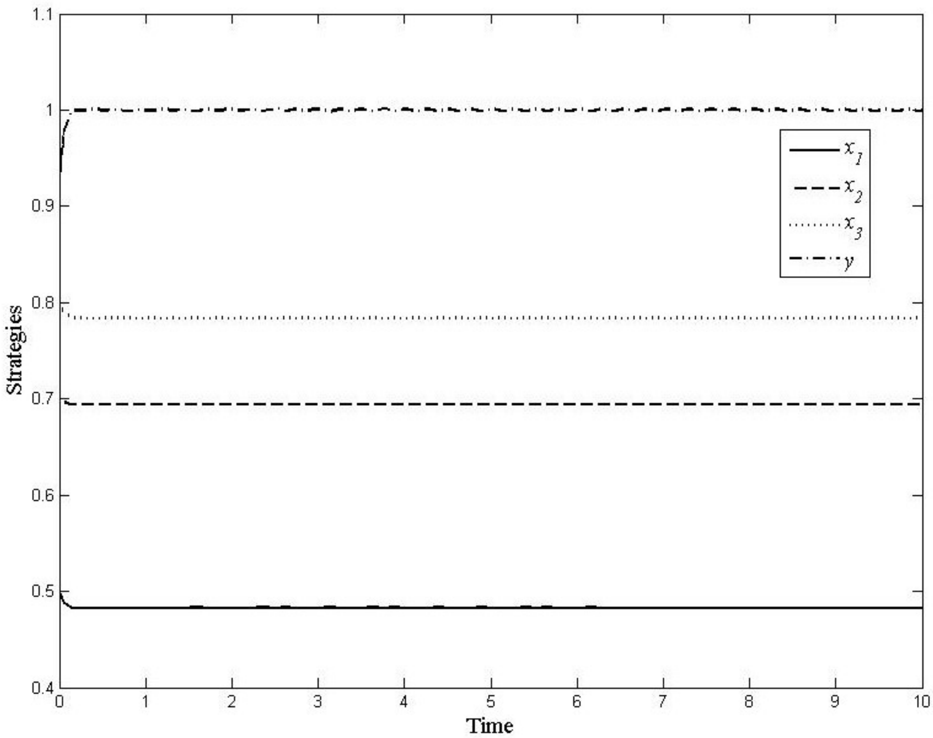

Figure 5. Strategy dynamics (4.3)-(4.4), corresponding to fish dynamics of Figure 4,

$$\text{for } z(0) > z^*$$

Case of effective penalty $(\beta > 1)$

Now we will see that the effective penalty for unfaithfulness can stabilize the cooperative, since the multi-strategy will then tend to a limit where, under the effective threat by the cooperative, all members are to complete faithful.

Theorem 3. If the penalty is effective ($\beta > 1$), and the oligopoly effect is weak enough, then any solution of the strategy dynamics starting from $(x(0), y(0))$ in $]0,1[^n \times]1/\beta, 1[$, will tend to the attractive solution $(\mathbf{1}, 1)$ of the game (i.e. dynamic equilibrium $(\mathbf{1}, 1)$ is an attractor for dynamics (4.3)-(4.4).)

Proof. Now in $]0,1[^n \times]1/\beta, 1[$, by the sign reasoning of the proof of the previous theorem, for b small enough we obtain that the right-hand side of equation (4.3) will be positive, because now $y > 1/\beta$. Hence x_i is strictly increasing, and so is y because of the positivity of the right-hand side of equation (4.4). Thus there exists $\lim_{t\to\infty}(x(t), y(t)) = (x^*, y^*) \in]0,1]^n \times]1/\beta, 1]$.

