Advances in Feedforward Control for Measurable Disturbances

José Luis Guzmán

Department of Informatics
Engineering and Systems Area
University of Almería (Spain)

joseluis.guzman@ual.es

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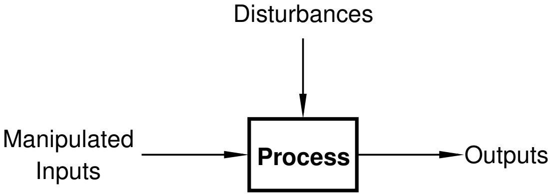
Outline

Outline

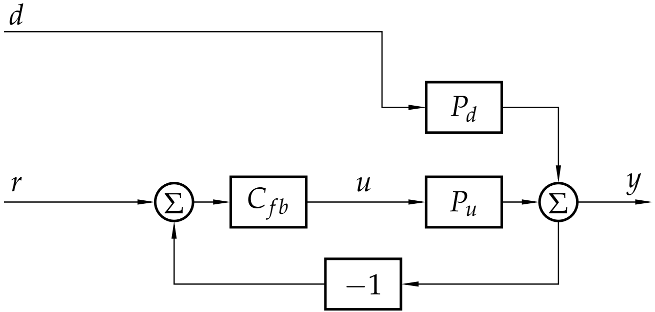
Introduction

What are load disturbances?

- Typically low frequency input signals which affect the output of processes but that cannot be manipulated

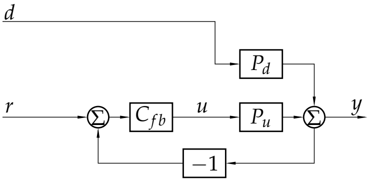


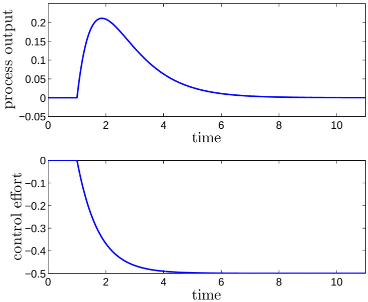
Motivation: feedback controller



Introduction

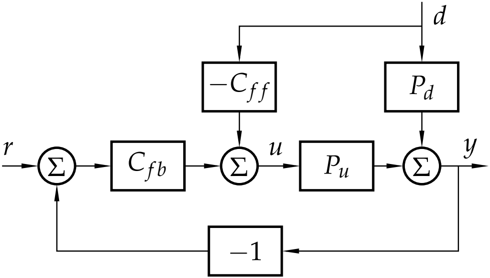
Motivation: feedback controller





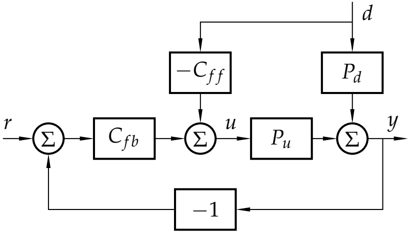
No reaction until there are discrepancies!

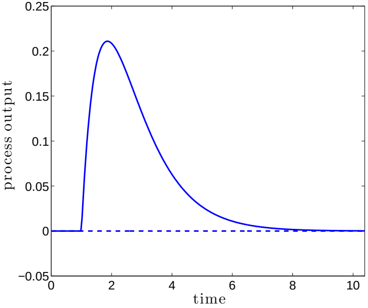
Motivation: feedforward compensator



$$Y = \frac{P_d - C_{ff}P_u}{1 + C_{fb}P_u}D, \qquad C_{ff} = \frac{P_d}{P_u}$$

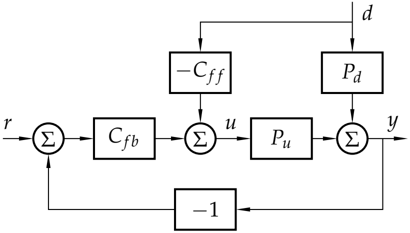
Motivation: feedforward compensator

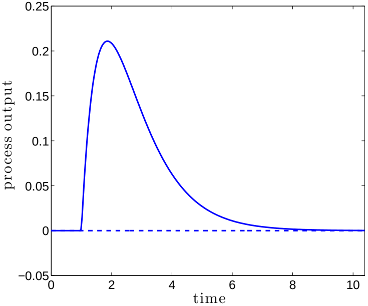




Ideal compensation: $C_{ff} = \frac{P_d}{P_u} = P_d\, P_u^{-1}$

Motivation: feedforward compensator





Ideal compensation: $C_{ff} = \dfrac{P_d}{P_u} = P_d P_u^{-1}$

Feedforward control problem

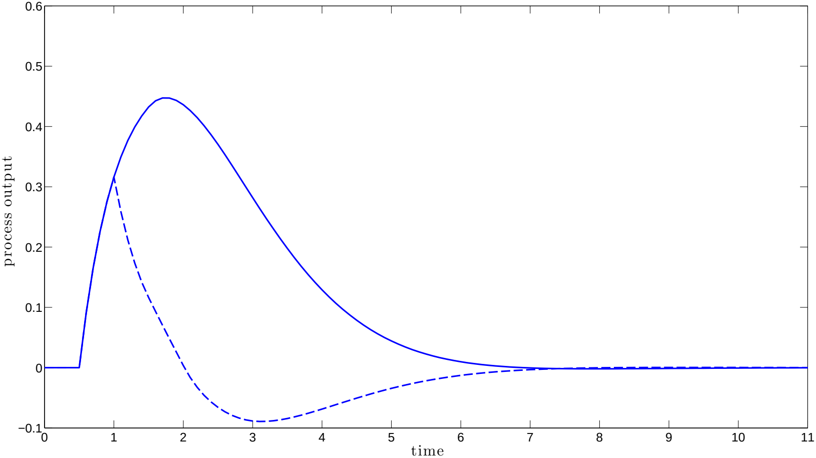
Perfect compensation is seldom realizable:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.
- Improper transfer functions.

