Severals Preys and Severals Predators.

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Abstract

In this paper we consider a biological system consisting of several preys and several predators. We study permanence and the existence of a global attractor for a such system.

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1 Introduction

In this paper, we study the following periodic Kolmogorov system

$$\begin{aligned} x_i' &= x_i f_i(t, x_1, \ldots, x_n, y_1, \ldots, y_m) \quad 1 \leq i \leq n \\ y_j' &= y_j g_j(t, x_1, \ldots, x_n, y_1, \ldots, y_m) \quad 1 \leq j \leq m. \end{aligned} \tag{1.1}$$

where $f_i, g_j : \mathbb{R} \times \mathbb{R}_+^n \times \mathbb{R}_+^m \to \mathbb{R}$ are continuous functions, which are T-periodic in t and locally lipschitz continuous in (x, y).

We shall assume that:

P_1) $f_i(t, x, y)$ is decreasing in $(x, y) \in \mathbb{R}_+^{n+m}$ and $g_j(t, x, y)$ is increasing in $x \in \mathbb{R}_+^n$ and decreasing in $y \in \mathbb{R}_+^m$.

P_2) There exist $\tau_i;\ \theta_j \in \mathbb{R}$ such that $f_i(\tau_i, x, y);\ g_j(\theta_j, x, y)$ are strictly decreasing in x_i, y_j respectively.$(i = 1, \ldots, n;\ j = 1, \ldots, m)$.

Our assumptions say that system (1.1) is a model for a biological community consisting of n preys and m predators. In the case $n = m = 1$, our system is the usual predator-prey system and has been extensively studied by many authors. For instance, see [1] for optimal results.

Under some additional assumptions, we shall prove in section three, that system (1.1) is permanent. Our result in this connection improves the main results in section three of [2], in which it was assumed $n = m = 1$, and the main results in section two of [3], in which it was assumed that (1.1) is Lotka-Volterra.

Finally in section four, we use a result in [4] to show that, under some restrictions on the partial derivatives of $f_i(t, x, y), g_j(t, x, y)$, with respect to (x, y), system (1.1) possesses a global attractor. Our result improves theorem 2 in [3].

2 Preliminaries

In this section we introduce some notations and we state, for reference purpose, some interesting properties of the periodic logistic equation,

$$x' = xF(t, x) \quad , \ x \geq 0. \tag{2.1}$$

Given $T > 0$, we denote by $\mathcal{C}_T$ the set of all continuous function $F : \mathbb{R} \times [0, \infty) \to \mathbb{R}$ such that:

a) $F(t, x)$ is T-periodic in t and locally lipschitz continuous in x.

b) $F(t, x)$ is decreasing in x.

c) $F(\tau, x)$ is strictly decreasing in x for some $\tau = \tau(F) \in \mathbb{R}$.

e) There exists $R = R(F) > 0$ satisfying $\int_0^T F(t, R)dt < 0$.

In [6] it were proved the following results.

Theorem 2.1 *If $F \in \mathcal{C}_T$ then equation (2.1) has a T-periodic solution U^F, which is globally asymptotically stable. That is, if u is a solution of (2.1) and $u(0) > 0$, then u is defined on $[0, \infty)$ and*

$$u(t) - U^F(t) \to 0 \quad as \quad t \to +\infty.$$

Moreover, $U^F > 0$ if $\int_0^T F(t,0)dt > 0$, and $U^F \equiv 0$ if $\int_0^T F(T,0)dt \leq 0$.

We say that U^F is the *"global attractor"* of (2.1).

Corollary 2.2 *Let $F,\ G \in \mathcal{C}_T$ and suppose $F \leq G$. Then $U^F \leq U^G$.*

Theorem 2.3 *Let $\{F_n\}$ be a sequence in $\mathcal{C}_T$ converging to $F \in \mathcal{C}_T$ uniformly on compact sets. Then $U^{F_n}(t) \to U^F(t)$, uniformly on $\mathbb{R}$.*

Theorem 2.3 will be used under a suitable form for us. To be precise, let us fix the following notations. Let $h : \mathbb{R} \times \mathbb{R}_+^p \times \mathbb{R}_+^q \times \mathbb{R}_+ \to \mathbb{R};\ \ p, q \geq 0$; be a continuous function satisfying the following properties:

A1) $h(t, \xi, \eta, x)$ is T-periodic in t.

A2) $h(t, \xi, \eta, x)$ is locally lipschitz continuous in x.

