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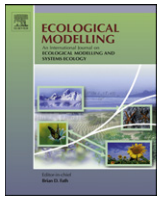

Open- and closed-loop equilibrium control of trophic chains

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ABSTRACT

If a nearly natural population system is deviated from its equilibrium, an important task of conservation ecology may be to control it back into equilibrium. In the paper a trophic chain is considered, and control systems are obtained by changing certain model parameters into control variables. For the equilibrium control two approaches are proposed. First, for a fixed time interval, local controllability into equilibrium is proved, and applying tools of optimal control, it is also shown how an appropriate open-loop control can be determined that actually controls the system into the equilibrium in given time. Another considered problem is to control the system to a new desired equilibrium. The problem is solved by the construction of a closed-loop control which asymptotically steers the trophic chain into this new equilibrium. In this way, actually, a controlled regime shift is realized.

1. Introduction

The concept of control of a trophic chain is used in different senses in the literature. A possible classification is: internal natural control, external natural control, and external control by management. (For an overview of the different types of ecosystem control see Fath, 2004). Our study joins the research line concerning external, human control of trophic chains.

The human influence on ecosystems, in particular on population systems, is an important issue in conservation ecology. Moreover, sustainability of economic and social development in a broader sense also involves conservation aspects of ecology. On the one hand, ecosystems are often exposed to a strong human intervention, such as economic activity, wildlife management, fisheries or environmental pollution. On the other hand, if the human activity breaks the equilibrium of the population system in question, we can try to control it back to the previous or a new equilibrium.

These problems make it necessary to extend the traditional approach of theoretical biology focusing only on a biological object, to the study of systems consisting of a biological object and man that monitors or/and controls the biological object. This, in dynamic situation, i.e. in case of a long-term human intervention, typically requires the approach of mathematical systems theory (in frequently used terms, state-space modelling), see Kalman et al. (1969) for the basic results of this theory, and Chen et al. (2004) for a recent reference. This methodology offers solutions not only to controlling but also to monitoring (i.e. observation) problems of population systems. While by now, mathematical systems theory became quite familiar to system engineers, observability and controllability analysis of dynamic models in population biology is relatively new. The results on controllability and observability in frequency-dependent population genetics models are mostly based on the sufficient conditions obtained in Varga (1989, 1990, 1992), for the control and observation of systems with invariant manifold. For the applications of these theorems see e.g. Kósa and Varga (1996), Scarelli and Varga (2002), López et al. (2004) and Varga (2008a).

For the control and monitoring problems of density-dependent population systems, the corresponding mathematical tools can be found in Lee and Markus (1971); conditions for controllability and observability problems for different *Lotka–Volterra type systems* have been obtained e.g. in Varga et al. (2003), Gámez et al. (2008), López et al. (2007). A recent general overview of the different applications of mathematical systems theory in population biology is Varga (2008b).

In the present paper ecological systems of *non-Lotka–Volterra* type will be considered, that form a *trophic chain* of type *resource – producer – primary consumer*, see e.g. Svirezhev and Logofet (1983), Yodzis (1989). Stability and observability results for such systems have been obtained in Shamandy (2005). We note that the monitoring of a somewhat different, four-level ecological *interaction chain* of type *resource – producer – primary user – secondary consumer* has been studied, applying the mathematical results on verticum type systems, published in Molnár (1987, 1988a,b,c,d,e, 1989, 1993), Molnár and Szigeti (1994).

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