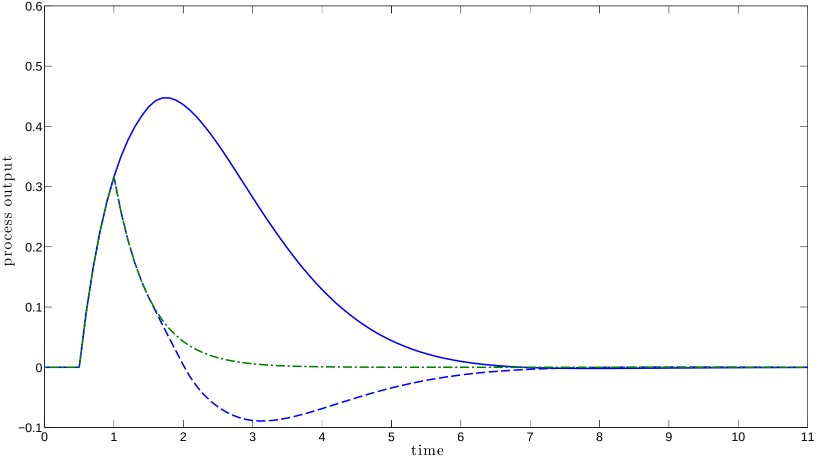
Classical solution

Ignore the non-realizable part of the compensator and implement the realizable one. In practice, static gain feedfoward compensators are quite common.

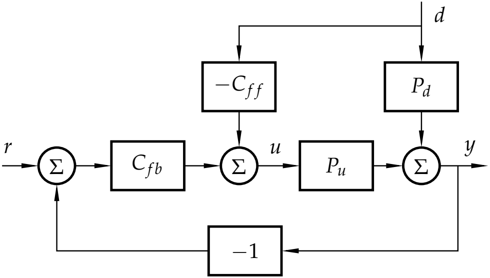
Motivation: non-ideal feedforward compensator



Motivation: non-ideal feedforward compensator



Motivation: residual term



$$Y = \frac{P_d - C_{ff}P_u}{1 + C_{fb}P_u}D, \qquad C_{ff} = \frac{P_d}{P_u}$$

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - C_{ff}P_u}{1+C_{fb}P_u}d$$

Other design strategies are required!

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff} P_u}{1 + L} d = \frac{P_d - C_{ff} P_u}{1 + C_{fb} P_u} d$$

Other design strategies are required!

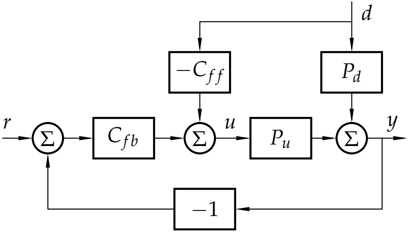
Motivation

Surprisingly there are very few studies in literature (we starting the project in 2010):

- D. Seborg, T. Edgar, D. Mellichamp, Process Dynamics and Control, Wiley, New York, 1989.
- F. G. Shinskey, Process Control Systems. Application Design Adjustment, McGraw-Hill, New York, 1996.
- C. Brosilow, B. Joseph, Techniques of Model-Based Control, Prentice-Hall, New Jersey, 2002.
- A. Isaksson, M. Molander, P. ModÈn, T. Matsko, K. Starr, Low-Order Feedforward Design Optimizing the Closed-Loop Response, Preprints, Control Systems, 2008, Vancouver, Canada.

Outline

The idea behind feedforward control from disturbances is to supply control actions before the disturbance affects the process output:



$$C_{ff} = \frac{P_d}{P_u}$$

Feedforward control problem

In industry, PID control is commonly used as feedback controller and four structures of the feedforward compensator are widely considered:

$$C_{fb} = \kappa_{fb}\left(1 + \frac{1}{s\tau_i} + s\tau_d\right)$$

Static: $C_{ff} = \kappa_{ff}$

Static with delay: $C_{ff} = \kappa_{ff} e^{-sL_{ff}}$

Lead-lag: $C_{ff} = \kappa_{ff}\dfrac{1 + s\beta_{ff}}{1 + s\tau_{ff}}$

Lead-lag with delay: $C_{ff} = \kappa_{ff}\dfrac{1 + s\beta_{ff}}{1 + s\tau_{ff}} e^{-sL_{ff}}$

Feedforward control problem

Then, if we consider that process transfer functions are modeled as first-order systems with time delay, i.e.

$$P_u = \frac{\kappa_u}{1+\tau_u} e^{-s\lambda_u}, \quad P_d = \frac{\kappa_d}{1+s\tau_d} e^{-s\lambda_d}$$

The following feedforward compensator can be considered:

Static: $C_{ff} = \frac{\kappa_d}{\kappa_u}$

Static with delay: $C_{ff} = \frac{\kappa_d}{\kappa_u} e^{-s(\lambda_d - \lambda_u)}$

Lead-lag: $C_{ff} = \frac{\kappa_d}{\kappa_u} \frac{1+s\tau_u}{1+s\tau_d}$

Lead-lag with delay: $C_{ff} = \frac{\kappa_d}{\kappa_u} \frac{1+s\tau_u}{1+s\tau_d} e^{-s(\lambda_d - \lambda_u)}$

Feedforward control problem

Lets consider the following example:

$$P_u(s) = \frac{1}{s+1}e^{-s}, \quad P_d(s) = \frac{1}{2s+1}e^{-2s}$$

Static: $C_{ff} = 1$

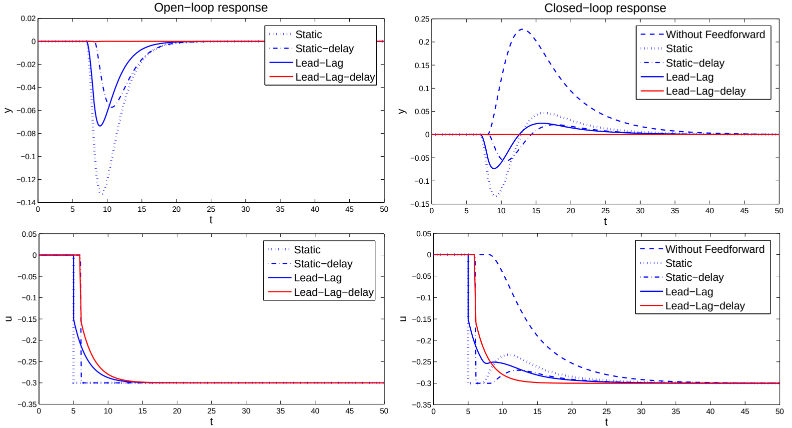
Static with delay: $C_{ff} = e^{-s}$

Lead-lag: $C_{ff} = \frac{1+s}{1+2s}$

Lead-lag with delay: $C_{ff} = \frac{1+s}{1+2s}e^{-s}$

C_{fb} is a PI controller tuned using the AMIGO rule, $\kappa_{fb} = 0.25$ and $\tau_i = 2.0$.