It is clear that if to equations (4.3) and (4.4) we join the population dynamic equations

$$\dot{z}_i = r_i z_i \left(1 - \frac{z_i}{K_i}\right) - E_i \rho_i z_i \quad (i \in \overline{1,n})$$

to system (4.3)-(4.4) by the substitution $L_i(t) = E_i \rho_i z_i(t) \ (t \geq 0)$, we obtain an autonomous system for (x, y, z) of the form

$$\left.\begin{aligned} \dot{x} &= u(x, y, z) \\ \dot{y} &= v(x, y, z) \\ \dot{z} &= w(x, y, z) \end{aligned}\right\} \qquad (4.7)$$

with $z := (z_1, z_2, ..., z_n)$. As it is known (see Appendix, (A.4)), from every initial population density $z_i(0) \in]0, K[$ z_i tends to the equilibrium: $\lim_{t\to\infty} z_i(t) = z_i^*$. Thus we have $\lim_{t\to\infty}(x(t), y(t), z(t)) = (x^*, y^*, z^*) \in]0,1]^n \times]1/\beta,1] \times]0, K[$. We will show that (x^*, y^*, z^*) is an equilibrium of system (4.7). Indeed, $\lim_{t\to\infty} \dot{x}(t) = \lim_{t\to\infty} u(x(t), y(t), z(t)) = u(x^*, y^*, z^*)$. Hence we get

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \dot{x}(s)ds = \lim_{t\to\infty} \frac{1}{t}[x(t) - x(0)] = u(x^*, y^*, z^*) = 0 ,$$

where boundedness of $x(t)$ $(t \geq 0)$ was applied. We similarly obtain $v(x^*, y^*, z^*) = 0$ and $w(x^*, y^*, z^*) = 0$. It is easy to see that $(x^*, y^*) = (\mathbf{1},1)$. Indeed, suppose that for some i, inequality $x_i^* < 1$ holds. Then

$$\begin{aligned} 0 = u_i(x^*, y^*, z^*) = x_i^*(1 - x_i^*)E_i\rho_i z_i^*(\beta y^* - 1) \\ [a - b\sum_{j=1}^n E_j\rho_j z_j^*(1 - x_j^*) - p - bE_i\rho_i z_i^*(1 - x_i^*)] > 0 \end{aligned},$$

which is a contradiction. Similar reasoning leads to $y_i^* = 1$.

Example 2. For an illustration of Theorem 3, we consider the parameter system of Example 1, except $\beta = 1.15$. In Figures 6-9 we show how the effective penalty stabilizes the cooperative: now the strategy dynamics leads to the attractive solution $(\mathbf{1},1)$ of game (3.3), which, as we have seen in Section 3, is a particular NE.