A3) $h(t, \xi, \eta, x)$ is increasing in ξ and decreasing in (η, x).

A4) $h(\tau, \xi, \eta, x)$ is strictly decreasing in x for some $\tau \in \mathbb{R}$.

Theorem 2.4 *Let $(\varphi, \psi) : \mathbb{R}_+ \to \mathbb{R}_+^p \times \mathbb{R}_+^q$ be a continuous function and let $(\Phi, \Psi) : \mathbb{R} \to \mathbb{R}_+^p \times \mathbb{R}_+^q$ be a T-periodic continuous function such that,*

$$(\varphi(t), \psi(t)) - (\Phi(t), \Psi(t)) \to (0, 0) \quad as \ \ t \to +\infty. \tag{2.2}$$

Suppose also that there exists $R > 0$ such that,

$$\int_0^T h(t, \Phi(t), \Psi(t), R)dt < 0. \tag{2.3}$$

If u is solution of the equation $x' = xh(t, \varphi(t), \psi(t), x)$ and $u(t_0) > 0$ for some t_0, then is defined on $[t_0, +\infty)$ and

$$u(t) - U(t) \to 0 \qquad as \quad t \to +\infty \ ,$$

where U is the global attractor of the logistic equation $x' = xh(t, \Phi(t), \Psi(t), x)$.

Proof. Let us define, for each $\delta \geq 0$

$$\Phi_i^\delta(t) = \max\{\Phi_i(t) - \delta, 0\} \quad 1 \leq i \leq p$$

$$\Psi_j^\delta(t) = \max\{\Psi_j(t) - \delta, 0\} \quad 1 \leq j \leq q$$

and let $\Phi^\delta = (\Phi_1^\delta, \ldots, \Phi_p^\delta)$; $\Psi^\delta = (\Psi_1^\delta, \ldots, \Psi_q^\delta)$. By (2.2) there exists $t_0 \geq 0$ such that,

$$\Phi_i^\delta(t) \leq \varphi_i(t) \leq \Phi_i(t) + \delta \qquad t \geq t_0,$$

$$\Psi_j^\delta(t) \leq \psi_j(t) \leq \Psi_j(t) + \delta \qquad t \geq t_0.$$

From this and A3) we have for $t \geq t_0$,

$$h(t, \Phi^\delta(t), \Psi(t) + \delta^q, x) \leq h(t, \varphi(t), \psi(t), x) \leq h(t, \Phi(t) + \delta^p, \Psi^\delta(t), x), \tag{2.4}$$

where $\delta^p = (\delta, \ldots, \delta)$; $\delta^q = (\delta, \ldots .\delta)$ with p-times and q-times resp.

On the other hand, $\{\Phi^\delta(t)\}$, $\{\Psi^\delta(t)\}$ converge uniformly to $\Phi(t)$ and $\Psi(t)$ as $\delta \to 0$ respectively, from this and (2.3) there exists a $\delta_1 > 0$ such that,

$$\int_0^T h(t, \Phi^\delta(t), \Psi(t) + \delta^q, R)dt < 0 \qquad \text{for} \quad 0 < \delta < \delta_1. \tag{2.5}$$

Let $(u_*)^\delta$, $(u^*)^\delta$ respectively the solutions of the following initial value problems,

$$x' = xh(t, \Phi^\delta(t), \Psi(t) + \delta^q, x) \quad x(t_0) = u(t_0) \tag{2.6}$$

$$x' = xh(t, \Phi(t) + \delta^p, \Psi^\delta(t), x) \quad x(t_0) = u(t_0). \tag{2.7}$$

By (2.5) and Theorem 2.1, $(u_*)^\delta$, $(u^*)^\delta$ are defined on $[t_0, +\infty)$ and by (2.4),

$$(u_*)^\delta(t) \le u(t) \le (u^*)^\delta(t) \quad \forall t \ge t_0. \tag{2.8}$$

Analogously,

$$h(t, \Phi^\delta(t), \Psi(t) + \delta^q, x) \le h(t, \Phi(t), \Psi(t), x) \le h(t, \Phi(t) + \delta^p, \Psi^\delta(t), x),$$

and by Corollary 2.2 , $(U_*)^\delta \le U \le (U^*)^\delta$, where $(U_*)^\delta$ and $(U^*)^\delta$ are the global attractors of (2.6) and (2.7) respectively. Hence, by (2.8) we have,

$$u(t)-U(t) \le (u^*)^\delta(t)-U(t) = (u^*)^\delta(t)-(U^*)^\delta(t)+(U^*)^\delta(t)-U(t), \quad \forall t \ge t_0.$$