Feedforward control problem



Motivation

Then, lets consider a delay inversion problem, i.e., $\lambda_d < \lambda_u$. Then, the resulting feedforward compensators are given by:

$$C_{ff} = K_{ff} = \frac{\kappa_d}{\kappa_u}$$

$$C_{ff} = \frac{\kappa_d}{\kappa_u}\frac{\tau_u s + 1}{\tau_d s + 1}$$

Motivation

Example:

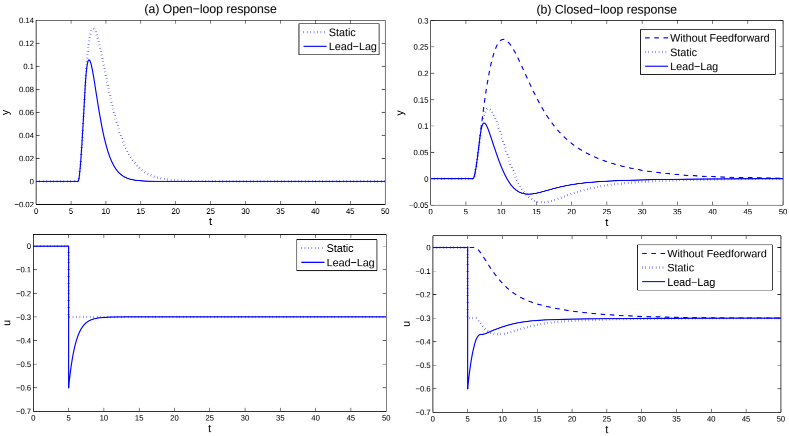
$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

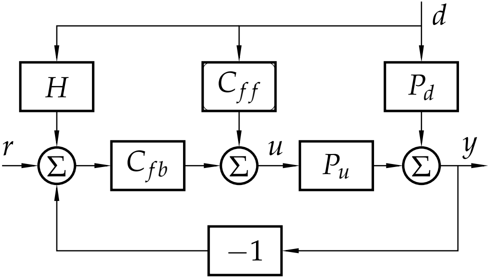
$$C_{ff} = 1, \quad C_{ff} = \frac{2s+1}{s+1}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.32$ and $\tau_i = 2.85$.

Feedforward control problem

Motivation





$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

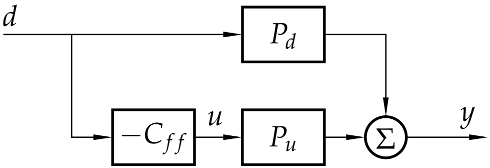
Outline

Cases to be evaluated in this research:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.

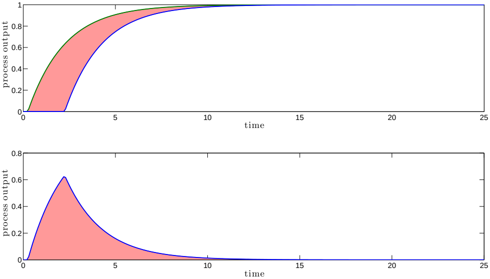
Nominal feedforward design: non-realizable delay

Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d \qquad C_{ff} = \frac{\kappa_d}{\kappa_u} \cdot \frac{\tau_u s + 1}{\tau_d s + 1}$$

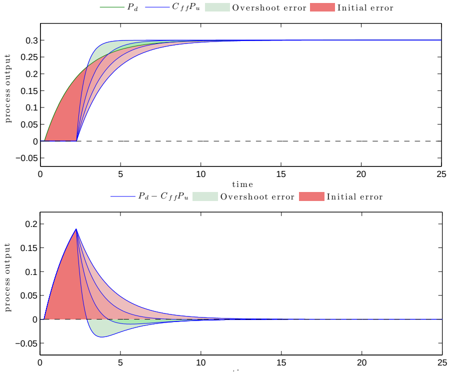
Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d \qquad C_{ff} = \frac{\kappa_d}{\kappa_u} \cdot \frac{\tau_u s + 1}{\tau_d s + 1}$$

Nominal feedforward design: non-realizable delay

Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d + u_{fb}P_u$$

Nominal feedforward design: non-realizable delay

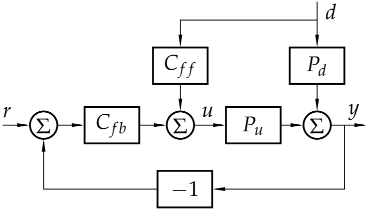
Objective

To improve the final disturbance response of the closed-loop system when delay inversion is not realizable ($\lambda_u > \lambda_d$)

Methodology

- Adapt the open-loop tuning rules to closed-loop design
- Obtain optimal open-loop tuning rules

First approach



$$P_k(s) = \frac{\kappa_k}{\tau_k s + 1} e^{-\lambda_k s} \qquad k \in [u, d] \qquad \lambda_u > \lambda_d$$

$$C_{fb}(s) = \kappa_{fb} \frac{\tau_i s + 1}{\tau_i s} \qquad C_{ff}(s) = \kappa_{ff} \frac{\beta_{ff} s + 1}{\tau_{ff} s + 1}$$

Nominal feedforward design: non-realizable delay

First approach

To deal with the non-realizable delay case, the first approach was to work with the following:

- Use the classical feedforward control scheme.
- Remove the overshoot observed in the response.
- Proposed a tuning rule to minimize Integral Absolute Error (IAE).
- The rules should be simple and based on the feedback and model parameters.

Nominal feedforward design: non-realizable delay

To remove the overshoot, the feedback control action is taken into account to calculate the feedforward gain, κ_{ff}.

$$\Delta u = \frac{\kappa_{fb}}{\tau_i} \int e dt = \frac{\kappa_{fb}}{\tau_i} IE \cdot d$$

So, in the new rule, the goal is to take the control signal to the correct stationary level $-\Delta u$ in order to take the feedback control signal into account and reduce the overshoot. The gain is therefore reduced to

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

Closed-loop design

Nominal feedforward design: non-realizable delay

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$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

Closed-loop design

Nominal feedforward design: non-realizable delay

IE estimation:

$$Y = (P_d - P_u C_{ff})D = P_d D - P_u C_{ff} D$$

$$y(t) - y_{sp} = \begin{cases} k_d\left(1 - e^{-\frac{t}{\tau_d}}\right)d & 0 \leq t \leq \lambda_b \\ k_d\left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-\lambda_b}{T_b}}\right)\right)d & \lambda_b < t \end{cases}$$

$$\lambda_b = \max(0, \lambda_u - \lambda_d), \quad T_b = \tau_u + \tau_{ff} - \beta_{ff}$$

Nominal feedforward design: non-realizable delay

IE estimation:

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$$\lambda_b = \max(0, \lambda_u - \lambda_d), \quad T_b = \tau_u + \tau_{ff} - \beta_{ff}$$