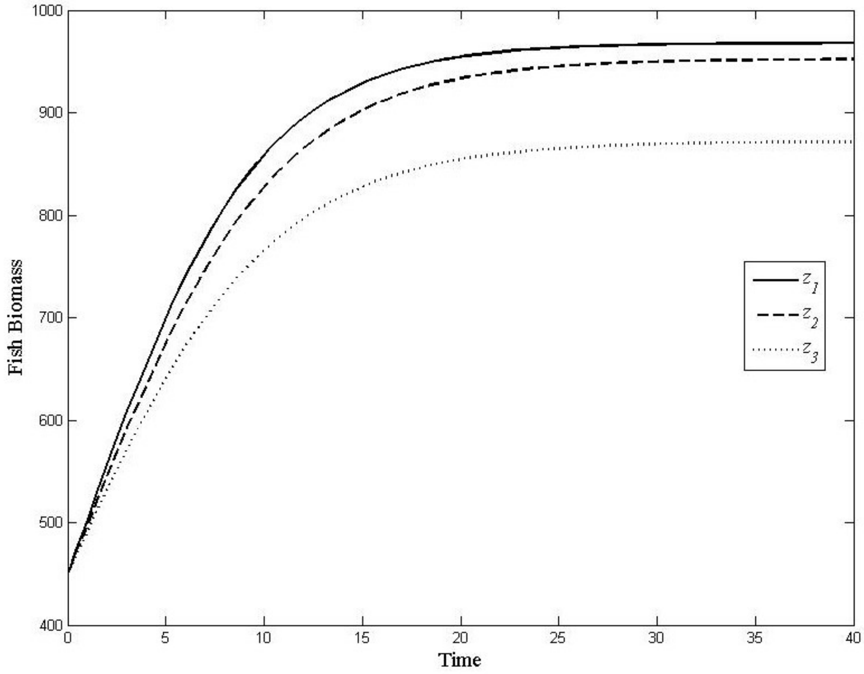

Figure 6. Solution of fish dynamics (4.6), for $\beta > 1$ and $z(0) < z^*$

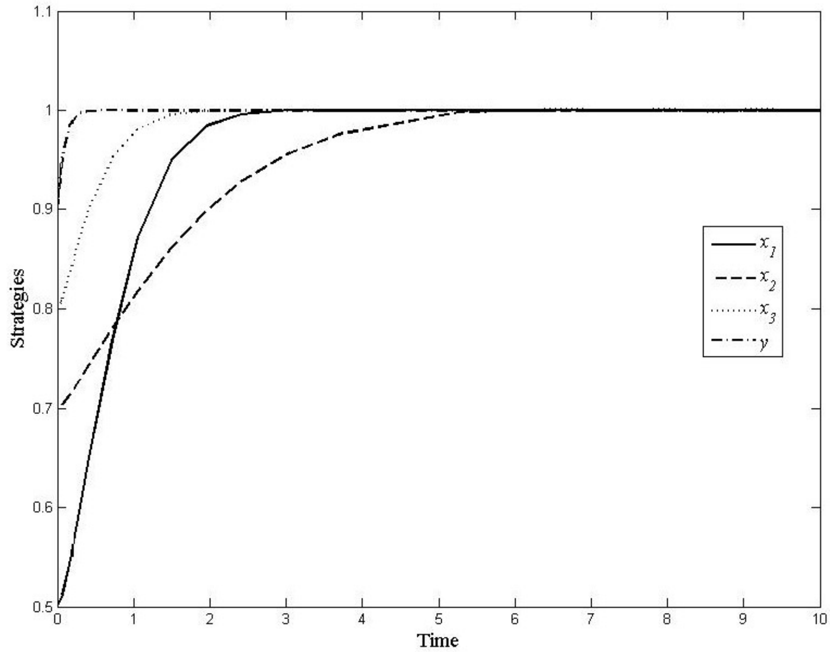

Figure 7. Strategy dynamics corresponding to fish dynamics of Figure 6, tending to the attractive solution $(\mathbf{1},1)$

