Perspectives on Control-Relevant Identification Through the Use of Interactive Tools

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Abstract

System identification is an essential part of control design. Control engineers must devote substantial effort to identification issues in order to obtain suitable models for closed-loop control. Control-relevant identification seeks to both simplify the modeling task and improve the usefulness of the model by taking into account controller requirements during system identification. The advantages of this methodology can be better understood and appreciated through the interactive software tool described in this paper. The **I**nteractive **T**ool for **C**ontrol **R**elevant **I**dentification (*ITCRI*) comprehensively captures the control-relevant identification process for the monovariable problem, from input design to closed-loop control, depicting these stages simultaneously and interactively in one screen. By simultaneously displaying both open- and closed-loop responses of the estimated models, *ITCRI* enables the user to readily assess how design variable choices during identification and control performance requirements impact model error and ultimately, closed-loop performance. Moreover, the work presents several examples which the aim to illustrate the tool and the considerations that arise when control requirements are taken into account during the identification stage.

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1. Introduction

System identification focuses on the building of dynamical models from data (Goodwin & Payne, 1977; Isermann & Münchhof, 2010; Juang, 1994; Keesman, 2011; Ljung, 1999; Pintelon & Schoukens, 2001; Walter & Pronzato, 1997). It is often considered the most challenging and time consuming step in control engineering practice and thus represents an important component in the professional training of any control engineer; to this end, flexible and simple-to-use software tools are essential. Classical system identification is focused on satisfying "open-loop" criteria that may lead to high-order models that are not be directly suitable for control system design. However, by taking into account controller requirements during system identification, it becomes possible to both simplify the modeling task and improve the usefulness of the model with respect to the intended application of control design; this is the essence of control-relevant identification (van den Hof & Callafon, 2003; Hjalmarsson, 2005). Thus, control-relevant identification examines the impact of the controller on the identification problem in order to develop a methodology that produces useful models in spite of the limitations previous stated (Rivera et al., 1992).

In recent years, the term interactivity is becoming common in the field of Automatic Control since advances in information technologies have provided powerful interactive software tools for training control engineers (Dormido, 2004; Casini et al., 2004; Nassirharand, 2008). Moreover, interactive software tools have been proven as particularly useful techniques with high impact on control education (Guzmán et al., 2005, 2008a) and engineer training (Guzmán et al., 2008b; Normey-Rico et al., 2009). Interactive tools provide a real-time connection between decisions made during the design phase and results obtained in the analysis phase of any control-related project. Prior work involving the authors has resulted in *ITSIE*, an **I**nteractive software **T**ool for **S**ystem **I**dentification **E**ducation (Guzmán et al., 2009a,b, 2011). *ITSIE* is focused exclusively on open-loop system identification; the current work goes beyond this to explore the problem of control-relevant identification.

This work presents a brand new **I**nteractive **T**ool for **C**ontrol **R**elevant **I**dentification (*ITCRI*). The objective of this tool is to help educate users with an interest in better understanding control-relevant identification for the monovariable problem. Thus, this tool is mainly aimed at control engineering students but, at the same time, it can be used by control engineers who wish to learn or extend their knowledge regarding this advanced control concept. Users, in one single environment, can discover all the aspects of this methodology, from the design of the input signal used for identification to how the prefilter affects the chosen controller. Therefore, users can, using a single visualization of the tool as a basis, analyze how the identified model plant fulfills the control requirements previously stated.

The tool incorporates control-relevant methods well-known by the authors (Rivera et al., 1992; Rivera & Gaikwad, 1996). These methods are not the only ways to perform control-relevant identification; sice it is possible to find similar methods and works in literature (Zang et al., 1995; Gevers, 2002). For this reason, the main novelties of this tool are twofold: i) the fact that it relies on interactivity to teach the main features of control-relevant identification and ii) the functionality provided by the interactive tool itself. Interactivity as presented in this tool is at a much higher level than in related packages such as CLOSID (van den Hof et al., 1996). Unlike CLOSID, which is focused primarily on implementing closed-loop identification, the interactive tool presented in this work is not a toolbox, but is coded in Sysquake, a Matlab-like language with fast execution and excellent facilities for interactive graphics (Piguet, 2004), but stand-alone executable files that do not require the Sysquake software are in the public domain and available for Windows, Mac, and Linux operating systems (`http://aer.ual.es/ITCRI/`).

The tool considers the control-relevant estimation of low-order ARX and Output Error models conforming to the IMC Prett-García PID tuning rules (Prett & García, 1988). To this aim, two prefiltered prediction-error estimation procedures are considered. The prefilters are systematically defined from closed-loop performance requirements and the setpoint/disturbance changes to be faced in the control problem. Moreover, this tool not only allows to design the prefilter but allows to study other design variables as the input signal and the model structure, as it can be seen in the following sections. The interactive tool enables understanding how the tuning parameter of the prefilter directly influences both the open- and closed-loop responses of the system. Validation criteria allow the user to evaluate: (i) how control-relevant modeling keeps the error low over a bandwidth defined by the control requirements specified by the user and (ii) how open-loop error in the model translates into closed-loop behaviour. In order to show the advantages of the interactive tool and to relate its results with theory, some illustrative examples (involving two classical problems from the control literature, a high order system and a first-order system with delay) and a case study involving a fluidized bed calciner) are presented.

The paper is organized as follows. First, a brief description of the theoretical background behind the tool is presented in Section 2, with a description of the control-relevant estimation algorithms in Section 3. In Section 4, the functionality of the tool is described. Some illustrative examples and a case study are presented in Section 5. Finally, Section 6 presents the main conclusions and future research work.

2. Theoretical Background

This section is devoted to describing the theoretical background behind the interactive tool. The aspects of the tool that are shared with the *ITSIE* tool (Guzmán et al., 2009a,b, 2011) are summarized, while concepts referring exclusively to control-relevant identification are emphasized.

2.1. Plant to be identified and controlled

The plant to be identified, and subsequently controlled, consists of a discrete-time system sampled at a value specified by the user (default value $T_s = 1$ min) and subject to noise and disturbances according to:

$$\begin{aligned} y(t) &= P_0(q)(u(t) + n_1(t)) + n_2(t) \\ &= P_0(q)\ u(t) + \nu(t) \end{aligned} \tag{1}$$

where:

- $y(t)$ is the measured output signal.
- $u(t)$ is the input signal that is designed by the user.
- $P_0(q)$ is the zero-order-hold-equivalent transfer function for $P_0(s)$ and q is the forward-shift operator.
- n_1 is a stationary white noise signal that allows to evaluate the effects of autocorrelated disturbances in the data.

- n_2 is a second stationary white noise signal that is introduced directly to the output.
- $\nu(t) = P_0(q)n_1(t) + n_2(t)$ is the global output noise signal.

2.2. Input signals for identification

This tool allows to use, among the several input signals which can be used for control-relevant identification, two of the most common ones: Pseudo-Random Binary Sequences (PRBS) and multisine signals. A PRBS is a binary signal generated by using shift register modulo 2 addition. One cycle of a PRBS sequence is determined by the number of registers n_r and the switching time T_{sw} which is an integer multiple of the sampling time T_s. The signal repeats itself after N_sT_{sw} units of time, where $N_s = 2^{n_r} - 1$. Multisine signals are deterministic, periodic signals, represented in the single input case by the equation:

$$u(k) = \rho \sum_{i=1}^{n_s} \sqrt{2\alpha_i}\; cos(\omega_i k T_s + \phi_i) \tag{2}$$
$$\omega_i = 2\pi i/N_sT_s, \quad n_s \leq N_s/2$$

where $u(k)$ is the value of $u(t)$ at discrete time k. The power spectrum of the multisine input is directly specified through the selection of the scaling factor ρ, the Fourier coefficients α_i, the number of harmonics n_s, and the signal length N_s.

In the tool, the input signal can be designed by means of direct parameter specification or by applying time constant-based guidelines. In practice, little is known about the process dynamics at the start of identification testing, and plant operating restrictions will discourage excessively long or very intrusive identification experiments. A guideline that provides a suitable estimate of the frequency band over which excitation is required is:

$$\frac{1}{\beta_s \tau_{\mathrm{dom}}^H} \leq \omega \leq \frac{\alpha_s}{\tau_{\mathrm{dom}}^L} \tag{3}$$

where:

- τ_{dom}^H is high estimate of the dominant time constant.
- τ_{dom}^L is low estimate of the dominant time constant.

- β_s is an integer factor representing the settling time of the process.
- α_s is a factor representing the closed-loop speed of response, written as a multiple of the open-loop response time.

Eq. (3) is used in the tool to specify design variables in both PRBS and multisine signals. Expressions for specifying T_{sw} and n_r based on Eq. (3) are developed in Rivera (1992):

$$T_{sw} \leq \frac{2.8\tau_{\text{dom}}^L}{\alpha_s}, \quad N_s = 2^{n_r} - 1 \geq \frac{2\pi\beta_s\tau_{\text{dom}}^H}{T_{sw}} \tag{4}$$

where n_r and N_s must be integer values. Similarly, Eq. (3) can also be used to specify design variables in multisine inputs, using guidelines found in Rivera et al. (1993):

$$N_s \geq \frac{2\pi\beta_s\tau_{dom}^H}{T_s}, \qquad n_s \geq \frac{N_s T_s \alpha_s}{2\pi\tau_{dom}^L} \tag{5}$$

In both cases, increasing α_s and β_s will widen the frequency band of emphasis in the input signal and increase the resolution of the input signal spectrum.

2.3. Data preprocessing

ITCRI data preprocessing supports mean subtraction, differencing, and substraction of baseline values, whereas mean detrending is applied by default.