IE estimation:

$$
\begin{aligned}
IE \cdot d &= \int_0^{\infty} (y(t) - y_{sp}) dt \\
&= k_d \int_0^{\lambda_b} \left(1 - e^{-\frac{t}{\tau_d}}\right) d\, dt + k_d \int_{\lambda_b}^{\infty} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) d\, dt \\
&= k_d \left[t + \tau_d e^{-\frac{t}{\tau_d}}\right]_0^{\lambda_b} d + k_d \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{\lambda_b}^{\infty} d \\
&= k_d \left(\lambda_b + \tau_d e^{-\frac{\lambda_b}{\tau_d}} - \tau_d - \tau_d e^{-\frac{\lambda_b}{\tau_d}} + T_b\right) d \\
&= k_d \left(\lambda_b - \tau_d + T_b\right) d
\end{aligned}
$$

Nominal feedforward design: non-realizable delay

IE estimation:

$$IE = \begin{cases} k_d(\tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d + \tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d < \lambda_u \end{cases}$$

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

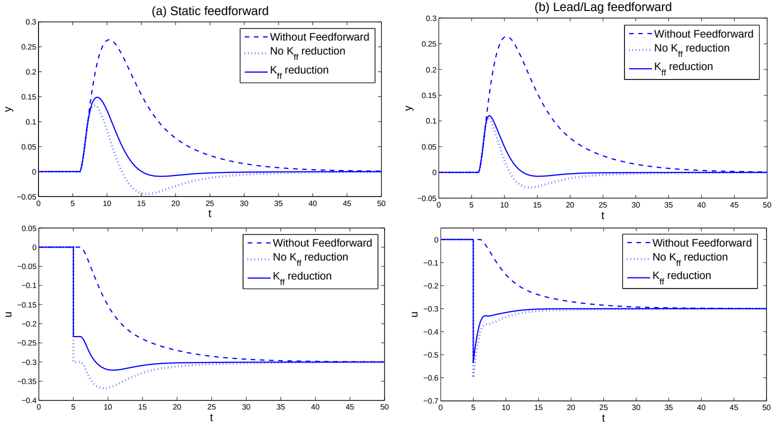
Lets consider the same previous example:

$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

$$C_{ff} = 1, \quad C_{ff} = \frac{2s+1}{s+1}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.32$ and $\tau_i = 2.85$.

Nominal feedforward design: non-realizable delay



The feedforward gain κ_{ff} has been reduced from 1 to 0.778 for the static feedforward and from 1 to 0.889 for the lead-lag filter.

Nominal feedforward design: non-realizable delay

Once the overshoot is reduced, the second goal is to design β_{ff} and τ_{ff} to minimize the IAE value. In this way, we keep $\beta_{ff} = \tau_u$ to cancel the pole of P_u and fix the zero of the compensator:

$$IAE = \int_0^\infty |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^\infty y(t)dt$$

where t_0 is the time when y crosses the setpoint, with $y_{sp} = 0$ and $d = 1$.

Nominal feedforward design: non-realizable delay

$$y(t) - y_{sp} = \begin{cases} k_d\left(1 - e^{-\frac{t}{\tau_d}}\right)d & 0 \le t \le \lambda_b \\ k_d\left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-\lambda_b}{T_b}}\right)\right)d & \lambda_b < t \end{cases}$$

$$IAE = \int_0^{\infty} |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^{\infty} y(t)dt$$

$$\frac{t_0}{\tau_d} = \frac{t_0 - \lambda_b}{T_b} \rightarrow t_0 = \frac{\tau_d \lambda_b}{\tau_d - T_b} = \frac{\tau_d}{\tau_u - \tau_{ff}}\lambda_b$$

$$T_b = \tau_u + \tau_{ff} - \beta_{ff}$$

Nominal feedforward design: non-realizable delay

$$
\begin{aligned}
IAE &= \int_0^{\lambda_b} \left(1 - e^{-\frac{t}{\tau_d}}\right) dt + \int_{\lambda_b}^{t_0} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) dt - \int_{t_0}^{\infty} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) dt \\
&= \left[t + \tau_d e^{-\frac{t}{\tau_d}}\right]_0^{\lambda_b} + \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{\lambda_b}^{t_0} - \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{t_0}^{\infty} \\
&= \lambda_b - \tau_d + T_b + 2\tau_d e^{-\frac{t_0}{\tau_d}} - 2T_b e^{-\frac{t_0-\lambda_b}{T_b}} \\
&= \lambda_b - \tau_d + T_b + 2\tau_d e^{-\frac{\lambda_b}{\tau_d - T_b}} - 2T_b e^{-\frac{\lambda_b}{\tau_d - T_b}} \\
&= \lambda_b - \tau \left(1 - 2e^{-\frac{\lambda_b}{\tau}}\right)
\end{aligned}
$$

with $\tau = \tau_d - \tau_{ff}$.

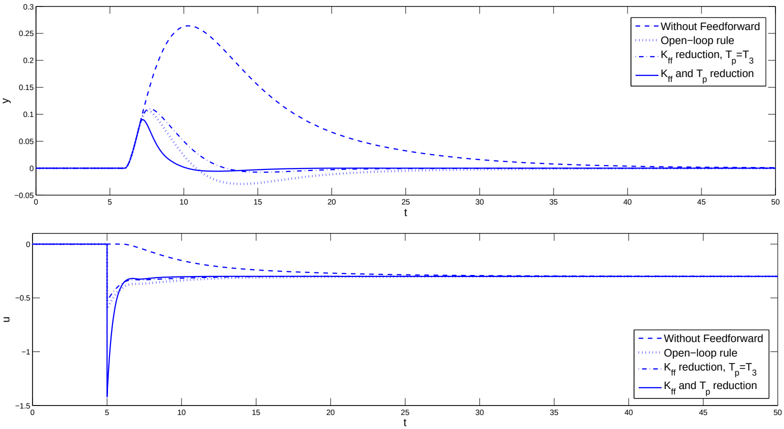
$$\frac{d}{d\tau}IAE = -1 + 2e^{-\frac{\lambda_b}{\tau}} + 2\frac{\lambda_b}{\tau}e^{-\frac{\lambda_b}{\tau}} = -1 + 2(1+x)e^{-x} = 0$$

where $x = \lambda_b/\tau$. A numerical solution of this equation gives $x \approx 1.7$, which gives

$$\tau_{ff} = T_b - \tau_d + \tau_u = \tau_d - \tau \approx \tau_d - \frac{\lambda_b}{1.7}$$