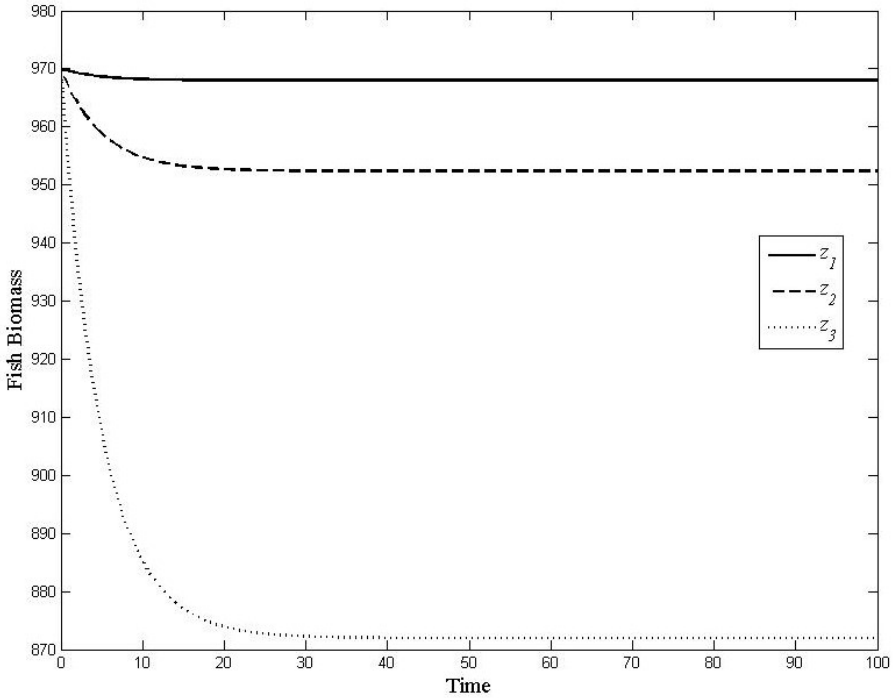

Figure 8. Solution of fish dynamics (4.6), for $\beta > 1$ and $z(0) > z^*$.

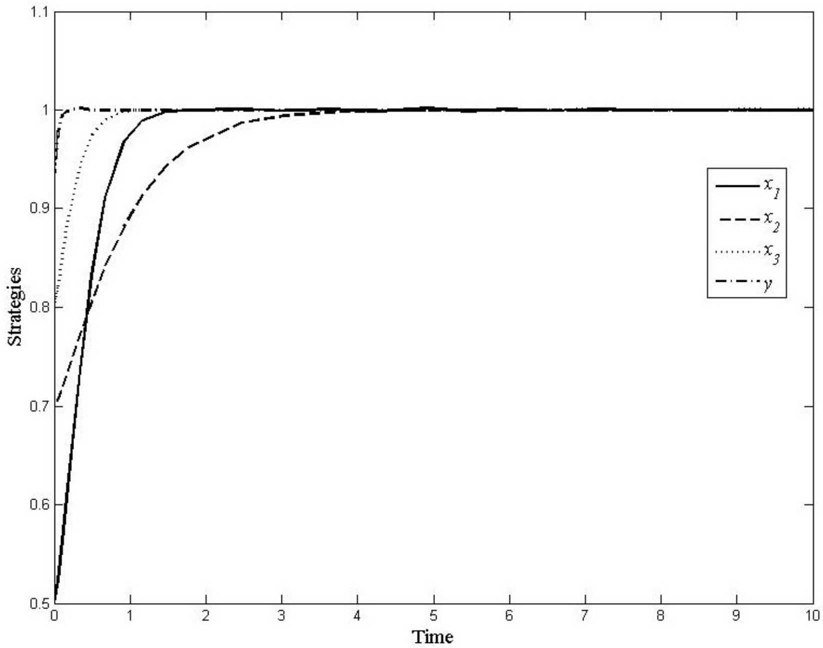

Figure 9. Strategy dynamics corresponding to fish dynamics of Figure 8, tending to the attractive solution $(\mathbf{1},1)$

5. Strategy dynamics with discrete-time delivery

In this section we suppose that instead of selling the captured fish immediately, the enterprises process and accumulate and sell them deep frozen. We want to see how much this change may also influence the cooperative and its members in their behaviour (strategy choice) according to strategy dynamics (4.3)-(4.4). To this end, counting with half month accumulation periods of length Δ, we consider M periods in the time interval [0,*T*], with $T = M\Delta$. Let us calculate the total catch of cooperative member *i*, during the time period $[m\Delta, (m+1)\Delta]$:

$$L_i^m = \int_{m\Delta}^{(m+1)\Delta} E_i \rho_i z_i(t)\, dt \quad (m{=}0,1,\ldots,M{-}1),$$

corresponding to the fishing effort model, see Appendix, (A.5) and (A.6).