Let us fix $\epsilon > 0$. By Theorem 2.3 there exists $0 < \delta_2 \le \delta_1$ such that,

$$(U^*)^{\delta_2}(t) - U(t) \le \frac{\epsilon}{2} \quad \forall t \ge 0.$$

On the other hand, by Theorem 2.1 there exist $t_1 \ge t_0$ verify,

$$(u^*)^{\delta_2}(t) - (U^*)^{\delta_2}(t) \le \frac{\epsilon}{2} \quad \forall t \ge t_1$$

hence, $u(t) - U(t) \le \epsilon \;\; \forall t \ge t_1$. Similarly, there exists $t_2 \in \mathbb{R}$ such that $u(t) - U(t) \ge -\epsilon \;\; \forall t \ge t_2$, and the proof is complete. ∎

3 Permanence.

In this section, we use iterative schemes as in [2], [5], [6], to show that system (1.1) is permanent. In addition to P_1)-P_2) we also assume that:

P_3) There exists $R > 0$ satisfying,

$$\int_0^T f_i(t, Re_i, 0)dt < 0 \qquad 1 \le i \le n$$

$$\int_0^T g_j(t, U^1(t), R\nu_j)dt < 0 \quad 1 \leq j \leq m.$$

Here $\{e_1, \dots, e_n\}$, $\{\nu_1, \dots, \nu_m\}$ denote the canonical vector basis of $\mathbb{R}^n$ and $\mathbb{R}^m$ respectively and $U^1 = (U_1^1, \dots, U_n^1) : \mathbb{R} \to \mathbb{R}_+^n$, where U_i^1; $1 \leq i \leq n$; is the global attractor of the equation,

$$z' = zf_i(t, ze_i, 0) \quad 1 \leq i \leq n, \tag{3.1}$$

See Theorem 2.1.

Associated to system (1.1), we have two sequences of nonnegative T-periodic functions $\{U^N = (U_1^N, \dots, U_n^N)\}$ and $\{V^N = (V_1^N, \dots, V_m^N)\}$, $N \in \mathbb{N}$, defined inductively as follows: $U^0 = V^0 \equiv 0$, and U_i^{N+1}; $1 \leq i \leq n$; is the global attractor of the logistic equation,

$$z' = zf_i(t, U_1^N(t), \dots, U_{i-1}^N(t), z, U_{i+1}^N(t), \dots, U_n^N(t), V^N(t)), \tag{3.2}$$

and V_j^N $1 \leq j \leq m$ the global attractor of the equation

$$z' = zg_j(t, U^{N+1}(t), V_1^N(t), \dots, V_{j-1}^N(t), z, V_{j+1}^N(t), \dots, V_m^N(t)). \tag{3.3}$$

Remark. The above scheme is obtained, using some ideas in Lopez-Gomez, Ortega and Tineo in ([2], section 3). In fact, the scheme in that paper is obtained from (3.2)-(3.3) when $m = n = 1$

We shall show that these sequences are well defined and satisfy:

$$\begin{aligned} 0 &\leq U^2 \leq U^4 \leq \dots \leq U^{2N} \leq U^{2N-1} \leq \dots \leq U^3 \leq U^1 \\ 0 &\leq V^2 \leq V^4 \leq \dots \leq V^{2N} \leq V^{2N-1} \leq \dots \leq V^3 \leq V^1. \end{aligned} \tag{3.4}$$

We recall that U^1 is the global attractor of (3.1). Hence, using P_3) and Theorem 2.1, we know that equation

$$w' = wg_j(t, U^1(t), w\nu_j) \quad 1 \leq j \leq m,$$

has a global attractor V_j^1.

