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Existence of a polycycle in non-Lipschitz Gause-type predator–prey models

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We study Gause-type predator–prey models when the interaction between predator and prey is not locally-Lipschitz continuous in the absence of one of them. We shall show that in this case there appears a polycycle, which affects the existence of limit cycles for the system. We apply the results to study the existence of limit cycles for a classical Gause system.

1. Introduction

A Gause-type predator–prey system is a system of the form

$$x' = F(x) - y\phi(x), \qquad y' = y\big(\psi(x) - \mu\big), \tag{1.1}$$

where $F(x)$ models the growth of the prey population in the absence of predators, $\phi(x)$ the kill rate in terms of the density of the prey, $\psi(x)$ the rate of reproduction of the predator in terms of the predation, and μ the death rate of predators. Detailed descriptions and biological explanations of the system may be found in [1,2,5,7,14,16].

For the growth of the prey population, it is usual to assume $F(x) = rx(1 - kx)$ for certain parameters $r, k > 0$, with the prey thus following the logistic equation. But Rosenzweig [17], for instance, also considers $F(x) = rx(\ln k - \ln x)$. In the present work, we shall only require there to be exactly two non-negative roots, 0 and $k > 0$, such that $F'(0) > 0$ and $F'(k) < 0$.

The kill rate of the prey and the rate of reproduction of the predator in terms of the prey, $\phi(x)$, $\psi(x)$, are usually continuous functions in $x \geqslant 0$ and continuously differentiable functions with strictly positive derivative in $x > 0$. For instance, $\phi(x) = x$ in the Lotka–Volterra model [13,18], $\phi(x) = x^\alpha$ in the Gause model [6,17], and $\phi(x) = \frac{px}{1+pqx}$, with $p, q > 0$, in models with a Holling Type II response [8,9]. Recent studies [2,10] have pointed out that under certain conditions it is not possible to assume that ϕ and ψ are proportional, so that neither shall we make this assumption here.

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