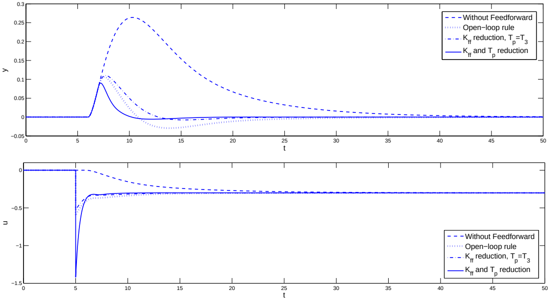
$$\tau_{ff} = \begin{cases} \tau_d & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \end{cases}$$

Gain and τ_{ff} reduction rule:



Nominal feedforward design: non-realizable delay

Gain and τ_{ff} reduction rule:



	No FF	Open-loop rule	κ_{ff} reduction	κ_{ff} &τ_{ff} reduction
IAE	9.03	1.76	1.37	0.59

First approach: Guideline summary

1. Set $\beta_{ff} = \tau_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \end{cases}$$

2. Calculate the compensator gain, κ_{ff}, as

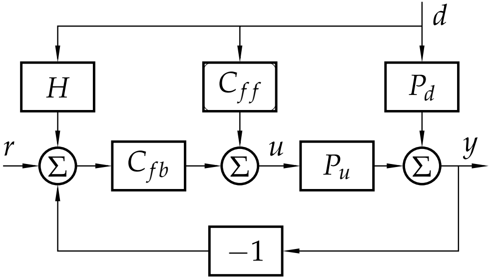
$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

$$IE = \begin{cases} k_d(\tau_{ff} - \tau_d) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d - \tau_d + \tau_{ff}) & \lambda_d < \lambda_u \end{cases}$$

Second approach: non-interacting structure

Nominal feedforward design: non-realizable delay

Second approach: non-interacting structure



$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

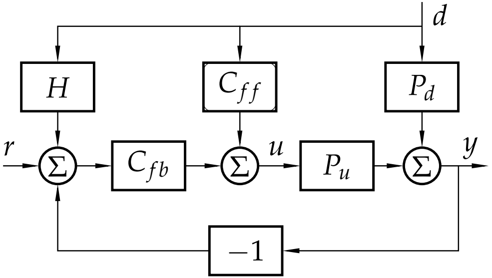
Nominal feedforward design: non-realizable delay

Second approach: non-interacting structure

To deal with the non-realizable delay case, the second approach was to work with the following:

- Use the non-interacting feedforward control scheme (feedback effect removed).
- Obtain a generalized tuning rule for τ_{ff} for moderate, aggressive and conservative responses.
- The rules should be simple and based on the feedback and model parameters.

Second approach: non-interacting structure



$$\frac{y}{d} = P_d - P_u C_{ff}$$

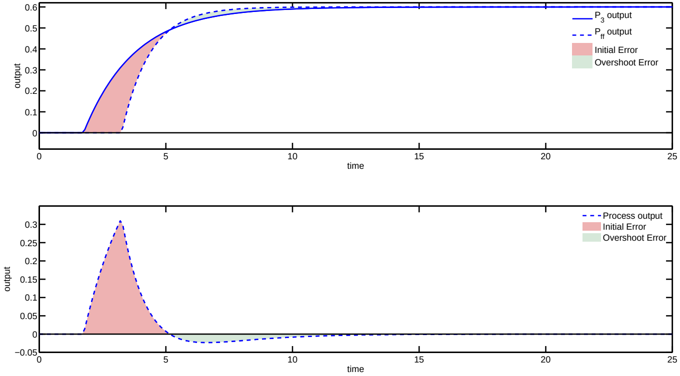
Second approach

The main idea of this second approach relies on analyzing the residual term appearing when perfect cancelation is not possible:

$$\frac{y}{d} = P_d - P_u C_{ff}$$

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

Nominal feedforward design: non-realizable delay



$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

Nominal feedforward design: non-realizable delay

From the previous analysis, it can be concluded that in order to totally remove the overshoot for the disturbance rejection problem by using a lead-lag filter, the settling times of both transfer functions must be the same:

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}, \quad \lambda_b = \lambda_d - \lambda_u$$

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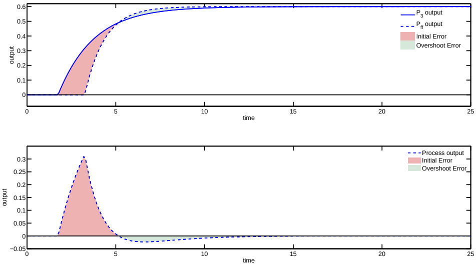
Notice that the new rule for τ_{ff} implies a natural limit on performance. If parameter τ_{ff} is chosen larger, performance will only get worse because of a late compensation. The only reasons why τ_{ff} should be even larger is to decrease the control signal peak:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{4}$$



Nominal feedforward design: non-realizable delay

So, considering the IAE rule obtained for the first approach, two tuning rules are available:

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}$$

$$\tau_{ff} = \tau_d - \frac{\lambda_u - \lambda_d}{1.7} = \tau_d - \frac{\lambda_b}{1.7}$$

And a third one (a more agreessive rule) can be calculated to minimize Integral Squared Error (ISE) instead of IAE such as proposed in the first approach.

ISE minimization:

$$\text{ISE} = \int_{\lambda_b}^{\infty} \left(e^{-\frac{(t-\lambda_b)}{\tau_{ff}}} - e^{-\frac{t}{\tau_d}} \right)^2 dt$$

$$= \int_{\lambda_b}^{\infty} \left(e^{-\frac{2(t-\lambda_b)}{\tau_{ff}}} - 2e^{-\frac{\tau_d(t-\lambda_b)+\tau_{ff}t}{\tau_d\tau_{ff}}} + e^{-\frac{2t}{\tau_d}} \right) dt$$

$$= -\frac{\tau_{ff}}{2} \left[e^{-\frac{2(t-\lambda_b)}{\tau_{ff}}} \right]_{\lambda_b}^{\infty} + 2\frac{\tau_d\tau_{ff}}{\tau_d+\tau_{ff}} \left[e^{-\frac{\tau_d(t-\lambda_b)+\tau_{ff}t}{\tau_d\tau_{ff}}} \right]_{\lambda_b}^{\infty} - \frac{\tau_d}{2} \left[e^{-\frac{2t}{\tau_d}} \right]_{\lambda_b}^{\infty}$$

$$= \frac{\tau_{ff}}{2} - 2\tau_d \frac{\tau_{ff}}{\tau_d+\tau_{ff}} e^{-\frac{\lambda_b}{\tau_d}} + \frac{\tau_d}{2} e^{-\frac{2\lambda_b}{\tau_d}}$$