Let us fix $1 \leq i \leq n$. By P_1) we have,

$$f_i(t, U_1^1(t), \ldots, U_{i-1}^1(t), z, U_{i+1}^1(t), \ldots, U_n^1(t), V^1(t)) \leq f_i(t, ze_i, 0) \tag{3.5}$$

and using $P_2), P_3)$, we conclude that the logistic equation

$$z' = zf_i(t, U_1^1(t), \ldots, U_{i-1}^1(t), z, U_{i+1}^1(t), \ldots, U_n^1(t), V^1(t)) \tag{3.6}$$

satisfies the assumptions in Theorem 2.1. We define U_i^2 as the global attractor of (3.6), and $U^2 = (U_1^2, \ldots, U_n^N)$. Note that by (3.5) and Corollary 2.2, we have

$$U^2 \leq U^1.$$

From this and P_1) we have,

$$g_j(t, U^2(t), V_1^1(t), \ldots, V_{j-1}^1(t), w, V_{j+1}^1(t), \ldots, V_m^N(t)) \leq g_j(t, U^1(t), w\nu_j) \tag{3.7}$$

and by the above arguments, the equation,

$$w' = wg_j(t, U^2(t), V_1^1(t), \ldots, V_{j-1}^1(t), w, V_{j+1}^1(t), \ldots, V_m^N(t)),$$

has a global attractor V_j^2. We define $V^2 = (V_1^2, \ldots, V_m^2)$. As above we have,

$$V^2 \leq V^1.$$

The proof of (3.4) follows now by induction (see [2]).

By (3.4), $\{U^{2N-1}\}$, $\{U^{2N}\}$, $\{V^{2N-1}\}$, $\{V^{2N}\}$; $\quad N \in \mathbb{N}$ are monotone and uniformly bounded sequences. So, we have well defined functions:

$$\begin{array}{lcl} \bar{U}(t) = \lim_{N\to\infty} U^{2N-1}(t) & ; & \underline{U}(t) = \lim_{N\to\infty} U^{2N}(t); \\ \bar{V}(t) = \lim_{N\to\infty} V^{2N-1}(t) & ; & \underline{V}(t) = \lim_{N\to\infty} V^{2N}(t). \end{array} \tag{3.8}$$

On the other hand, for $1 \leq i \leq n$; $1 \leq j \leq m$; we have

$$
\begin{aligned}
(U_i^{N+1})' &= U_i^{N+1} f_i(t, U_1^N(t), \dots, U_{i-1}^N(t), U_i^{N+1}, U_{i+1}^N(t), \dots, U_n^N(t), V^N(t)) \\
(V_j^{N+1})' &= V_j^{N+1} g_j(t, U^{N+1}(t), V_1^N(t), \dots, V_{j-1}^N(t), V_j^{N+1}, V_{j+1}^N, \dots, V_m^N(t))
\end{aligned}
$$

and hence, $\{(U^N)'\}$ $\{(V^N)'\}$ are uniformly bounded.

By Ascoli's Theorem, there exist subsequence of $\{U^{2N-1}\}$; $\{U^{2N}\}$; $\{V^{2N-1}\}$ and $\{V^{2N}\}$ respectively, which converge uniformly on $\mathbb{R}$. From this, the limits in (3.8) are uniform on $t \in \mathbb{R}$.

Finally, by an elementary result about limits and derivatives, we conclude that

$$
\begin{aligned}
\bar{U}_i' &= \bar{U}_i f_i(t, \underline{U}_1(t), \dots, \underline{U}_{i-1}(t), \bar{U}_i, \underline{U}_{i+1}(t), \dots, \underline{U}_n(t), \underline{V}(t)) \\
\underline{U}_i' &= \underline{U}_i f_i(t, \bar{U}_1(t), \dots, \bar{U}_{i-1}(t), \underline{U}_i, \bar{U}_{i+1}(t), \dots, \bar{U}_n(t), \bar{V}(t)) \\
\bar{V}_j' &= \bar{V}_j g_j(t, \bar{U}(t), \underline{V}_1(t), \dots, \underline{V}_{j-1}(t), \bar{V}_j, \underline{V}_{j+1}(t), \dots, \underline{V}_m(t)) \\
\underline{V}_j' &= \underline{V}_j g_j(t, \underline{U}(t), \bar{V}_1(t), \dots, \bar{V}_{j-1}(t), \underline{V}_j, \bar{V}_{j+1}(t), \dots, \bar{V}_m(t)).
\end{aligned}
\tag{3.9}
$$

Let $(u(t), v(t))$ be a solution of (1.1) such that $u(0) > 0$; $v(0) > 0$ and let u_i^1; $1 \leq i \leq n$; be the solution of (3.1) determined by the initial condition $u_i^1(0) = u_i(0)$. By Theorem 2.1, u_i^1 is defined on $[0, \infty)$ and by P_1),

$$
u_i' \leq u_i f_i(t, u_i e_i, 0) \qquad 1 \leq i \leq n.
$$

That is, u_i is a subsolution of (3.1) and hence,

$$
u(t) \leq u^1(t) \quad \forall t \geq 0, \quad t \in domain(u). \tag{3.10}
$$

In particular, u is defined on $[0, \infty)$.