2.4. Digital PID controller design

An algorithm for digital PID controller design which is based on the Internal Model Control (IMC) design procedure for discrete-time models (Morari & Zafiriou, 1997), is presented in Prett & García (1988). These PID controllers possess the feature that they have a single adjustable parameter $\delta = \exp(-T_s/\lambda)$ which is directly linked to the closed-loop speed of response λ. In *ITCRI*, second-order plants without integrator are identified according to the tuning rules summarized in Table 1, resulting in Prett & García (1988) controllers of the general form:

$$\Delta u(k) = K_c[e_c(k) - \tau_I e_c(k-1) + \tau_D e_c(k-2)] + \tau_F \Delta u(k-1) \tag{6}$$

where $\Delta u(k)$ is the change in controller output, that is, $\Delta u(k) = u(k) - u(k-1) = (1-q^{-1})$ and $e_c(k) = r(k) - y(k)$ is the setpoint tracking error. The parameters K_c, τ_I, τ_D and τ_F are coefficients of the difference equation in Eq. (6) and are not equivalent to the continuous PID controllers parameters. $\hat{P}(q)$ refers to the estimated plant model and $\tilde{\eta}$ is the complementary sensitivity operator (Morari & Zafiriou, 1997).

Table 1: Prett-García Digital PID Controller Parameters for Low-Order Models. ($0 < \delta < 1$, $\delta = \exp(-T_s/\lambda)$ is an adjustable parameter; T_s is the sampling time).

	$\hat{P}(q)$	$\tilde{\eta}(q)$	KK_c	τ_I	τ_D	τ_F
$0 \leq \beta < 1$	$\frac{K(q-\beta)}{(q^2-\alpha_1 q+\alpha_2)}$	$\frac{1-\delta}{q-\delta}$	$1-\delta$	α_1	α_2	β
$\beta \geq 1$	$\frac{K(q-\beta)}{(q^2-\alpha_1 q+\alpha_2)}$	$\frac{1-\delta}{q-\delta}\frac{q-\beta}{1-\beta q}$	$\frac{1-\delta}{-\beta}$	α_1	α_2	$\frac{\delta(1+\beta)}{\beta} - 1$
$\beta < 0$	$\frac{K(q-\beta)}{(q^2-\alpha_1 q+\alpha_2)}$	$\frac{1-\delta}{q-\delta}\frac{q-\beta}{(1-\beta)q}$	$\frac{1-\delta}{1-\beta}$	α_1	α_2	$\frac{(1-\delta)\beta}{1-\beta}$

2.5. Model structure selection and parameter estimation

The interactive tool allows to work with AutoRegresive model with eXternal input (ARX) models and Output Error (OE) models. Both type of models belong to the general family of prediction-error (PEM) models which corresponds to

$$A'(q)y(t) = \frac{B'(q)}{F'(q)}u(t-nk) + \frac{C'(q)}{D'(q)}e(t) \quad (7)$$

$$y(t) = \hat{P}(q)u(t) + \hat{H}(q)e(t) \quad (8)$$

In Eq. (8) $\hat{H}(q)$ is the noise model and $e(t)$ is the prediction error, usually a white noise disturbance. $A'(q)$, $B'(q)$, $C'(q)$, $D'(q)$ and $F'(q)$ are polynomials in q, where the roots of $A'(q)$ and $F'(q)$ are the poles of the plant whereas the roots of $B'(q)$ are the zeros of the plant. The two PEM models used in *ITCRI* for control-relevant identification are shown in Table 2.

Choosing a suitable model structure is a relevant point in the identification procedure and a prior knowledge about the system to be modelled is a valuable help. The ARX model is the simplest model incorporating an input signal for identification. The estimation of the ARX model is the most efficient of the polynomial estimation methods because it is the result of solving linear regression equations in analytic form. Moreover, the solution

Table 2: Prediction-error model structures evaluated in *ITCRI*.

Method	$\hat{P}(q)$	$\hat{H}(q)$	A'	B'	C'	D'	F'
ARX	$\frac{B'(q)}{A'(q)}q^{-nk}$	$\frac{1}{A'(q)}$	$A'(q)$	$B'(q)$	1	1	1
Output Error	$\frac{B'(q)}{F'(q)}q^{-nk}$	1	1	$B'(q)$	1	1	$F'(q)$

is unique, i.e., the solution always satisfies the global minimum of the loss function. The ARX model therefore is preferable, especially when the model order is high. The disadvantage of the ARX model is that disturbances are part of the system dynamics. The estimated plant model $\hat{P}(q)$ and the noise model $\hat{H}(q)$ have the same set of poles, the roots of the $A'(q)$ polynomial. This coupling can be unrealistic but this disadvantage can be reduced with a good signal-to-noise ratio.

When the disturbance $e(t)$ of the system is not white noise, the coupling between the estimated plant model and the noise model can bias the estimation of the ARX model. In order to minimize the equation error is advisable to set the model order higher than the actual model order, especially when the signal-to-noise ratio is low. However, increasing the model order can change some dynamic characteristics of the model, such as its stability.

On the other hand, the OE model allows to describes the system dynamics separately due to there are not any shared poles between the estimated plant model and the noise model. That is, no parameters are used for modelling the disturbance characteristics, $\hat{H}(q) = 1$, see Table 2. However, it requires nonlinear optimization in the identification procedure and the minimization can get stuck at a false local minimum, especially when the order is high and the signal-to-noise ratio is low. However, this kind of models are better to use when it is not necessary to estimate the noise model and it affects only the output (Juang, 1994; Ljung, 1999).

Control-relevant identification in *ITCRI* is accomplished via prefiltered prediction error estimation,

$$\arg \min_{\hat{P},\hat{H}} \frac{1}{N} \sum_{i=1}^{N} e_F^2(i) \tag{9}$$

where $e_F(t) = L(q)e(t)$ is the prefiltered prediction error, and $L(q)$ is the prefilter. The use of Parseval's Theorem enables a frequency-domain analysis of bias effects in PEM estimation that allows deep insights into the selection of the prefilter and other identification design variables. As the number of observations $N \to \infty$, the least-squares estimation problem denoted by (9) can be written as:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} e_F^2(i) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_{e_F}(\omega) d\omega \tag{10}$$

where $\Phi_{e_F}(\omega)$, the prediction-error power spectrum is

$$\Phi_{e_F}(\omega) = \frac{|L(e^{j\omega})|^2}{|\hat{H}(e^{j\omega})|^2} \left(|P_0(e^{j\omega}) - \hat{P}(e^{j\omega})|^2 \Phi_u(\omega) + |P_0(e^{j\omega})|^2 \sigma_{n_1}^2 + \sigma_{n_2}^2 \right) \tag{11}$$

Eq. (11) helps explain systematic bias effects in identification, which can be readily explored in *ITCRI*. This includes issues relating to the spectral content in the input signal, bias that is introduced (or removed) by the choice of model structure (particularly the noise model), and the associated multi-objective optimization problem resulting from varying magnitudes of the noise variances $\sigma_{n_1}^2$ and $\sigma_{n_2}^2$. Most importantly, Eq. (11) shows that prefiltering acts as a frequency-dependent weight on the goodness-of-fit in prediction-error estimation. How to properly design this prefilter to take into account closed-loop performance requirements is the focus of the ensuing section.

2.6. Control-Relevant Parameter Estimation

The model structures required by the controllers in Table 1 are often times too simple to describe the entire dynamic behaviour of the plant. However, control requirements can narrow the regions of time and frequency over which an adequate model fit is necessary. Therefore, the objective of the control-relevant identification process is to obtain improved models over the frequency band of importance of the control problem. To fulfill this objective, a control-relevant prefilter from the 2-norm closed-loop objective function is developed, which acts as a frequency-dependent weight on the parameter estimation problem and systematically incorporates control requirements in the parameter estimation problem (Rivera et al., 1992).

Control-relevance thus requires that one defines the control problem for which the model is intended. In the interactive tool, the control-relevant estimation is exclusively focused on a plant model $\hat{P}$ to be used for single degree-of-freedom feedback control using the tuning rules given in Prett &

García (1988). The control objective is to minimize the 2-norm of the control error $e_c(t) = (r(t) - y(t))$, that is, the difference between the reference $r(t)$ and the measured output signal $y(t)$:

$$\|e_c\|_2 = \left(\sum_{k=0}^{\infty} e_c^2(k)\right)^{1/2} \tag{12}$$

The feedback controller $C(q)$, that is assumed to be a single degree-of-freedom, is designed on the basis of $\hat{P}(q)$. Resulting in the following nominal response transfer function:

$$\tilde{\eta}(q) = \frac{\hat{P}(q)C(q)}{1 + \hat{P}(q)C(q)} \tag{13}$$

$$\tilde{\epsilon}(q) = (1 - \tilde{\eta}(q)) = \frac{1}{1 + \hat{P}(q)C(q)} \tag{14}$$

where $\tilde{\epsilon}$ is the sensitivity operator of the closed-loop system (Morari & Zafiriou, 1997). When $C(q)$ is implemented on the plant $P_0(q)$, the deterioration in control performance caused by plant/model mismatch is

$$e_c(q) = \frac{\tilde{\epsilon}(q)}{1 + \tilde{\eta}(q)e_m(q)}(r(q) - d(q)) \tag{15}$$

where $e_m(q) = \left(P_0(q) - \hat{P}(q)\right)\hat{P}^{-1}(q)$ is the multiplicative error between the true plant and the calculated model which can describe an uncertain actuator, and $d(q)$ is the disturbance signal. Stability of $C(q)$ on $\hat{P}(q)$ does not ensure stability with regards to $P_0(q)$. A computationally simpler stability requirement used for stability is the small gain theorem:

$$|\tilde{\eta}\left(e^{j\omega}\right) e_m\left(e^{j\omega}\right)| \leq 1 \qquad \forall - \pi \leq \omega \leq \pi \tag{16}$$

