A Note for Cyclic 3-Dimensional Competitive Systems.

Manuel Gámez

Departamento de Estadística y Matemática Aplicada. Universidad de Almería. Spain.

Bertha Granados

Departamento de Matemáticas, Facultad de Ciencias, Universidad de Los Andes. Merida, Venezuela

Abstract

In this paper we consider the class $\mathcal{C}$ of all T-periodic competitive and dissipative 3-dimensional systems which have a cyclic in the boundary and such that the origin is a source. We use the ideas in [1] in order to give a correct proof of theorem 1.2 of [1] where we study the coexistence state for these system.

Key Words: Competitive systems, coexistence states, dissipative systems.

1 Introduction

Let $\mathcal{C}$ be the class of all T-periodic competitive and dissipative systems which have a cyclic in the boundary and such that the origin is a source.

In a recent paper by Tineo [1], it was proved that the property of having a coexistence state is a generic property in $\mathcal{C}$ with respect to a suitable topology.

Unfortunately the proof in [1] contains a mistake, but fortunately the ideas in that paper are correct and we use these in order to give a correct proof of theorem 1.2 of [1]. Our proof is based in on improvement of theorem 1.1 in that paper.

To be more precise, let us consider the system

$$x_i' = x_i F_i(t,x), \quad x = (x_1, x_2, x_3), \qquad 1 \le i \le 3, \tag{1.1}$$

where $F_1, F_2, F_3 : \mathbb{R} \times \mathbb{R}^3_+ \to \mathbb{R}$ are continuous functions which are T-periodic in t and locally Lipschitz continuous in x. We shall assume that the following hypotheses hold:

H_1) System (1.1) is competitive. That is, $F_i(t,x)$ is decreasing with respect to x_j for all $i \neq j$.

H_2) System (1.1) is dissipative.

H_3) $\int_0^T F_i(t,0)dt > 0$ for all i. This condition implies that the trivial solution is a source.

We say that (1.1) is $\tau-$cyclic (resp. $\sigma-$cyclic) if the species x_i (resp. x_{i+1}) is carried to extinction by x_{i+1} (resp. x_i) in the subsystem obtained from (1.1) by letting $x_{i-1} = 0, \quad i \in \mathbb{Z}$. (Here and henceforth we shall use the mod 3 notation).

Remark. If (H_1) holds then (H_2) is equivalent to saying the system

$$z' = zF_i(t, ze_i) \tag{1.2}$$

is dissipative for $i \le i \le 3$. Here and henceforth, (e_1, e_2, e_3) denotes the canonical vector basis of $\mathbb{R}^3$. Thus, if (H_1)-(H_3) holds then (1.2) has a minimal positive T-periodic solution that shall denote by v_i.

In [1] is was proved that if (1.1) is $\tau-$cyclic then,

$$I_i := \int_0^T F_{i+1}(t, v_i(t)e_i)dt \geq 0 \geq J_i := \int_0^T F_i(t, v_{i+1}(t)e_i)dt$$

Also in theorem 1.1 it was proved that if (H_1)-(H_3) hold and (1.1) is $\tau-$cyclic, then this system has a coexistence state if $I_i > 0 \quad \forall i$. We shall complete this result as follows.

Theorem 1.1 *If $J_i < 0 \quad \forall i$, then (1.1) has a coexistence state.*

Using this result we prove our main result.

Theorem 1.2 *Let F satisfying (H_1)-(H_3). If F is $\tau-$cyclic then the system*

$$x_i = F_i^\epsilon(t, x), \tag{1.3}$$

is $\tau-$cyclic and has a coexistence state for any $\epsilon > 0$, where

$$F_i^\epsilon(t, x) := F_i(t, x) + \epsilon[F_i(t, x_{i+1}e_{i+1}) - F_i(t, 0)] \quad \epsilon \in (0, 1). \tag{1.4}$$

In [1], the author define,

$$F_i^\epsilon(t, x) := F_i(t, x) + \epsilon[F_i(t, 0) - F_i(t, x_{i-1}e_{i-1})] \quad \epsilon \in (0, 1).$$

but, in this case, system (1.3) is not, in general, competitive.

2 The Proofs

Proof of Theorem 1.1 Let Δ be 2-cell given by theorem 2.1 in [1] and let $\pi : \Delta \to \Delta$ the Poincare map of (1.1). We know that π is an orientation-preserving homeomorphism onto Δ. The proof of our result follows as in [1], applying the main result in [2] to π^{-1}. ■

Proof of Theorem 1.2 Obviously, (1.3) is competitive for all $\epsilon > 0$. On the other hand, if $z_{i+1} = 0$, we have

$$\begin{aligned} F^{\epsilon}_{i+1}(t,x) &= F_{i+1}(t,x) + \epsilon[F_{i+1}(t, x_{i+2}e_{i+2}) - F_{i+1}(t,0)] \leq F_{i+1}(t,x), \\ F^{\epsilon}_{i+2}(t,x) &= F_{i+2}(t,x) + \epsilon[F_{i+2}(t, x_{i+3}e_{i+3}) - F_{i+2}(t,0)] = \\ &= F_{i+2}(t,x) + \epsilon[F_{i+2}(t, x_i e_i) - F_{i+2}(t,0)] = F_{i+2}(t,x) \geq F_{i+2}(t,x), \end{aligned}$$

and by proposition 4.1 of [1], (1.3) is $\tau-$cyclic.

Note now that, the minimal positive T-periodic solution of the logistic equation $z' = zF^{\epsilon}_i(t, ze_i)$ is also v_i, since $F^{\epsilon}_i(t, ze_i) \equiv F_i(t, ze_i)$. Finally

$$\int_0^T F^{\epsilon}_i(t, v_{i+1}(t)e_{i+1})dt = (1+\epsilon)J_i - \epsilon \int_0^T F_i(t,0)dt \leq -\epsilon \int_0^T F_i(t,0)dt < 0,$$

and the proof follows from Theorem 1.1 ■

Remark. Let $F^{\epsilon}(t,x)$ be defined by (1.4). It is easy to show that $F^{\epsilon}(t,x) \to F(t,x)$ as $\epsilon \to 0^+$ uniformly on $\mathbb{R} \times K$ for any compact subset K of $\mathbb{R}$. We shall construct a family $\{F^{\epsilon}, \quad \epsilon > 0\}$, satisfying the conclusions of Theorem 1.2, such that $F^{\epsilon}(t,x) \to F(t,x)$ as $\epsilon \to 0^+$ uniformly on $\mathbb{R} \times \mathbb{R}^3_+$. To this end let us define

$$\varphi_i(t,w) = \begin{cases} v_i(t) - \frac{1}{v_i(t)}[w - v_i(t)]^2 & if \quad 0 \leq w \leq v_i(t) \\ v_i(t) & if \quad w \geq v_i(t) \end{cases}$$

and

$$F^{\epsilon}(t,x) = F_i(t,x) + \epsilon[F_i(t, \varphi_{i+1}(t, x_{i+1})e_{i+1}) - F_i(t,0)].$$

Using the arguments in Theorem 1.2, if is easy to show that this family $\{F^{\epsilon}\}$ has the required properties.

References

[1] A. Tineo, *Existence of Coexistence States: A Generic Property for Cyclic 3-Dimensional Competitive Systems.*J. Math. Anal. Appl. 258 (2001), 13-21.

[2] J. Campos, R. Ortega R. and Tineo A., *Homeomorphisms of the disk trivial dynamics and extinction of competitive system.* J. Differential Equations 138, N 1 (1997), 157-170.