By the substitution of L_i^m, from the strategy dynamics and the fishing effort models, we obtain an autonomous system for (x^m, y^m, z^m) of the form

$$\left.\begin{aligned} \dot{x}^m &= u(x^m, y^m, z^m) \\ \dot{y}^m &= v(x^m, y^m, z^m) \\ \dot{z}^m &= w(x^m, y^m, z^m) \end{aligned}\right\} \qquad (5.1)$$

At the end of each period, the endpoint of the solution of (5.1) is taken as initial value for the next period. The following example illustrates the behaviour of the above model.

Example 3. Starting from basic parameter system of Example 2, with $\beta = 1.15$. For the case of "real-time delivery" we obtain strategy dynamics as shown in Figure 10. For the illustration of the above construction, let us set $\Delta = 0.5$ (say half a month), *M*=10, hence *T*=5. Figure 11 suggests that in case of "discrete-time delivery" the strategy dynamics also tends to the original attractive solution (**1**,1) of the game, however the convergence is slower than in the case of "real-time delivery".

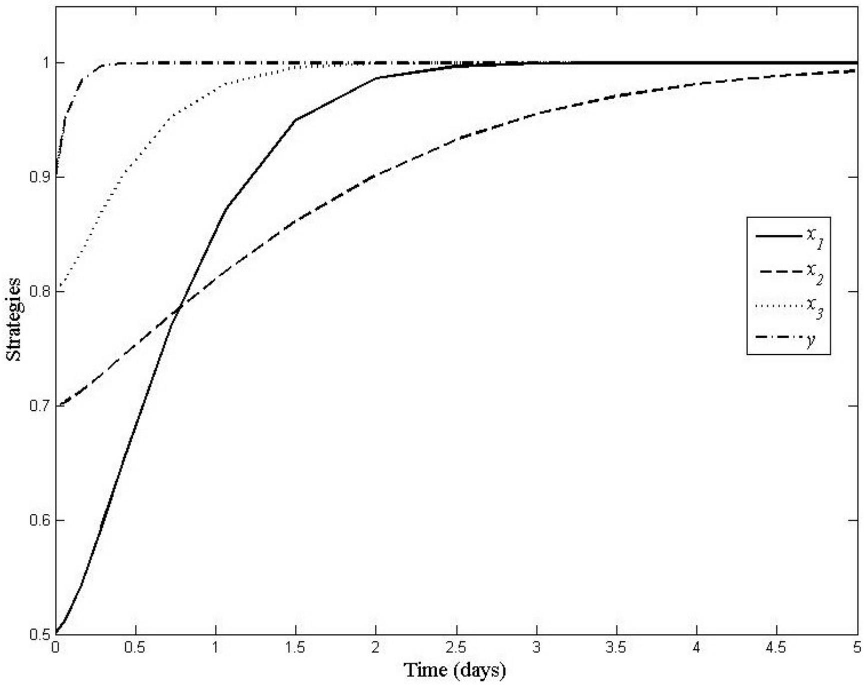

Figure 10. Strategy dynamics for the case of “real-time delivery”, with $z(0) < z^*$ in the fish dynamics