ISE minimization:

$$\frac{d\,\text{ISE}}{d\,\tau_{ff}} = \frac{1}{2} - 2\tau_d e^{-\frac{\lambda_b}{\tau_d}} \left(\frac{1}{\tau_d + \tau_{ff}} + \frac{-\tau_{ff}}{(\tau_d + \tau_{ff})^2} \right) = \frac{1}{2} - \frac{2\tau_d^2}{(\tau_d + \tau_{ff})^2} e^{-\frac{\lambda_b}{\tau_d}} = 0$$

$$\tau_{ff}^2 + 2\tau_d \tau_{ff} + \tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}}) = 0$$

$$\tau_{ff} = \frac{-2\tau_d + \sqrt{4\tau_d^2 - 4\tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}})}}{2} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1 \right)$$

Nominal feedforward design: non-realizable delay

Thus, three tuning rules are available:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{4}$$

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{1.7}$$

$$\tau_{ff} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1 \right)$$

which can be generalized as:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{\alpha}$$

Second approach: Guideline summary

1. Set $\beta_{ff} = \tau_u$, $\kappa_{ff} = k_d/k_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_b \leq 0 \\ \tau_d - \frac{\lambda_b}{\alpha} & 0 < \lambda_b < 4\tau_d \\ 0 & \lambda_b \geq 4\tau_d \end{cases}$$

2. Determine τ_{ff} with $\lambda_b/\tau_d < \alpha < \infty$ using:

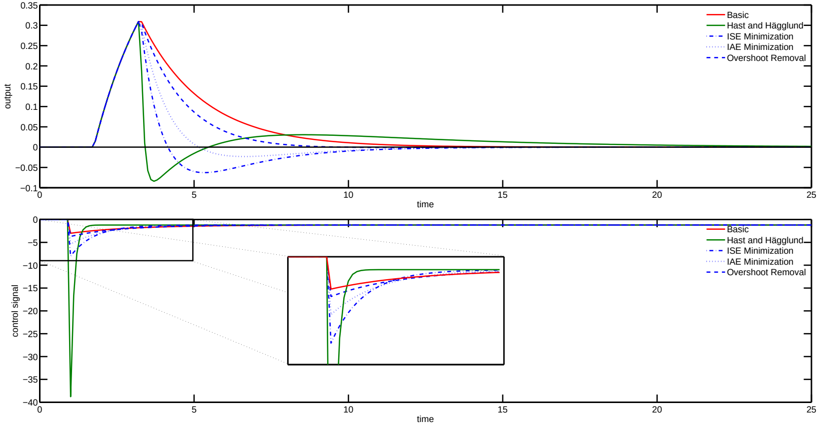
$$\alpha = \begin{cases} \dfrac{\lambda_b}{2\tau_d\left(1-\sqrt{e^{-\lambda_b/\tau_d}}\right)} & \text{aggressive (ISE minimization)} \\ 1.7 & \text{moderate (IAE minimization)} \\ 4 & \text{conservative (Overshoot removal)} \end{cases}$$

Example:

$$P_u(s) = \frac{0.5}{5s+1}e^{-2.25s}, \quad P_d(s) = \frac{1}{2s+1}e^{-0.75s}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.9$ and $\tau_i = 4.53$.

Nominal feedforward design: non-realizable delay



	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	**0.0739**	0.6423	38.7800	2.5710	0.8979
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	**0.5641**	5.3680	**0.9113**	**0.8315**
Overshoot Removal	0.1277	0.6833	**3.6920**	0.9323	0.8870

$$J_1(F,B) = \frac{1}{2}\left(\frac{\text{ISE}(F)}{\text{ISE}(B)} + \frac{\text{ISC}(F)}{\text{ISC}(B)}\right), \quad \text{ISC} = \int_0^\infty u(t)^2 \, \mathrm{d}t$$

$$J_2(F,B) = \frac{1}{2}\left(\frac{\text{IAE}(F)}{\text{IAE}(B)} + \frac{\text{IAC}(F)}{\text{IAC}(B)}\right), \quad \text{IAC} = \int_0^\infty |u(t)| \, \mathrm{d}t$$

Outline

Performance indices for feedforward control

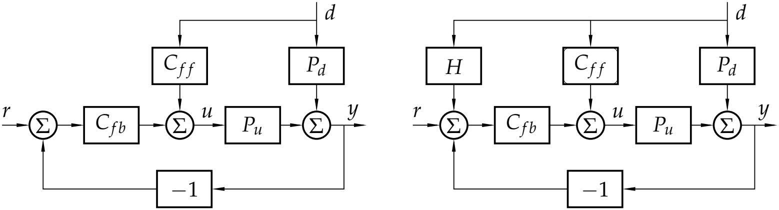
Objective

To proposed indices such that the advantage of using a feedforward compensator with respect to the use of a feedback controller only can be quantified.

Methodology

- Propose different indices
- Calculate the indices based on the process parameters

The two feedforward schemes are considered:



Performance indices for feedforward control

Assumptions:

$$P_u = \frac{\kappa_u}{1+\tau_u} e^{-s\lambda_u}, \quad P_d = \frac{\kappa_d}{1+s\tau_d} e^{-s\lambda_d}$$

Only, the non-inversion delay problem is analyzed:

$$\text{Lead-lag:} \quad C_{ff} = \frac{\kappa_d}{\kappa_u} \frac{1+s\tau_u}{1+s\tau_d}$$

Assumptions:

$$C_{fb} = \kappa_{fb}\left(1 + \frac{1}{s\tau_i}\right)$$

The lambda tuning rule is considered:

$$\kappa_{fb} = \frac{\tau_i}{\kappa_u(\lambda_u + \tau_{bc})}, \qquad \tau_i = \tau_u$$

where τ_{bc} is the closed-loop time constant.

Performance indices for feedforward control

The following index structure is proposed

$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}},$$

where IAE_{FB} is the integrated absolute value of the control error obtained when only feedback is used, and IAE_{FF} is the corresponding IAE value obtained when feedforward is added to the loop.

As long as the feedforward improves control, i.e. $IAE_{FF} < IAE_{FB}$, the index is in the region $0 < I_{FF/FB} < 1$.