When Eq. (16) holds, Eq. (15) can be approximated by a first term Taylor series if $|\tilde{\eta}\left(e^{j\omega}\right) e_m\left(e^{j\omega}\right)| \ll 1$ over the bandwidth defined by $\tilde{\epsilon}(q)(r - d)$:

$$e_c(q) \approx \tilde{\epsilon}(q)\left(1 - \tilde{\eta}(q)e_m(q)\right)(r(q) - d(q)) \tag{17}$$

The control objective function that appears in Eq. (12) can be approximated by substituting Eq. (17) into Eq. (12). Once expressed the approximation in the frequency domain via Parseval's Theorem:

$$\|e_c\|_2 \approx \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} |\tilde{\epsilon}|^2 |1 - \tilde{\eta} e_m|^2 |r - d|^2 d\omega \right)^{1/2} \tag{18}$$

$$\le \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} |\tilde{\epsilon}|^2 |r - d|^2 d\omega \right)^{1/2} + \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} |\tilde{\epsilon}|^2 |\tilde{\eta} e_m|^2 |r - d|^2 d\omega \right)^{1/2} \tag{19}$$

The statement of the control-relevant parameter estimation problem is obtained by minimizing the contribution arising from identification error:

$$\min_{\hat{P}} \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} |\tilde{\epsilon}\left(e^{j\omega}\right)|^2 |\tilde{\eta}\left(e^{j\omega}\right)|^2 \ |r - d|^2 |e_m\left(e^{j\omega}\right)|^2 d\omega \right)^{1/2} \tag{20}$$

Equation (20) is the problem whose solution is solved in the time domain by means of prefiltered ARX and OE estimation. As presented in Rivera et al. (1992), the relationship between Eq. (11) and (20) leads to a general definition for the control-relevant prefilter:

$$L(q) = \hat{H}(q)\hat{P}^{-1}(q)\tilde{\epsilon}(q)\tilde{\eta}(q)(r(q) - d(q)) \tag{21}$$

It is important to highlight the components that form the prefilter $L(q)$:

- The closed-loop transfer functions $\tilde{\eta}(q)$ and $\tilde{\epsilon}(q)$ that define the closed-loop speed of response.
- The setpoint/disturbance direction $(r(q) - d(q))$.
- The identified plant and noise models $\hat{P}(q)$ and $\hat{H}(q)$.

Since $\hat{P}(q)$ is initially unknown, the implementation of the prefilter is inherently iterative. However, in *ITCRI* two algorithms to calculated the prefilter are implemented: (i) a rigorous iterative implementation that is applied to an ARX high-order model and (ii) a simplified non-iterative alternative that is applied directly to the data. These are summarized in the ensuing section.

3. Control-relevant estimation algorithms

The *ITCRI* tool evaluates two alternate procedures for arriving at a control-relevant low-order model conforming to the Prett-García PID tuning rules. In both cases, prefiltering is applied. These are described below:

Direct one-step approach using input/output data.
: ARX-[2 2 1] or OE-[2 2 1] models are obtained directly from the prefiltered input-output data. Where ARX-[2 2 1] refers to an ARX model with two poles, two zeros and one sample delay. Equivalently, OE-[2 2 1] refers to an OE model with two poles, two zeros and one sample delay (MATLAB notation).

Iterative approach from a full-order estimated model.
: A high-order ARX model is obtained first, followed by control-relevant model reduction to an ARX-[2 2 1] or OE-[2 2 1] model structure. The control-relevant model reduction step is accomplished via iterative prefiltered estimation.

The reader is referred to Rivera et al. (1992) where the iterative and direct (single-pass) algorithms are presented with some examples; moreover, a more detailed description of the iterative case appears in Rivera & Gaikwad (1996). A summary of the procedures is enclosed below.

3.1. Single-pass prefilter applied to data

This algorithm requires that the user specify up-front reasonable estimates for the dominant plant time constant and desired closed-loop speed of response, and substitute these into (21). For $\tilde{\eta}$, the following structure is used:

$$\tilde{\eta}(q) = q^{-nk} f(q) \tag{22}$$

where the order of $f(q)$ is dictated by the control design procedure. In *ITCRI*, the second-order filter structure:

$$f(q) = \frac{(1-\delta)^2 q^2}{(q-\delta)^2} \tag{23}$$

is used, where $\delta = \exp(-1.555 T_s/\tau_{cl})$, with τ_{cl} being the anticipated closed-loop time constant. Furthermore, *a priori* knowledge of the plant dominant time constant is used to approximate $\hat{P}$ as:

$$\hat{P}(q) = \frac{q^{-nk+1}}{(q-\alpha)} \tag{24}$$

where $\alpha = e^{-Ts/\tau_{dom}}$ and τ_{dom} is an estimate of the dominant time constant of the system. For OE estimation, $\hat{H} = 1$, while for ARX models, $\hat{H}$ can be approximated with the same dominant time constant guess made for $\hat{P}$:

$$\hat{H}(q) = \frac{q}{(q-\alpha)} \tag{25}$$

3.2. Iterative prefiltering approach

The iterative prefiltering approach is split in two steps. The first step consists of estimating a full-order PEM model that meets classical validation criteria (e.g., white residuals uncorrelated with the input). In *ITCRI*, this full-order model is estimated via high-order ARX estimation, which can be consistently estimated if a persistently exciting input is used (Ljung, 1999). The second step consists of model reduction, in which the impulse between u and y of the full-order model is reduced to a restricted complexity form as summarized in Table 1. The impulse response of the full-order plant can be adequately represented by a FIR model:

$$\begin{aligned} y(t) &= B(q)\,u(t-n_k), \\ B(q) &= b_1 + b_2 q^{-1} + \ldots + b_{n_b} q^{-n_b+1} \end{aligned} \tag{26}$$

where n_b is chosen big enough to capture the transfer function dynamics and n_k refers to the delay estimated in the high-order model. In fact, with a n_b big enough the delay dynamics of the high-order model are included by default. The goal is to approximate Eq. (26) with a low-order ARX model, see Eq. (27). It is important to highlight here, that the noise term, $e(t)$, does not appear in Eq. (26) because the low-order ARX model captures the dynamics of the high-order model without noise involved.

$$A'(q)y(t) = B'(q)u(t-1) + e_r(t) \tag{27}$$

where

$$\begin{aligned} A'(q) &= 1 + a'_1 q^{-1} + \ldots + a'_{n'_a} q^{-n'_a} \\ B'(q) &= b'_1 + b'_2 q^{-1} + \ldots + b'_{n'_b} q^{-n'_b+1} \end{aligned}$$

and n'_a and n'_b are low-numbered integers (1 or 2); in *ITCRI*, $n'_a = 2$ and $n'_b = 2$. In this method, the prediction error $e_r(t)$ represents the model reduction error. The objective minimized in ARX identification is the squared filtered prediction error ($e_f(t) = L(q)e_r(t)$) which for $N \to \infty$ can be written equivalently in the frequency domain as:

$$V = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|B(e^{j\omega}) - \frac{B'(e^{j\omega})}{A'(e^{j\omega})}\right|^2 |A'(e^{j\omega})|^2 |L(e^{j\omega})|^2 \Phi_u(\omega) d\omega \tag{28}$$

where $\Phi_u(\omega)$ represents the power spectra for the input. Because the model reduction step is applied to a noise-free data set (i.e., the full-order model's impulse response), the influence of noise n_1 and n_2 is greatly reduced, in contrast to more general PEM estimation as seen in (11). The definition of the prefilter is obtained by comparing the frequency-domain expressions of the prefiltered ARX problem that appear in Eq. (28) to that of the control-relevant parameter estimation problem in Eq. (20). Since $u(t)$ is an impulse, ($\Phi_u(\omega) = 1 \ \forall \ \omega$) this leads to:

$$L(q) = B'(q)^{-1}\tilde{\epsilon}(q)\tilde{\eta}(q)(r(q) - d(q)) \tag{29}$$

Thus, the iterative method to calculate the prefilter for open-loop stable systems is composed of five steps:

1. *Performance specification.* From Table 1, the user chooses the structure for $\hat{P}$ and $\tilde{\eta}$. The user must only specify the value for the closed-loop time constant λ, which in turn defines the value of the filter adjustable parameter according to $\delta = \exp(-T_s/\lambda)$.
2. *Initialization.* In the first iteration, i.e., $i = 1$, $y(t)$, the finite impulse response, and $u(t)$ are filtered using $L(q)$ defined according to Eq. (29) with:

   $$B'(q) = 1 \qquad \tilde{\eta}(q) = \frac{(1-\delta)}{q-\delta} \qquad r(q) - d(q) = \frac{q}{q-1}$$

   One must now perform ARX estimation using $y_F(t)$ and $u_F(t)$ (the prefiltered output and input) to obtain an initial estimate for the reduced-order model $\hat{P}$.
3. *Iteration.* Use the models $\hat{P}(q)$ and $\hat{H}(q)$ obtained from initialization, $i = 1$, or from the previous iteration, $i - 1$, to update $B'(q)$, $\tilde{\eta}(q)$ and thus, define a new $L(q)$. Proceed then to prefilter $y(t)$ and $u(t)$ and redo ARX estimation.

4. *Termination.* This step determines when convergence has been reached and, therefore, the iteration is finished. For this aim, two criteria are used. If the difference between the objective function, V, in the current iteration, i, and the one in the previous iteration, $i-1$, does not change by a specified amount, that is:

$$|V_i - V_{i-1}| \leq TOL \tag{30}$$

   and the parameters of $\hat{P}$ change by less than a user-defined tolerance, TOL, then terminate, go to Point 5. Otherwise, complete another iteration, i.e., return to Point 3.
5. *Validation.* Once iterations have converged, one must verify that: (i) the estimated model is stable and, (ii) the small gain condition in Eq. (16) has been satisfied. Failure to satisfy these criteria implies that either the closed-loop speed of response must decrease, or the order of the model must increase, in both cases the user must return to Point 1.