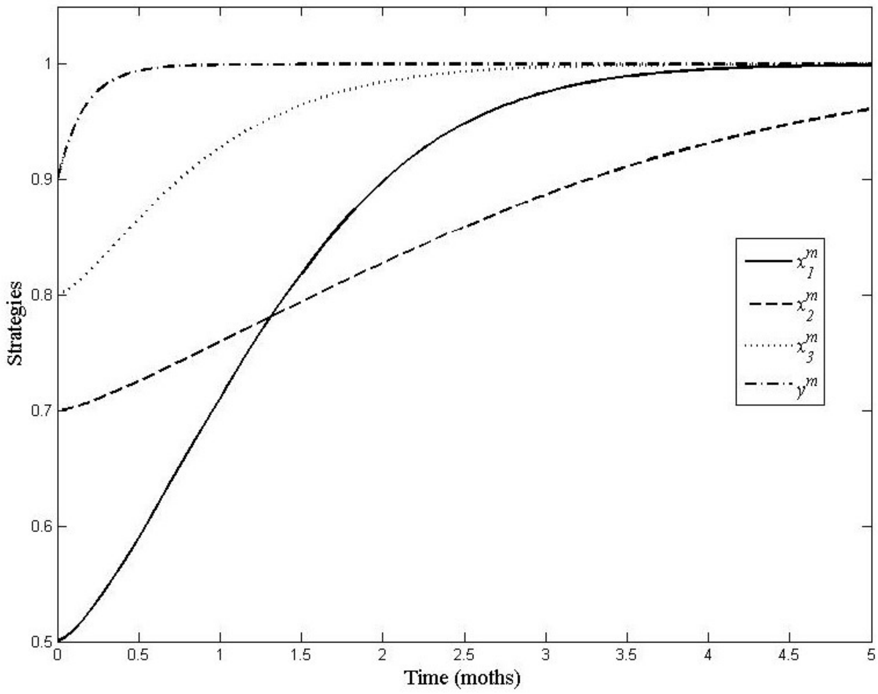

Figure 11. Strategy dynamics for the case of “discrete-time” delivery, with $z(0) < z^*$ in the fish dynamics

6. Discussion and outlook

It has been shown that an earlier approach of the authors concerning the conflict between a marketing cooperative and its unfaithful members, can be extended to the case when the offer in the oligopoly market where the unfaithful members sell a part of their production, is determined by an “exosystem” describing a time-varying production, in our case it is the dynamic model of fishing.

An important point is that, unlike the previous model, where the parameters of the resulting multi-person game were constant, in the present case, due to the time-varying oligopoly market, they change with time, according to a fishing effort model. Another specificity of the considered situation is, that the harvested fish preferably should be commercialized “in real time”, that is immediately. In the case of continuous time-dependent delivery, we have also proved that the corresponding time-varying partial adaptive dynamics is also appropriate for the description of the development of the strategy choice. In case of an effective punishment for unfaithfulness, the appropriate time-dependent strategy choice also tends to the solution of the game.

In addition to the “real-time delivery”, we have also adapted our model to the case when deep frozen fish is commercialized, operating with discrete-time sale, accumulating the product for given time periods. All this also means that our model may also be valid for certain production and commercialization of certain long season vegetables, sold either fresh, i.e. in “real time”, or deep frozen, i.e. in discrete-time moments.

As a further development of the presented model, the conflict between a marketing cooperative and its members could be combined with the conflict of several enterprises fishing in the same water.

Finally, we note that the methods of mathematical systems theory and optimal control models (see e.g. Guiro *et al.*, 2009; Gámez *et al.*, 2012) can be also applied in the fishery management context of the present paper.

Acknowledgement. This work was partially supported by Hungarian Scientific Research Fund GINOP, Grant No. 2.3.2-15-2016-00057 (to J.G.).

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Appendix

For the reader convenience below we summarize some basics of the classical fishing effort model by Schaefer (1954), see also Clark (1990), adding some results of analytical calculations used in our simulations.

Starting from the logistic model, suppose that the fishing is proportional to the present biomass (stock) of a given species. Then the fishing effort model considered in subsection 1.2 is

$$\dot{z} = rz\left(1 - \frac{z}{K}\right) - E\rho z\,. \tag{A.1}$$

We have two equilibrium points, the trivial equilibrium 0 and with the condition $E < \frac{r}{\rho}$, the non trivial equilibrium

$$z^* = K\left(1 - \frac{E\rho}{r}\right) > 0\text{, with } 0 < z^* < K\,. \tag{A.2}$$

To each effort E, there corresponds a sustainable catch:

$H(E) = E\rho z^* = E\rho K\left(1 - \frac{E\rho}{r}\right)$. $H(E) = 0$ is a quadratic equation, with roots 0 and $\frac{r}{\rho}$, and the function H attains a maximum at $E = \frac{r}{2\rho}$, that is, $H\left(\frac{r}{2\rho}\right) = \frac{rK}{4} = M$ is the maximum production of biomass (or maximum sustainable yield, MSY) of the population described with the logistic dynamics. Therefore, if we start fishing in the equilibrium, the effort $\frac{r}{2\rho}$ is an optimal and sustainable strategy. A routine calculation shows that, in correspondence with Figure 1 of subsection 1.2,

if $0 < z(0) < z^*$, the solution of (A.1) is

$$z(t)=\frac{\dfrac{z^*\cdot z(0)}{z^*-z(0)}}{\dfrac{z(0)}{z^*-z(0)}+e^{-(r-E\rho)t}}\quad (t\geq 0), \tag{A.2}$$

and if $z^* < z(0) < K$, the solution of (A.1) is

$$z(t)=\frac{\dfrac{z^*\cdot z(0)}{z(0)-z^*}}{\dfrac{z(0)}{z(0)-z^*}-e^{-(r-E\rho)t}}\quad (t\geq 0). \tag{A.3}$$

Obviously, in both cases we have

$$\lim_{t\to\infty} z(t) = z^*. \tag{A.4}$$

As Figure A1 shows, the convergence is monotonically increasing in the first case, and monotonically decreasing in the second one. Hence the solutions in both cases are also bounded.

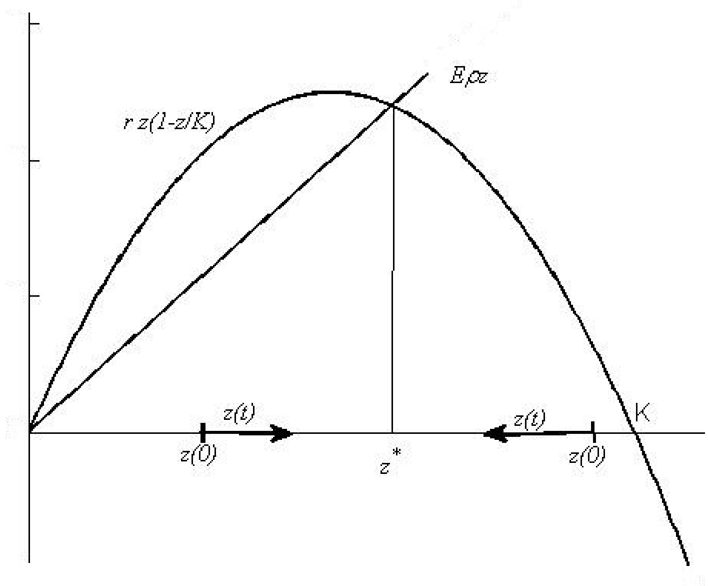

Figure A1. Convergence of biomass to equilibrium in the fishing effort model

For Section 5, concerning the case of discrete-time delivery, from (A.2) and (A.3), we easily calculate the total biomass caught in time interval $[m\Delta,(m+1)\Delta]$:

For $0 < z < z^*$,

$$\int_{m\Delta}^{(m+1)\Delta} z(t)dt = \frac{z^*}{r-E\rho} \cdot \ln \frac{\dfrac{z(0)}{z^*-z(0)} + e^{-(r-E\rho)(m+1)\Delta}}{e^{-(r-E\rho)} \cdot \left[\dfrac{z(0)}{z^*-z(0)} + e^{-(r-E\rho)m\Delta}\right]}, \tag{A.5}$$

and for $z^* < z < K$,

$$\int_{m\Delta}^{(m+1)\Delta} z(t)dt = \frac{z^*}{r-E\rho} \cdot \ln \frac{\dfrac{z(0)}{z(0)-z^*} - e^{-(r-E\rho)(m+1)\Delta}}{e^{-(r-E\rho)} \cdot \left[\dfrac{z(0)}{z(0)-z^*} - e^{-(r-E\rho)m\Delta}\right]}. \tag{A.6}$$