Calculation of IAE_{fb}

In the feedback only case, the transfer function between disturbance d and process output y is

$$G_{y/d}(s) = \frac{P_d(s)}{1 + P_u(s)C_{fb}(s)} = \frac{\kappa_d \dfrac{e^{-s\lambda_d}}{1+s\tau_d}}{1 + \kappa_u \dfrac{e^{-s\lambda_u}}{1+s\tau_u} \kappa_{fb} \dfrac{1+s\tau_i}{s\tau_i}}$$

Assuming that $r = 0$ and d is a step with magnitude A_d and using the final value theorem, the Integrated Error (IE) value becomes (note that $e = -y$, with $r = 0$)

$$IE_{FB} = \int_0^\infty e(t)dt = \lim_{s\to 0} s \cdot \frac{1}{s} E(s) = \lim_{s\to 0} -G_{y/d}(s)\frac{A_d}{s} = -\frac{\tau_i \kappa_d}{\kappa_u \kappa_{fb}} A_d$$

Calculation of IAE_{fb}

The magnitude of the IE value can be set equal to the IAE value provided that the controller is tuned so that there are no oscillations:

$$IAE_{FB} = \frac{\tau_i \kappa_d}{\kappa_u \kappa_{fb}} A_d$$

Finally, considering the lambda tuning rule, it becomes

$$IAE_{FB} = \kappa_d A_d (\lambda_u + \tau_{bc})$$

Calculation of IAE_{FF} for classical FF scheme

In this case, the transfer function from the disturbance to the error is

$$G_{y/d}(s) = -\frac{P_d(s) + P_u(s)C_{ff}(s)}{1 + P_u(s)C_{fb}(s)} = \frac{\kappa_d \dfrac{e^{-s\lambda_d}}{1+s\tau_d} - \kappa_d \dfrac{e^{-s\lambda_u}}{1+s\tau_d}}{1 + \kappa_u \dfrac{e^{-s\lambda_u}}{1+s\tau_u} \kappa_{fb} \dfrac{1+s\tau_i}{s\tau_i}}$$

Considering the lambda tuning rule and that the delays are approximated as

$$e^{-\lambda_u s} \cong 1 - \lambda_u s, \qquad e^{-\lambda_d s} \cong 1 - \lambda_d s$$

It results in:

$$G_{y/d}(s) = -\frac{\kappa_d(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)s^2}{(1+\tau_d s)(1+\tau_{bc} s)}$$

After some considerations and basic calculations, the IAE_{FF} estimation can be obtained as follows

$$IAE_{FF} = \begin{cases} 2\frac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)\left(\frac{\tau_{bc}}{\tau_d}\right)^{-\frac{\tau_{bc}}{\tau_{bc}-\tau_d}} & \tau_{bc} \neq \tau_d \\ 2\frac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)e^{-1} & \tau_{bc} = \tau_d \end{cases}$$

Calculation of IAE_{FF} for non-interacting FF scheme

In this case, the IAE_{FF} estimation can be obtained in a straightforward manner, as the effect from the feedback controller is removed.

The IAE result obtained in the non-invertible delay case can be reformulated as

$$\begin{aligned} IAE_{FF} &= \kappa_d A_d \left((\lambda_u - \lambda_d) - (\tau_d - \tau_u - \tau_u + \tau_u) \left(1 - 2e^{-\frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}} \right) \right) \\ &= \kappa_d A_d \left(1 - \frac{\tau_d - \tau_u - \tau_u + \tau_u}{\lambda_u - \lambda_d} \left(1 - 2e^{-\frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}} \right) \right) (\lambda_u - \lambda_d) \\ &= \kappa_d A_d \left(1 - \frac{1}{a} + \frac{2}{a} e^{-a} \right) (\lambda_u - \lambda_d) \\ &= \kappa_d A_d \alpha (\lambda_u - \lambda_d) \end{aligned}$$

where

$$\alpha = 1 - \frac{1}{a} + \frac{2}{a} e^{-a}, \;\; a = \frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}$$

Performance indices for feedforward control

Analysis and discussion on the indices

- Feedback control without feedforward:

$$IAE_{FB} = \kappa_d A_d(\lambda_u + \tau_{bc})$$

- Feedforward with classical control scheme and classical tuning:

$$IAE_{FF} = 2\frac{\kappa_d A_d}{\tau}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d) f(\tau_{bc}/\tau_d) \quad (1)$$

where

$$f(\tau_{bc}/\tau_d) = \begin{cases} \left(\dfrac{\tau_{bc}}{\tau_d}\right)^{-\dfrac{\tau_{bc}}{\tau_{bc}-\tau_d}} & \tau_{bc} \neq \tau_d \\ e^{-1} & \tau_{bc} = \tau_d \end{cases} \quad (2)$$

- Feedforward with non-interacting control scheme:

$$IAE_{FF} = \alpha \kappa_d A_d(\lambda_u - \lambda_d)$$

where α can vary based on the τ_{ff} value.

Index interpretation

For the classical feedforward control case, the index becomes

$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}} = 1 - \frac{2(\lambda_u - \lambda_d)}{\tau_d} f(\tau_{bc}/\tau_d)$$

For the noninteracting feedforward control scheme, the index is given by

$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}} = 1 - \frac{\alpha(\lambda_u - \lambda_d)}{\lambda_u + \tau_{bc}}$$

Performance indices for feedforward control

Example 1

$$P_u(s) = \frac{e^{-2s}}{10s+1} \qquad P_d(s) = \frac{e^{-s}}{5s+1}$$

Using lambda tuning with $\tau_{bc} = \tau_u = 10$ gives the PI controller parameters $\kappa_{fb} = 0.83$ and $\tau_i = 10$.

The feedforward compensators are defined as

$$C_{ff}(s) = \frac{10s+1}{5s+1}$$

for the classical feedforward control scheme and as

$$C_{ff} = \frac{10s+1}{4.4s+1}$$

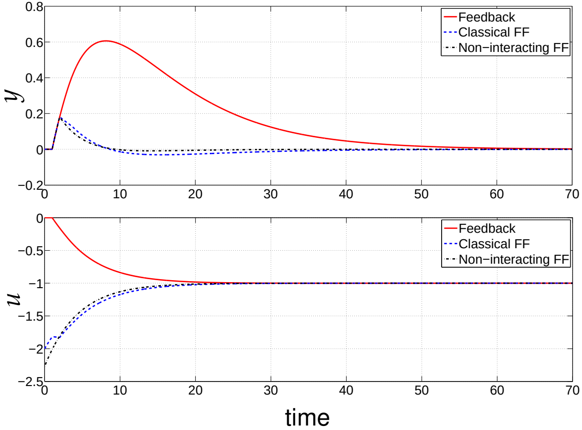
for the non-interacting feedforward control scheme (to minimize IAE).