Let us define v_j^1; $1 \leq j \leq m$; as the solution of the IVP

$$
w' = w g_j(t, u^1(t), w\nu_j) \qquad w(0) = v_j(0). \tag{3.11}
$$

Since, u^1 is bounded on $[0, \infty)$, there exists $M > 0$ such that $u_i^1 \le M$ for all $i = 1, \ldots, n$; $\forall t \le 0$ and hence,

$$g_j(t, u^1(t), w\nu_j) \le g_j(t, \vec{M}, w\nu_j) \le g_j(t, \vec{M}, 0) \le K,$$

for some constant K, where $\vec{M} := \sum_{i=1}^n M e_i$.

From this, v_j^1 is defined on $[0, \infty)$. Moreover, by P_1) and (3.10), $v_j' \le v_j g_j(t, u^1(t), v_j \nu_j)$; $\quad 1 \le j \le m$. That is, v_j is a subsolution of (3.11) and hence, $v(t) \le v^1(t)$; $\quad \forall t \ge 0$ and $t \in$ domain(v). In particular v is defined on $[0, \infty)$.

Note that by Theorem 2.1 $v_j^1(t) - V_j^1(t) \to 0$ as $t \to +\infty$ $\;(1 \le j \le m)$ and thus, v is bounded on $[0, \infty)$.

Let u_i^2; $\;1 \le i \le n$; be the solution of the IVP

$$z' = z f_i(t, u_1^1(t), \ldots, u_{i-1}^1(t), z, u_{i+1}^1(t), \ldots, u_n^1(t), v^1(t)), \qquad z(0) = u_i(0).$$

Using the above arguments we can prove that, u_i^2 is defined on $[0, \infty)$ and $u_i(t) \ge u_i^2(t)$ $\;\forall t \ge 0$ and $t \in$ domain(u). Analogously, it is easy to verify that $v_j(t) \ge v_j^2(t)$ $\;\forall t \ge 0$ and $t \in$ domain(v), where v_j^2 is the solution of the IVP

$$w' = w g_j(t, u^2(t), v_1^1(t), \ldots, v_{j-1}^1(t), w, v_{j+1}^1(t), \ldots, v_m^1(t)) \quad w(0) = v_j(0).$$

Inductively we can construct two sequences $\{u^N = (u_1^N, \ldots, u_n^N)\}$ and $\{v^N = (v_1^N, \ldots, v_m^N)\}$, defined on $[0, \infty)$, as follows: $u^0 = v^0 \equiv 0$,

$$\begin{aligned}(u_i^N)' &= u_i^N f_i(t, u_1^{N-1}(t), \ldots, u_{i-1}^{N-1}(t), u_i^N, u_{i+1}^{N-1}(t), \ldots, u_n^{N-1}(t), v^{N-1}(t)) \\ (v_j^N)' &= v_j^N g_j(t, u^N(t), v_1^{N-1}(t), \ldots, v_{j-1}^{N-1}(t), v_i^N, v_{j+1}^{N-1}(t), \ldots, v_m^{N-1}(t)) \\ u_i^N(0) &= u_i(0); \quad v_j^N(0) = v_j(0); (i = 1, \ldots, n; j = 1, \ldots, m; \quad N \in \mathbb{N})\end{aligned} \tag{3.12}$$

It is not hard to show that,

$$
\begin{aligned}
0 &\leq u^2 \leq u^4 \leq \ldots \leq u^{2N} \leq u \leq u^{2N-1} \leq \ldots \leq u^3 \leq u^1 \\
0 &\leq v^2 \leq v^4 \leq \ldots \leq v^{2N} \leq v \leq v^{2N-1} \leq \ldots \leq v^3 \leq v^1.
\end{aligned} \tag{3.13}
$$

Using induction and Theorem 2.4 it is easy to show the following result.

Corollary 3.1 *For all $N \in \mathbb{N}$, we have*

$$u^N(t) - U^N(t) \to 0 \quad as \quad t \to +\infty,$$

$$v^N(t) - V^N(t) \to 0 \quad as \quad t \to +\infty,$$

where $u^N; v^N; U^N; V^N$ are defined in (3.12),(3.2) and (3.3).