It is important to highlight here that, the iterative method as described is the full step by step version, which involves user input at several points. However, in the interactive software tool presented in this paper, the iterative process is automated and totally transparent for the user. The only information that the user has to provide is the desired closed-loop speed response, as the model structure is fixed by the requirement to obtain a Prett-García controller.

3.3. Model validation

ITCRI provides classical methods for validation which include simulation, crossvalidation, residual analysis on the prediction errors (for full-order ARX modeling), and step responses. The percent output variance obtained by each model on the crossvalidation data set is also reported. For control-relevant validation, a valuable metric is to compare the multiplicative error e_m with the prefilter $L(q)$; a good control-relevant model will display low $|e_m|$ over the bandwidth denoted by $L(q)$. Ultimately, the most informative piece of control-relevant model validation is the closed-loop response resulting from the estimated model, which in the *ITCRI* tool is contrasted simultaneously with the open-loop response.

4. Interactive Tool Description

This section is devoted to describe the main features of the interactive tool. However, it is important to mention that interactivity, which is one of the most important features of the tool, cannot be fully appreciated through written text alone. Thus, the reader is cordially invited to download the tool at `http://aer.ual.es/ITCRI/` (see Fig. 1) and personally experience its interactive features. The tool is standalone and does not require a Sysquake license in order to execute.

ITSIE interactive tool user interface demonstrating four cycles of a PRBS input applied to a simulated fifth-order system. The time-constant guidelines from Section 2 are used to define input parameters. An OE-[2 2 1] model is compared with an ARX-[5 7 1] model obtained from exhaustive order selection on a crossvalidation data set.

The plant to be identified can be loaded indicating the transfer function for both the model and the prefilter. This can be done from the menu option *Mode* → *Simulation*. The graphical distribution has been designed according to the most important steps in a control-relevant identification. It is described as follows (see Fig. 1):

- *Input signal definition.* In the main screen, at the top left corner, there is a section called Input signal parameters. Here, the user can choose the type of the input signal (PRBS or multisine) and by means of the checkbox called Guidelines to decide between specifying the input signal directly or following the guidelines given in Guzmán et al. (2009a,b, 2011). For instance, if the PRBS is selected without activating the checkbox Guidelines, a text edit and two sliders appear to modify the number of cycles (N Cycles), the number of registers (N Reg), and the switching time (Tsw). At the bottom left corner, there are two graphics namely Input signal and Power Spectrum or AutoCorrelation depending on the chosen option. The graph above, Input signal, shows one cycle of the input signal, the graph below represents the input signal correlation or the input signal power spectrum depending on the chosen option in the radio buttons at the top right of the graph. The input signal can be modified dragging on both graphics too. Once an input signal has been configured, the final input signal is shown in Full input signal graph, located at the bottom of the central part of the main screen. When the checkbox Filtered Data is activated, the input signal is filtered too.

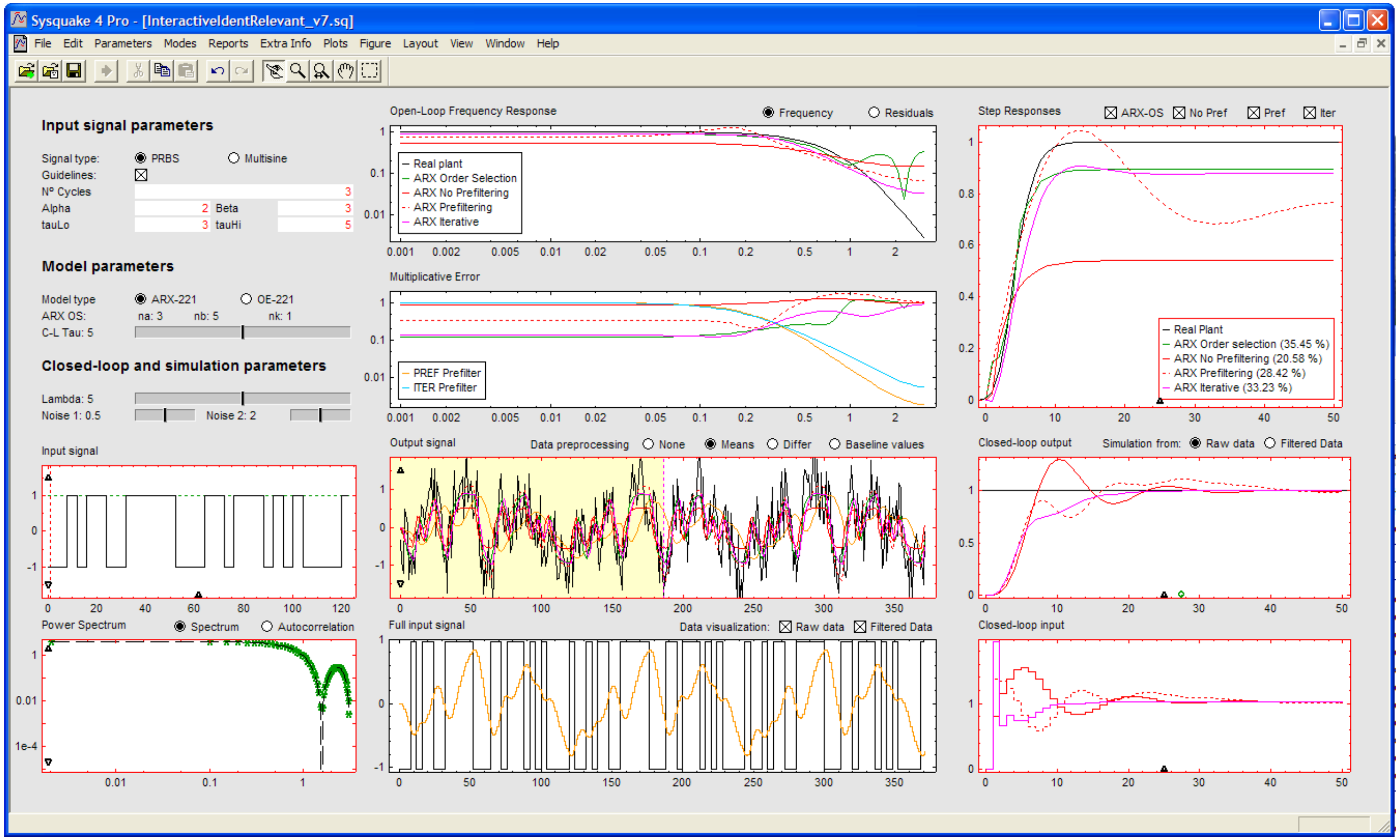

Figure 1: Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for a simulated fifth-order system explained in Section 5. At lower right charts, the results of three low order ARX models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are compared.

- *Process definition.* Below the section Input signal parameters, there is another section called Model parameters, where there are two radio buttons that allow to choose between ARX and OE, i.e. the type of model used for control-relevant identification. The order of the model, see Table 2, is limited to $n_a = 2$, $n_b = 2$ and $nk = 1$ for ARX model, and $n_f = 2$, $n_b = 2$ and $nk = 1$ for OE model. By default, the tool calculates a high-order ARX model, ARX Order selection, to compare with the low-order models calculated through control-relevant identification. Note that, the n_a, n_b and nk values of this high-order model appear also in the section Model parameters. Depending on the type of model used for control-relevant identification, one or two sliders will appear to determine the values of the two parameters needed for single pass prefiltered estimation (Prefiltering): the dominant plant-time constant (O-L Tau), only for the OE model, and the desired closed-loop speed of response (C-L Tau) for both the ARX and the OE models. Once a plant structure is selected, the full input signal applied to the simulated plant with noise is shown in black in the graph called Output signal located at the center of the main screen. This input signal is used to obtain the simulated "real data", which are then used as real process data in the estimation and validation process. In this graph, an interactive magenta vertical dashed line defines the estimation (yellow area) and validation data (white area) sets.

- *Closed-loop specification.* In the section Closed loop and simulation parameters, at the center of the left side of the main screen, the parameter λ for the IMC filter time constant (first-order filter only) which is used by the Prett-Garcia controller (Prett & García, 1988), is specified through a slider called Lambda. Below this slider, other two sliders called Noise 1 and Noise 2 determine the level of noise in the data, n_1, and in the output signal, n_2, respectively.

- *Model validation.* The magenta-colored vertical line of the Output signal graphic is interactively used to define the estimation and validation data sets. The validation data is used for crossvalidation purposes. Model validation results are displayed in other two different graphics: Step Responses and Correlation function of residuals. Note that, this last one only appears if the checkbox Residuals is activated. The Step Responses graph, which is located at the upper right-hand side of the tool, shows the step responses for the following models: (i) ARX Order selection: an ARX high-order model, green solid line, (ii) Non-Prefiltering: depending on the chosen type of model, an ARX or OE low-order model without prefiltering, red or blue solid line respectively, (iii) Prefiltering: depending on the chosen type of model, an ARX or OE low-order model prefiltered with the single-pass prefilter implementation, red or blue dashed line respectively, and (iv) Iterative: an ARX low-order model prefiltered with the iterative prefilter implementation, magenta solid line. Together with the step response of the models, a legend representing its goodness of fit in % is shown. Confidence intervals can be also shown in this graphic activating this option from the Parameters menu. In the Correlation function of residuals graphic, at the left of the Step responses graphic, the same color distribution explained previously is used to represent the results of each model. Moreover, above of this graphic there are two radio buttons that allow to commutate between this graphic and others two called Open-Loop Frequency Response and Multiplicative Error. In the first one, the frequency response of the calculated models is shown. In the second one, the frequency response of the multiplicative error produced by each model is shown together with the frequency response of both the iterative and the single-pass prefilters.

