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Observer design for phenotypic observation of genetic processes

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Abstract

In the paper a single-locus sexual population is considered, where the phenotypic selection process is described by an evolutionary game. First, in order to recover the genetic process from different observations, observability is guaranteed by the linearization method developed in earlier papers of the authors for systems with invariant manifold. Then, based on a known result of nonlinear systems theory, an observer system is constructed that makes it possible to asymptotically recover the solution of the original system from the observation. In the numerical illustrations the selection is described by a "rock-scissors-paper" type game widely studied in evolutionary game theory. For the corresponding evolutionary dynamics a Hopf bifurcation result is also obtained.

Keywords: Sexual population; Evolutionary game; Hopf bifurcation; Observer system

1. Introduction

In the applications of mathematical systems theory, the reconstruction of the state process from available measurements is an important issue for several reasons. In biology, this problem naturally arises in relation with phenotypic observation of genetic processes. In static situation, in Garay and Garay [9] biological conditions were given for the allele frequency—phenotype frequency correspondence to be one-to-one. In a dynamical situation, less restrictive conditions can guarantee that from the observation of time-dependent phenotypic characteristics the allele process can be uniquely recovered. In engineering practice, under the condition of observability and stability, for the effective approximate calculation of the state process from the observation, an auxiliary system, the so-called observer is constructed, the solution of which asymptotically produces the state process of the original system, in most cases with an exponential rate of convergence.

In earlier papers [17,18] observers have been constructed for different density-dependent population system models. In these models, for an observability analysis, it was enough to apply the classical sufficient condition of Lee and Markus [15] for local observability. Since for frequency-dependent selection processes the dynamic model has an invariant manifold, in order to guarantee local observability, we need to apply the linearization method developed in Varga [20] for systems with invariant manifold. To effectively recover the genetic process from the phenotypic observation, an observer system will be designed, applying the results of Sundarapandian [19]. In the next section we shortly recall the basic concepts and theorems of the above observability and observer design methodology. In Section

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