Performance indices for feedforward control

Example 1

Control scheme	IAE^r	IAE^e	$I_{FF/FB}$
Feedback	11.99	12	–
Classical FF	1.21	1.2	0.9
Non-interacting FF	0.63	0.63	0.95

Example 1



y
0.8
0.6
0.4
0.2
0
−0.2
0
10
20
30
40
50
60
70
Feedback
Classical FF
Non−interacting FF
u
0
−0.5
−1
−1.5
−2
−2.5
time

Example 2

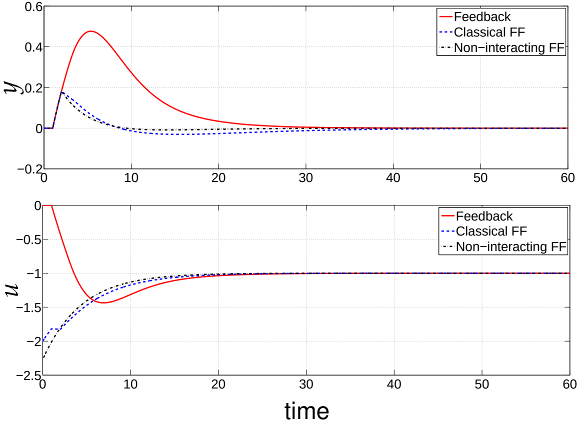
The differences between the pure feedback scheme and the feedforward schemes can be reduced by retuning the PI controller to obtain a more aggressive response. Lets retune the PI controller only for the case when pure feedback is used, by using $\tau_{bc} = 0.25\tau_u$.

Performance indices for feedforward control

Example 2

Control scheme	IAE^r	IAE^e	$I_{FF/FB}$
Feedback	4.5	4.5	–
Classical FF	1.21	1.2	0.73
Non-interacting FF	0.63	0.63	0.86

Example 2

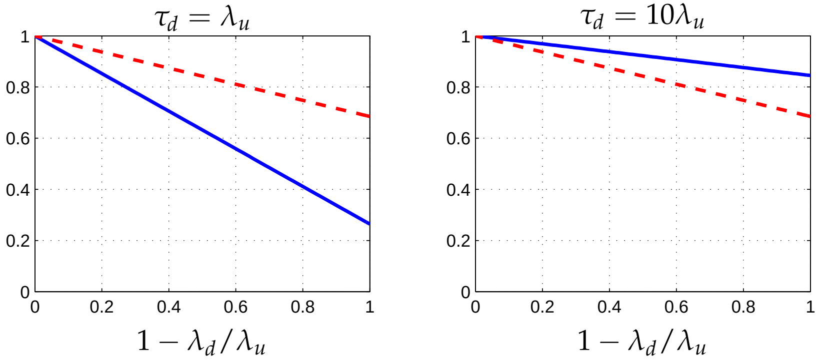


0.6
0.4
0.2
0
−0.2
y
0
10
20
30
40
50
60
Feedback
Classical FF
Non−interacting FF
0
−0.5
−1
−1.5
−2
−2.5
u
time

Example 3

Assume that $\tau_{bc} = \tau_u = \lambda_u$. It means that we have a process model $P_u(s)$ where the delay is equal to the time constant and that the lambda tuning rule is used with $\tau_{bc} = \tau_u$. Two different values of the time constant $\tau_d = \eta\lambda_u$, where η = 1 or 10.

Example 3

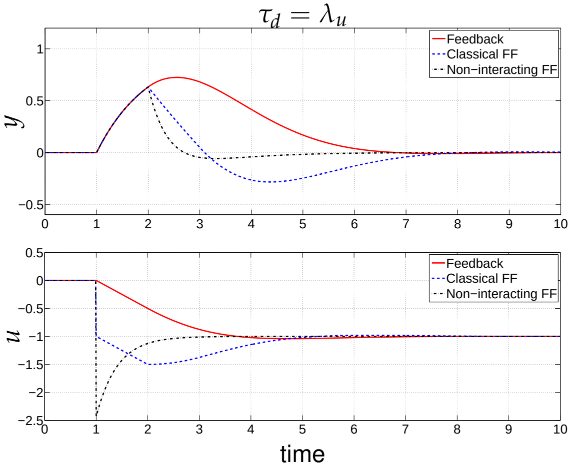


The index $I_{FF/FB}$ for the classical scheme (blue solid line) and the noninteracting scheme (dashed red line).

Example 3

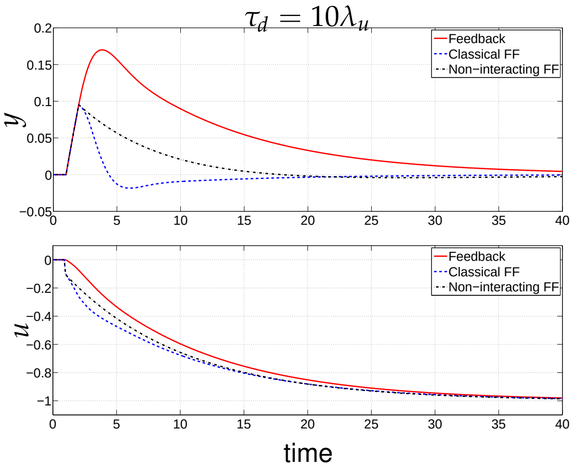
τ_d	Control scheme	IAE^r	IAE^e	$I^r_{FF/FB}$	$I^e_{FF/FB}$
λ_u	Feedback	2.04	2.0		
	Classical FF	1.43	1.47	0.30	0.26
	Non-interacting FF	0.63	0.63	0.69	0.69
$10\lambda_u$	Feedback	2.00	2.0		
	Classical FF	0.34	0.31	0.83	0.85
	Non-interacting FF	0.63	0.63	0.69	0.69

Example 3



$\tau_d = \lambda_u$
Feedback
Classical FF
Non−interacting FF
y
u
time

Example 3


$\tau_d = 10\lambda_u$
y
0.2
0.15
0.1
0.05
0
−0.05
Feedback
Classical FF
Non−interacting FF
0
5
10
15
20
25
30
35
40
u
0
−0.2
−0.4
−0.6
−0.8
−1
Feedback
Classical FF
Non−interacting FF
time

Outline

Conclusions

- The motivation for feedforward tuning rules was introduced.
- The feedback effect on the feedforward design was analyzed.
- The different non-realizable situations were studied.
- The two available feedforward control schemes were used.
- Simple tuning rules based on the process and feedback controllers parameters were derived.
- Performance indices for feedforward control were proposed.

Future research

What else can be done?

- **DTC with feedforward action.** Extension to MIMO processes
- **Experimental results.** Validate the theoretically claimed benefits
- **Distributed parameter systems.** Feedforward tuning rules to deal with resonance dynamics

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Collaborators

1. Carlos Rodríguez (UAL, Spain)
2. Tore Hägllund (Lund, Sweden)
3. Manuel Berenguel (UAL, Spain)
4. Julio Normey (Florianopolis, Brazil)
5. Antonio Visioli (Brescia, Italy)
6. Max Veronesi (Milano, Italy)

End of the presentation

Thank you for your attention