- *Closed-loop response.* At the lower right corner of the tool, there are two graphs that show the closed-loop response of the resulting feedback control system. The upper graph, where the output of the closed-loop is shown, is called Closed-loop output. Moreover, it is possible to simulate disturbances or noise on the closed-loop responses trough a vertical green or black solid line, respectively. The lower graph called Closed-loop input shows the output of the calculated IMC controllers. This graph, which contains digitally sampled signals, is displayed as stairstep-like graph due to the use of zero-order hold for these digital signals. In both graphs, it is important highlight two facts: i) the same color distribution previously explained is used to represent the results for each resulting model and ii) the time-scale of these graphs is independent to the one used in Step Responses graph due to time-constant of both open- and closed-loop responses may be different.

As an interactive tool, there is no one single standard procedure for making use of ITCRI. However, the flowchart shown in Fig. 2 provides useful

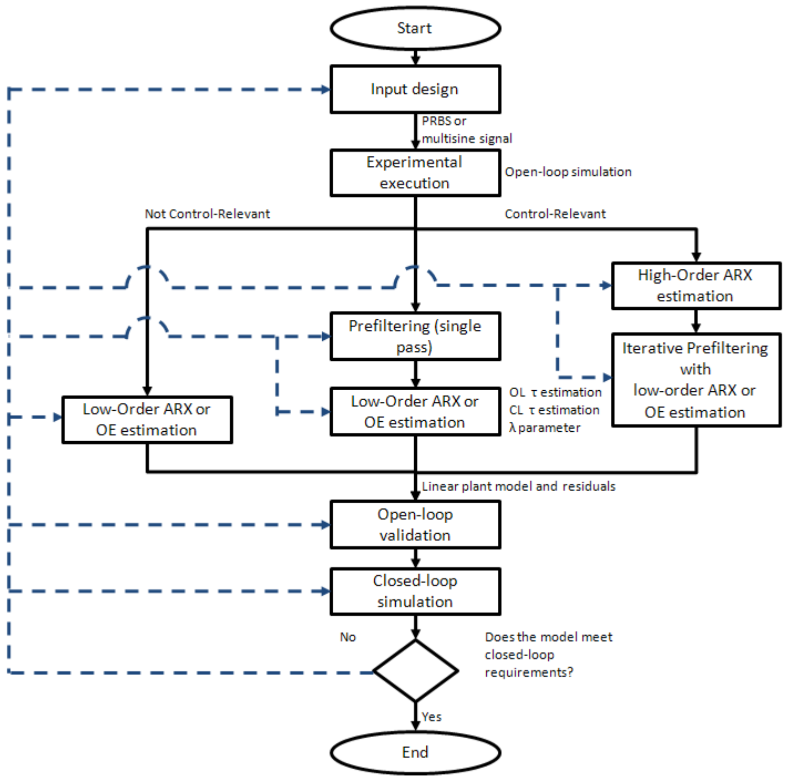

Figure 2: Flow diagram of Interactive Software Tool for Control-Relevant Identification *ITCRI*.

guidance regarding the structure and proper utilization of the tool.

5. Illustrative Examples and Case Study

In this section, two illustrative examples and a case study are developed to demonstrate the functionality and benefits of the interactive tool. The two examples correspond to representative transfer function models from the control literature, while the case study is based on an industrial process model. In the process of working with the tool, the reader should note how a model with a potentially poor fit in the open-loop can result in good closed-loop performance, provided that control-relevant emphasis (through prefiltering or other means) is used to improve the goodness-of-fit in the regions of time and frequency that contribute the most to the closed-loop response.

5.1. Fifth order example

In this example, a simulated fifth-order system is considered. The system is represented by the transfer function:

$$P_0(s) = \frac{1}{(s+1)^5} \tag{31}$$

with a default sample time of $T_s = 1$ min. Results of this comparison are shown in Fig. 1 where an ARX-like model has been chosen. A PRBS input signal is used for identification, with parameters: $m = 3$ (number of cycles), $\alpha_s = 2$, (factor representing the closed-loop speed of response), $\beta_s = 3$ (factor representing the settling time of the process), $\tau^L_{\text{dom}} = 3$ (low estimate of the dominant time constant) and $\tau^H_{\text{dom}} = 5$ (high estimate of the dominant time constant). Moreover, the noise on the output signal, $n_2(t)$ in Eq. (1), is augmented to a value of 2, whereas the noise on the disturbance ($n_1(t)$ in Eq. (1)) is set to 0.5.

A high-order ARX model, with a structure of ARX-[3 5 1], is obtained from this identification signal. Its open-loop response is shown in the Step Responses graph (ARX Order selection), at the upper right-hand side of the tool, together with the response of three ARX low-order models (ARX-[2 2 1]): (i) Non-Prefiltering, an ARX model without prefiltering, (ii) Prefiltering, an ARX low-order model prefiltered with the single-pass prefilter implementation, and (iii) Iterative, an ARX low-order model prefiltered with the iterative prefilter implementation. The validation criteria indicates the poor fit of these models. This is due to the high value of the noise signals n_1 and n_2, since ARX model estimation involves a tradeoff between the fit to the noise model and the fit to the transfer function. Notice that the ARX Order selection model displays the highest goodness of fit in %. Regarding closed-loop parameters, the filter parameter λ of the IMC controllers is set to a value of $\lambda = 5$. The closed-loop time constant estimation used in the control-relevant prefilter (Prefiltering) model is also set to $\tau_{cl} = 5$.

The inputs and outputs of the resulting feedback system are shown in Closed-loop input and Closed-loop output graphs, respectively. Notice the poor performance of the closed-loop system without prefilter (red solid line in the graphs), with a large overshoot of 30 % of the setpoint change magnitude. This fact is due to the high level of the noise in the data, which does not allow a good fit of the open-loop model Non-Prefiltering. From the Step Responses graph, it is possible to note how there is a substantial mismatch in the static gain between the Non-Prefiltering model and the real plant.

In the case of the Prefiltering model, the prefilter is calculated with the single-pass algorithm, PREF Prefilter, and applied directly to the noisy input/output data in order to calculate an ARX model, Prefiltering. The frequency response of both the prefilter and the multiplicative error associated with the ARX model can be observed in the Multiplicative Error graph, where it is possible to note how the prefilter enables the ARX model to achieve the control requirements imposed by specifiying $\tau_{cl} = 5$. Although the Prefiltering model displays a poor fit with an open-loop response that resembles an underdamped system, the closed-loop response from this model is much better than the previous model (resulting from Non-Prefiltering) with a substantial reduction in overshoot as a result of control-relevant modeling.

The third model, Iterative, is calculated from the high-order ARX model (ARX Order selection) through the iterative prefiltering method, ITER Prefilter. Its frequency response, together with the multiplicative error associated with the Iterative model, are shown in the Multiplicative Error graph. With the iterative approach, it is possible to calculate an ARX model that better fulfills control requirements in comparison to the Prefiltering model. The multiplicative error for the Iterative model (magenta line) is the lowest of all control-relevant reduced-order models, matching closely the error of the high-order ARX model (green line) up to a few multiples past the bandwidth of the iterative prefilter (ITER Prefilter, cyan solid line). The closed-loop controlled variable response (magenta solid line) displays no overshoot, very little oscillation, and has the fastest settling time of all reduced-order controllers evaluated.

We note that the ITER Prefilter (cyan solid line) has a lower gain in the high frequencies than the PREF Prefilter (gold solid line). For this reason, both the open-loop response of the Iterative model and the closed-loop performance of its feedback system are superior compared to the other methods.

We conduct an additional evaluation of the tool with the transfer function in Eq. (31), this time using Output Error (OE) model structures. The parameters of the PRBS signal used for identification and the values for $n_1(t)$ and $n_2(t)$ in Eq. (1) remain the same as in the previous ARX test. However, in this case the magnitude of the PRBS signal has been reduced to 0.75, see Input signal graph in Fig. 3. Since a OE model has been chosen, the dominant plant-time constant (O-L Tau) required by the prefilter is set to $\tau_{dom} = 2$. Moreover, the closed-loop time constant τ_{cl} used in the Prefiltering model, as well as the filter parameter λ used by the IMC controllers, are set to 2 as well.

In this case, a high-order ARX model with structure of ARX-[2 6 1] is obtained from this identification signal. Its open-loop response is shown in the Step Responses graph (ARX Order selection), at the upper right-hand side of the tool, together with the response of two OE low-order models (OE-[2 2 1]): (i) Non-Prefiltering, an OE model without prefiltering and (ii) Prefiltering, an OE low-order model prefiltered with the single-pass prefilter implementation. A fourth open-loop response corresponding to an ARX low-order model prefiltered with the iterative prefilter implementation Iterative, is shown. The goodness of fit obtained from these models is worse than the ones obtained from the models of the previous test; this is largely due to lower value for the input signal magnitude, compared to the ARX test.

In the Closed-loop output graph, it is possible to appreciate the poor performance of the closed-loop system without prefilter Non-Prefiltering (blue solid line in the graph), with a large overshoot of 20 % of the setpoint change magnitude. Regarding the prefiltered models two consideration aspects have to be taken into account. On the one hand, the Prefiltering model has the best closed-loop response (blue dashed line in the Closed-loop output graph) than the previous model without prefilter, with a smaller overshoot around 6 % of the setpoint change magnitude. On the other hand, the Iterative model has a similar closed-loop response (magenta line) than the Prefiltering model although a little bit worse. Additionally, it spends more time to reach the setpoint, $t = 12$, and it has an overshoot around 9 % of the setpoint change magnitude.