Theorem 3.2 *Let $(u(t), v(t))$ be a positive solution of (1.1).Then, (u, v) is defined on a terminal interval of $\mathbb{R}$ and,*

$$\limsup_{t\to\infty}[u_i(t) - \bar{U}_i(t)] \leq 0 \leq \liminf_{t\to\infty}[u_i(t) - \underline{U}_i(t)], \quad 1 \leq i \leq n; \quad t \geq t_0,$$

$$\limsup_{t\to\infty}[v_j(t) - \bar{V}_j(t)] \leq 0 \leq \liminf_{t\to\infty}[v_j(t) - \underline{V}_j(t)], \quad 1 \leq j \leq m; \quad t \geq t_0.$$

Proof. We know that, (u, v) is defined on $[0, \infty)$.

Let us fix $\epsilon > 0$. By (3.13),

$$u_i(t) - \bar{U}_i(t) \leq u_i^{2N-1}(t) - \bar{U}_i(t); \quad 1 \leq i \leq n,$$

which we can write in the following form:

$$u_i(t) - \bar{U}_i(t) \leq u_i^{2N-1}(t) - U_i^{2N-1}(t) + U_i^{2N-1}(t) - \bar{U}_i(t).$$

Since $\{U^{2N-1}\}$ converges uniformly to $\bar{U}$ (see (3.8)), there exists $N_1 \geq 0$ such that,

$$U_i^{2N-1}(t) - \bar{U}_i(t) \leq \frac{\epsilon}{2} \quad \forall t \in \mathbb{R}. \tag{3.14}$$

On the other hand, by Corollary 3.1, there exists $t_1 \in \mathbb{R}$ such that,

$$u^{2N_1-1}(t) - U^{2N_1-1}(t) \leq \frac{\epsilon}{2} \quad \forall t \geq t_1.$$

From this and (3.14), we obtain

$$u_i(t) - \bar{U}_i(t) \leq \epsilon \quad \forall t \geq t_1,$$

and so $\limsup_{t\to+\infty}[u_i(t) - \bar{U}_i(t)] \leq 0$. The rest of the proof is similar. ■

Corollary 3.3 *Let (u, v) be a positive T-periodic solution of (1.1). Then,*

$$\begin{aligned} \underline{U}_i \leq u_i \leq \bar{U}_i; &\quad i = 1, \ldots, n \\ \underline{V}_j \leq v_j \leq \bar{V}_j &\quad j = 1, \ldots, m. \end{aligned}$$

Proof. Let us fix $\epsilon > 0$. By Theorem 3.2, there exists $t_\epsilon \geq 0$ such that

$$\begin{aligned} \underline{U}_i(t) - \epsilon \leq u_i(t) \leq \bar{U}_i(t) + \epsilon \quad \forall t \geq t_\epsilon \\ \underline{V}_j(t) - \epsilon \leq v_j(t) \leq \bar{V}_j(t) + \epsilon \quad \forall t \geq t_\epsilon \end{aligned} \tag{3.15}$$

since u, v, $\underline{U}$, $\bar{U}$, $\underline{V}$, $\bar{V}$ are T-periodic, (3.15) hold for all $t \in \mathbb{R}$, and the proof follows easily. ■

Remark. Let us define,

$$\begin{aligned} &\hat{q}_j := \textstyle\int_0^T q_j(t, U^1(t), 0)dt \\ &\hat{\alpha}_i := \textstyle\int_0^T f_i(t, U_1^1(t), \ldots, U_{i-1}^1(t), 0, U_{i+1}^1(t), \ldots, U_n^1(t), V^1(t))dt \\ &\hat{\beta}_j := \textstyle\int_0^T g_j(t, U^2(t), V_1^1(t), \ldots, V_{j-1}^1(t), 0, V_{j+1}^1(t), \ldots, V_m^1(t))dt \\ &(i = 1, \ldots, n\ ;\ j = 1, \ldots, m) \end{aligned}$$

If $\hat{q}_j > 0$; $\hat{\alpha}_i > 0$; $\hat{\beta}_j > 0$ then, by Theorem 2.1, U^2, $V^2 > 0$.