It is important to highlight that the Iterative model is more consistent with respect to different processes noise, although in this case the Prefiltering model has the best results. The interested reader is cordially invited to test several simulations with the same parameters but with different processes of the noise just doing click into the Output signal graph at the center of the interactive tool. These tests probe that the Iterative model is more consistent than the Prefiltering model since its performance remains equal along the tests. Moreover, when the ratio between the magnitudes of the signal identification and the noise is high, as happens in this case, both prefiltered models are more consistent than the non-prefiltered one, Non-Prefiltering, which closed-loop performance changes significantly with the processes of the noise.

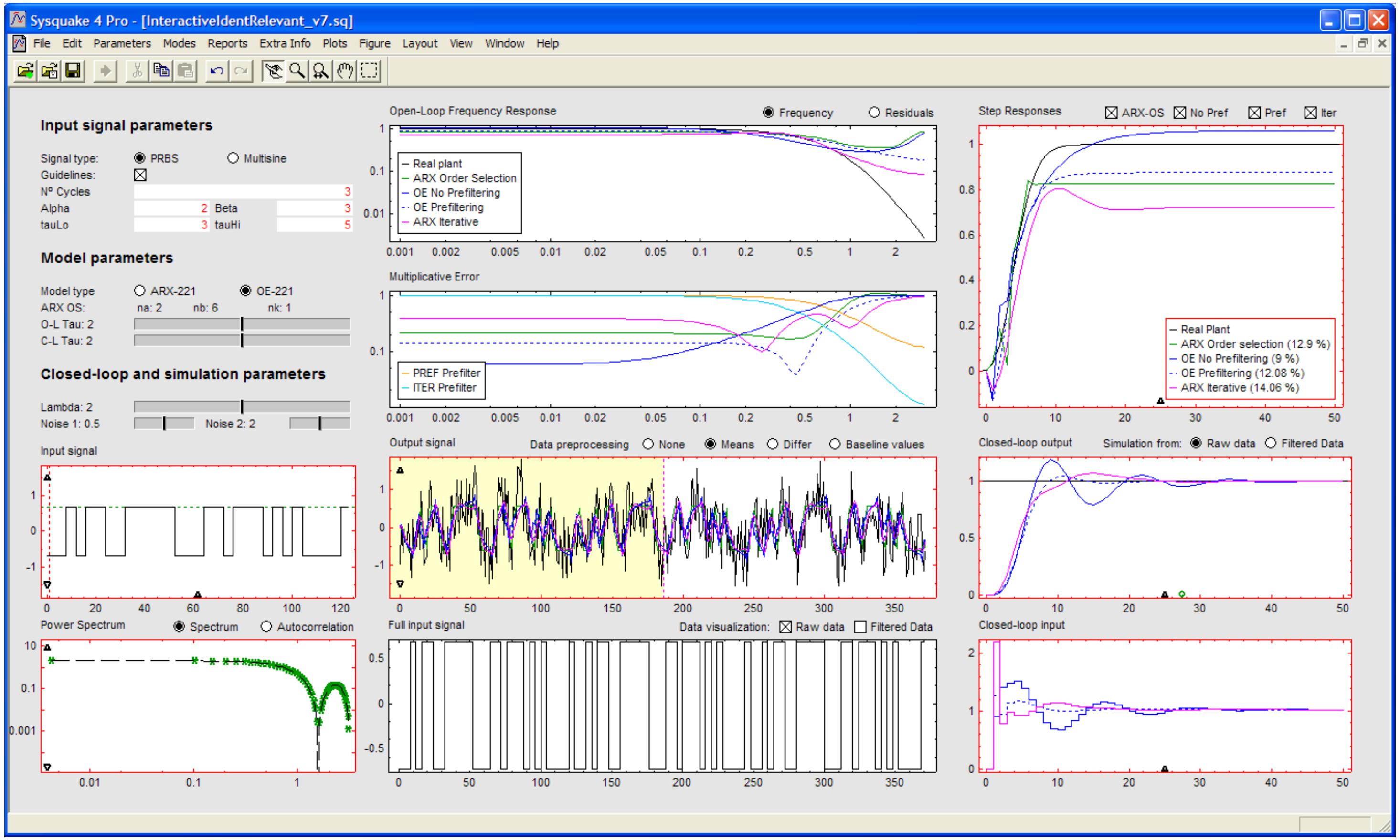

Figure 3: Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for a simulated fifth-order system explained in Section 5. At lower right charts, the results of three low order OE models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are compared.

5.2. First order plant with delay example

This example is meant to show the advantages of control-relevant identification in a system with significant delay:

$$P_0(s) = \frac{1}{(10s+1)} e^{-10s} \simeq \frac{1}{(10s+1)} R_{10,10}(s) \quad (32)$$

which must be reduced into a plant without delay in order to conform to the IMC Prett-García tuning rules in Table 1. In order to work with the tool, the Model Configuration feature is used to introduce a Padé approximation in lieu of a pure delay. $R_{10,10}(s)$ is the tenth order Padé approximation of e^{-10s} (see Eq. (33) and Table 3)

$$R_{10,10}(s) = \frac{g_{10}s^{10} - g_9 s^9 + g_8 s^8 - g_7 s^7 + g_6 s^6 - g_5 s^5 + g_4 s^4 - g_3 s^3 + g_2 s^2 - g_1 s + g_0}{g_{10}s^{10} + g_9 s^9 + g_8 s^8 + g_7 s^7 + g_6 s^6 + g_5 s^5 + g_4 s^4 + g_3 s^3 + g_2 s^2 + g_1 s + g_0} \quad (33)$$

Table 3: Coefficients of $R_{10,10}(s)$.

g_{10}	g_9	g_8	g_7	g_6	g_5	g_4	g_3	g_2	g_1	g_0
1	11	59.4	205.9	504.5	908.1	1211	1176	793.9	335.2	67.04

The default sampling time of $T_s = 1$ min is used, while the Order selection limits in the menu Parameters have been augmented to 15.

A minimum crest factor multisine input has been chosen for the identification signal, its parameter values have been set to: i) $Max_p = 200$, maximum L2p-norm of the multisine signal, ii) $m = 4$ (number of cycles) and iii) $N_s = 228$ (signal length) and iv) $n_s = 40$ (number of harmonics). Furthermore, the input magnitude of this identification signal is ± 1, the values of the noise on the disturbance, $n_1(t)$, and the noise on the output signal, $n_2(t)$, are set to 1 and 0.2, respectively. A high-order ARX model, with a structure of ARX-[2 9 8], is obtained from this identification signal. Its open-loop response is shown in the Step Responses graph (ARX Order selection), at the upper right-hand side of the tool. This model obtains the best goodness of fit of entire set of models presented in the graph, i.e. two OE low-order models (Non-Prefiltering and Prefiltering) and an ARX low-order

Iterative. It is important to highlight that the open-loop responses of the low-order prefiltered models, Prefiltering and Iterative, display non-minimum phase dynamics in which the pure delay is approximated through inverse response. On the contrary, the open-loop response of the Non-Prefiltering model does not include the inverse response and ultimately fails to control the system for the specified control requirements.

In this example, the dominant plant-time constant (O-L Tau) for OE models is set to $\tau_{dom} = 10$. Both the closed-loop time constant estimate τ_{cl} used for the Prefiltering option as well as the filter parameter λ for the IMC controllers are set to 10. The inputs and outputs of the feedback system using those parameters are shown in Closed-loop input and Closed-loop output graphs, respectively. Notice the unstable response of the Non-Prefiltering model's closed-loop response (blue solid line in the graphs), which is produced due to the improperly modeled delay in the reduced model. However, both low-order prefiltered models, Prefiltering and Iterative, which approximate the time delay through a Right-Half Plane zero, show good performance and fulfill desired control requirements.

Examining the problem in the frequency domain through the Multiplicative Error graph is possible to observe how the Non-Prefiltering model has higher multiplicative error gain (blue solid line) in the intermediate frequency range than the Prefiltering and Iterative models (dashed blue line and magenta solid line respectively) which show lower error in the bandwidth of the ITER and PREF prefilters (cyan and gold solid lines, respectively). Because the control-relevant models are better fits in this frequency range of importance to the problem, they are able to generate better closed-loop performance than the unprefiltered OE model.

5.3. Fluidized Bed Calciner Case Study

This case study is meant to demonstrate the tool with a complex model from an application in the chemical process industry: a fluidized bed calciner. A fluidized bed calciner system (which a generic schematic shown in Fig. 5) consists of a bed of heated particles, kept fluidized by air. The bed is kept at high temperature by in-bed combustion of fuel. Solid feed material, as a very fine powder, is sprayed into the bed where it is calcined and sicks to the bed particles leading to their growth. Product particles are withdrawn from the bed at a controlled rate to maintain a constant bed mass. Seed particles, obtained by crushing part of the product, are intermittently added to the bed to maintain the cumulative mass fraction (Ramanathan et al., 1989).

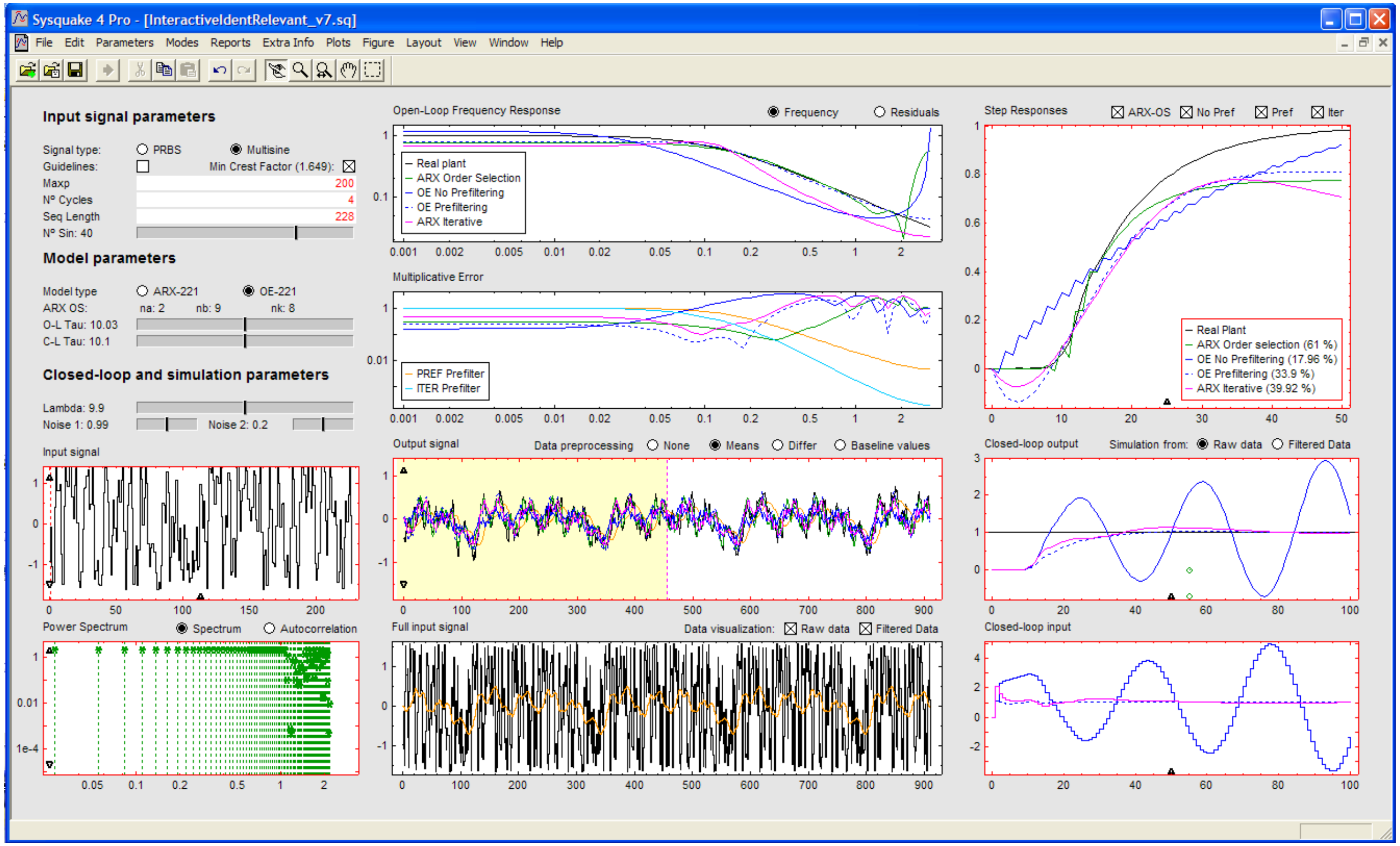

Figure 4: Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for the delay system example. At lower right charts, the results of three low order OE models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are shown. It is possible to note the unstable behaviour of the model without prefiltering.

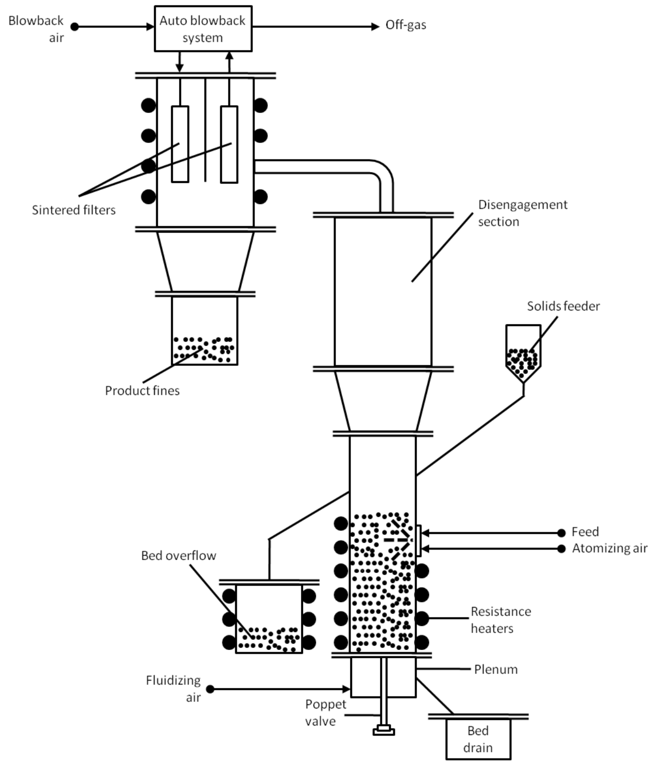

Figure 5: Scheme of a fluidized bed calciner.

Control of particle size distribution is better achieved through control of the cumulative mass fraction above a cut-point size, since it is easily measured.

The transfer function which relates the particle size distribution with the cumulative mass fraction above a cut-point size Z_c, has the following form in continuous time (Ramanathan et al., 1989):

$$P_0(s) = \frac{P_1(s) - P_2(s)e^{-Z_c s}}{Q(s)} \tag{34}$$

where $P_1(s)$, $P_2(s)$ and $Q(s)$ are polynomials in the Laplace variable s, with the order of $P_1(s)$ and $P_2(s)$ being less or equal than the order of $Q(s)$. This irrational transfer function corresponds to a quasirational distributed system (QRDS; (Ramanathan et al., 1989)), and is obtained from a partial differential equation (PDE). Although this kind of transfer function is not widely seen in the control literature, it represents a large class of processes with a wide range of dynamic behaviour (Curtain & Morris, 2009), such as solar collector fields (Álvarez et al., 2007) and tubular heat exchangers (Cohen & Johnston, 1956). A QRDS does not exhibit the simple delayed response characteristic of lumped parameter systems with time delays unless $P_1(s) = 0$. Some QRDS exhibit nonminimum phase behaviour, and can yield poor performance and stability properties in the closed-loop with restricted complexity controllers (e.g., PID controllers).

For this work, $Z_c = 2$ has been used which corresponds to a typical industrial choice (Moran & Wall, 1965) with a steady-state cumulative mass fraction of 85 % above 20 mesh. With this choice of cut-point size, the polynomials in Eq. (34) become:

$$P_0(s) = \frac{((8s^3 + 36s^2 + 60s + 38)/38)e^{-2s} - 1}{s(s+1)(s^2 + 3s + 3)} \tag{35}$$

Results of this system are shown in Fig. 6. A PRBS signal of amplitude ± 1 (see Input signal graph), is used for identification in this example. With the Guidelines option checked, the signal parameters are: $m = 4$, $\alpha_s = 2$, $\beta_s = 3$, $\tau^L_{\text{dom}} = 1$ and $\tau^H_{\text{dom}} = 2$. Moreover, the noise on the disturbance ($n_1(t)$ in Eq. (1)) as well as the noise on the output signal ($n_2(t)$ in Eq. (1)) are set to a value of 0.2. For the closed-loop response, both the filter parameter λ for the IMC controllers and the closed-loop time constant estimate τ_{cl} used in the control-relevant prefilter (Prefiltering) are set to a value of 4.

A high-order ARX model with a structure of ARX-[1 9 4] is obtained from this identification signal with a sample time of $T_s = 0.2$. As in the previous examples, its open-loop response (ARX Order selection), together with the response of three ARX low-order models (ARX-[2 2 1]), Non-Prefiltering, Prefiltering and Iterative), are shown in the Step Responses graph. The ARX Order selection model produces a step response that matches well the peculiar dynamics of the calciner plant, with the highest goodness of fit (54.54 %); the remaining low-order ARX models result in goodness of fit values that range from poor to acceptable. Since low-order ARX model estimation involves a tradeoff between the fit to the noise model and the fit to the transfer function (as indicated by Eq. (11)), the poor model estimate obtained from no prefiltering is to be expected. The use of direct single-pass prefiltering results in improvements, but does not match an iterative approach that relies on first obtaining an adequate high-order model (which essentially removes noise from the data) prior to accomplishing control-relevant model reduction.