Corollary 3.4 *If $U^2 > 0$ and $V^2 > 0$, then (1.1) has at least one strictly positive T-periodic solution.*

Proof. If U^2 and $V^2 > 0$, the system (1.1) is permanent and the proof follows from lemma 1 in [7]. ■

This result generalizes theorem 1 in [3]. To show this, let us consider the following predator-prey Lotka-Volterra system,

$$\begin{aligned} x_i' &= x_i \left[b_i(t) - \sum_{k=1}^{n} a_{ik}(t) x_k(t) - \sum_{k=1}^{m} c_{ik}(t) y_k(t) \right], \quad 1 \leq i \leq n, \\ y_j' &= y_j \left[-r_j(t) + \sum_{k=1}^{n} d_{jk}(t) x_k(t) - \sum_{k=1}^{m} e_{jk}(t) y_k(t) \right], \quad 1 \leq j \leq m, \end{aligned}$$

where $b_i, r_i, a_{ij}, c_{ij}, d_{ij}, e_{ij} : \mathbb{R} \to \mathbb{R}_+$ are T-periodic continuous function such that $a_{ii} > 0, e_{jj} > 0 \quad \forall i, j$. By theorem 4.1 d) in [5] we have, $U_i^2 \geq \alpha_i$ and $V_j^2 \geq \beta_j$ for $1 \leq i \leq n$ and $1 \leq j \leq m$, where $\alpha_i,\ \beta_j$ are the constants defined in [3]. Thus, the proof of our claim is complete.

4 Global Stability.

In this section we obtain sufficient conditions about the global attractivity of a periodic solution of (1.1). In addition to $P_1) - P_3)$, we also assume that f_i, g_j have partial derivatives with respect to (x, y) which are defined and continuous in $\mathbb{R} \times \mathbb{R}_+^{n+m}$.

Theorem 4.1 *Assume that, there are positive constants $p, c_1, \ldots, c_n, d_1, \ldots, d_m$ such that, for $1 \leq i \leq n$ and $1 \leq j \leq m$,*

$$c_i \frac{\partial f_i}{\partial x_i}(t, x, y) + \sum_{k=1 k \neq i}^{n} c_k \left| \frac{\partial f_k}{\partial x_i}(t, x, y) \right| + \sum_{j=1}^{m} d_j \left| \frac{\partial g_j}{\partial x_i}(t, x, y) \right| \leq -p,$$

$$d_j \frac{\partial g_j}{\partial y_j}(t, x, y) + \sum_{k=1 k \neq j}^{m} d_k \left| \frac{\partial g_k}{\partial y_j}(t, x, y) \right| + \sum_{i=1}^{n} c_i \left| \frac{\partial f_i}{\partial y_j}(t, x, y) \right| \leq -p,$$

Then, there exists a T-periodic solution (U, V) of (1.1) such that

$$(u(t), v(t)) - (U(t), V(t)) \to (0, 0) \quad as \quad t \to +\infty,$$

for any positive solution (u, v) *of (1.1).*

Proof. By Theorem 3.2 we know that (u, v) is defined and bounded on a terminal interval of $\mathbb{R}$. On the other hand by assumption, we know that, system (1.1) satisfy condition (0.2) in [5]. The proof follows now from theorem 1.5 in [5]. ■

The above result generalizes theorem 2 in [3], in which it was assumed that (1.1) is Lotka-Volterra and that $c_i = d_j = 1$ for all i, j.

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References

[1] Burton T.A. and Huston V., *Permanence for Non-Autonomous Predator-Prey Systems.*Diff.and Int. Equations, Vol.4, N 6 (1991), 1269-1280.

[2] Lopez-Gomez J.; Ortega R. and Tineo A., *The Periodic Predator-Prey Lotka Model.*Avances in Differential Equations.Vol.1,N 3 (1996), 403-423.

[3] Pinghua Y.and Rui X., *Global Attractivity of the Periodic Lotka-Volterra System.*Jour. Math. Anal. and Appl., 233 (1999), 221-232.

[4] Tineo A., *Global On the Asymptotic Behavior of Some Population Models.* Jour. Math. Anal. and Appl.,Vol 167, N 2 (1992), 516-529.

[5] Tineo A., *Iterative Schemes for the N-Competing Species Problem.* Jour. of Differ. Equations. Vol 116, N 1 (1995), 1-15.

[6] Tineo A., *Iterative Schemes for Some Population Models*. Nonli. World. 3 (1996), 695-708.

[7] Zanolin F., *Pernanence and Positive Periodic Solution for Kolmogorov Competing Species Systems*. Results in Mathematics, 21 (1992), 224-250.