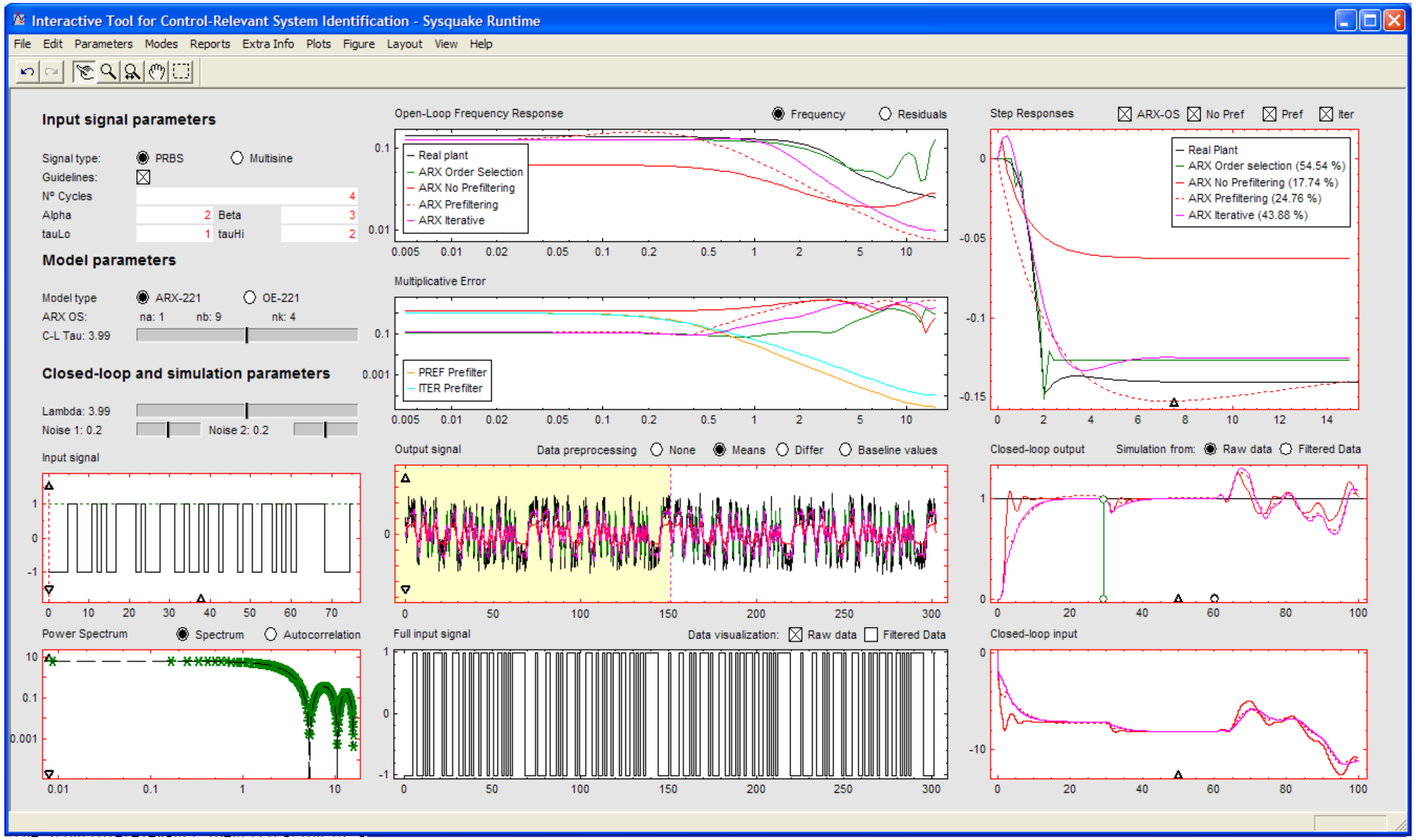

Figure 6: Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for the calciner system example. At right charts, both results, open-loop and closed-loop, of two low-order ARX models (prefiltered with single-pass prefilter and prefiltered with iterative prefilter) are compared.

Closed-loop responses for both setpoint tracking and disturbance rejection are shown in the Closed-loop input and Closed-loop output graphs. The response from the Non-Prefiltering model displays large overshoot and significant oscillations; see the dashed red line in the Closed-loop output graph. The use of the single pass prefiltered model Prefiltering lowers the overshoot substantially, but oscillations still remain. The Iterative model results in overdamped closed-loop behaviour with gentle manipulated variable moves. At the end of the closed-loop response, since time equal to 60 until the end, a white noise is introduced in the closed-loop response in order to give an idea of the sensitivity of these controllers towards noise.

The control adequacy of these various models can be understood by examining the multiplicative error amplitude $|e_m|$ shown in the Multiplicative Error graph. $|e_m|$ is high over all frequencies for the Non-Prefiltering model, while the prefiltered models Prefiltering and Iterative reduce multiplicative error over the bandwidth of the prefilter. The multiplicative error for the Iterative model (magenta line) is the lowest of all the reduced-order models, matching closely the error of the high-order ARX model (green line) up to a few multiples past the bandwidth of the iterative prefilter (ITER Prefilter, cyan solid line). Consequently, the best closed-loop results are obtained from this model.

Finally, it is important to highlight that in the majority of the examples presented throughout this section, the model calculated with the Iterative method displays the best results in terms of both open- and closed-loop responses. Nevertheless, the ARX high-order model from which the Iterative model is calculated can fail when reproducing the most relevant dynamics of the real plant if the identification signal, either multisine or PRBS type, does not satisfy the requirement of persistent excitation with respect to the full-order plant. Under these circumstances, the single pass **Prefiltering** option may still provide acceptable results, as only the lower persistence of excitation requirements for the reduced-order model need to be satisfied.

6. Conclusions

Control-relevant identification involves an interplay between system identification and control design. In this paper, an interactive tool which performs the main stages of control-relevant identification has been developed. The tool provides a diverse series of functional modes which make it possible for control users to apply concepts and become proficient in various aspects of control-relevant identification, with a low learning curve. The tool is freely available from `http://aer.ual.es/ITCRI/`.

The interactive tool allows the user to compare the closed-loop results from different models which have been developed with and without control-relevant prefiltering. Moreover, the user can examine other considerations too; these include the effects of model structure (between ARX and OE models), closed-loop speed-of-response, noise magnitude, experiment length, input signal power, and so forth. The user can discover that some models resulting from identification are not suitable for control, since they have not been designed taking into account control requirements. Several examples have been presented in order to show the benefits of this identification methodology and to prove the functionality and capabilities of the interactive tool. Two of them are based on classical examples which can be found in the control literature; one of them is a high-order transfer function and the other one is a low-order transfer function with delay. These two examples can be useful for teaching this identification methodology to control students or control engineers. The last example is based on a practical application, a fluidized bed calciner, which is described by an irrational transfer function and consequently necessitates some form of model reduction to a structure amenable for control. This case study interactively demonstrates the advantages of control-relevant identification for real-life plants.

Two control-relevant methodologies were examined. In principle, the iterative approach is the most desirable, because the initial step of high-order ARX modeling will reduce the effects of noise in the subsequent model reduction stage. However, under conditions of low noise, the direct single pass approach will yield equivalent results, with less effort. The iterative approach will demand a higher order of persistence of excitation in the identification experiment in support of high-order ARX estimation. Future efforts in developing this interactive tool include an extension to multivariable problems and an evaluation of control-relevant identification under closed-loop identification conditions.

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Appendix A. Nomenclature

α_i	Fourier coefficients
α_s	Input signal parameter which represents the closed-loop speed of response
β_i	Zeros of the plant
$\hat{\beta}_i$	Images of the zeros of the plant
β_s	Input signal parameter which represents the settling time of the process
δ	Adjustable parameter, discrete IMC filter
$\tilde{\epsilon}$	Sensitivity operator
$\tilde{\eta}$	Complementary sensitivity operator
λ	Desired closed-loop speed of response
ν	Global output noise signal
ρ	Magnitude of the input signal for identification
τ_{cl}	Closed-loop speed-of-response
τ_{dom}	Dominant time constant
τ_{dom}^H	High estimate of the dominant time constant
τ_{dom}^L	Low estimate of the dominant time constant
Φ_{eF}	Power spectra of the prediction-error
Φ_u	Power spectra of input
Φ_ν	Power spectra of disturbance
ω	Frequency (radians/time)
$\Vert \cdot \Vert_2$	2-norm objective function
$A'(q)$	Autoregressive polynomial, ARX model structure
$B'(q)$, $F'(q)$	Polynomials describing the model structure for inputs
$C'(q)$, $D'(q)$	Polynomials describing the noise model
C	Feedback controller
d	Disturbance time series
$e(t)$	Prediction error
e_c	Feedback or control error
e_F	Prefiltered prediction error
e_m	Multiplicative error
g_x	Coefficient of the Padé approximat
$\hat{H}(q)$	Noised model
i	Current iteration in the iterative method
$L(q)$	Prefilter
m	Number of cycles of the identification signal

N	Number of observations in the identification
N_s	Signal length
nk	Time delay, prediction error models
$n_1(t)$	An unmeasured disturbance in the data
$n_2(t)$	An unmeasured disturbance in the output signal
n_a	Order of the $A'(q)$ polynomial
n_b	Order of the $B'(q)$ polynomial
n_f	Order of the $F'(q)$ polynomial
n_r	Number of register
n_s	Number of harmonics
$P_0(q)$	True plant model
$\hat{P}(q)$	Estimated plant model
$P_1(s)$	Polynomial in the Laplace variable s in the numerator of $P_0(s)$ without associated delay in the fluidized bed calciner example
$P_2(s)$	Polynomial in the Laplace variable s in the numerator of $P_0(s)$ with associated delay in the fluidized bed calciner example
$Q(s)$	Polynomial in the Laplace variable s which is the denominator of $P_0(s)$ in the fluidized bed calciner example
q	Forward shift operator
q^{-1}	Backward shift operator
r	Reference setpoint
t	Time
T_s	Sampling time
T_{sw}	Switching time
u	Input time series
V	Objective function, least square
y	True plant output variable
Z_c	Cumulative mass fraction above a cut-point size, calciner plant

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Figure captions:

Figure 1. Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for a simulated fifth-order system explained in Section 5. At lower right charts, the results of three low order ARX models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are compared.

Figure 2. Flow diagram of Interactive Software Tool for Control-Relevant Identification ITCRI.

Figure 3. Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for a simulated fifth-order system explained in Section 5. At lower right charts, the results of three low order OE models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are compared.

Figure 4. Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for the delay system example. At lower right charts, the results of three low order OE models (without prefiltering, prefiltered with single-pass prefilter and prefiltered with iterative prefilter) in closed loop are shown. It is possible to note the unstable behaviour of the model without prefiltering.

Figure 5. Scheme of a fluidized bed calciner.

Figure 6. Main screen of Interactive Software Tool for Control-Relevant Identification *ITCRI*, displaying results for the calciner system example. At right charts, both results, open-loop and closed-loop, of two low-order ARX models (prefiltered with single-pass prefilter and prefiltered with iterative prefilter) are compared.