ARTICLE TEMPLATE

Feedforward tuning rules for measurable disturbances with PID control: a tutorial

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ABSTRACT

Feedforward control can be considered as the most well-known control approach to deal with measurable disturbances. It started to be used almost 100 years ago, and since then it is being used in most industrial processes. It is a very simple technique that has been used typically as a complement to PID control, although it can be combined with any feedback controller. The feedforward control design has traditionally been performed assuming perfect cancellation of the disturbance signal, and when this solution was not possible because of non-realizable problems, static design solutions were usually implemented. In the last decade, this issue has been researched and analyzed, and new designs for feedforward control have appeared to face these problems. As a result, a set of new simple tuning rules have been obtained providing considerable improvements in the control system performance. This paper presents a summary of these contributions and a short history of feedforward control. The main objective is to highlight the advantages of this control approach and to provide a tutorial about a group of simple tuning rules with remarkable practical application.

KEYWORDS

disturbances; feedforward control; tuning rules; PID control; regulation problem; tutorial

1. Introduction

Load disturbances, together with model uncertainties, are one of the main reasons for feedback control. Disturbances are exogenous signals that move the controlled process variable away from the desired setpoint. Most industrial process are affected by disturbances, e.g., flow variations in the input of a steam engine, solar radiation changes affecting a solar power plant, road slopes in a car speed control, etc.

A regulation control problem is classically known as the design of a feedback control law to reduce the disturbance effects on the process variable. In that problem, it is assumed that the load disturbance is unmeasurable and therefore no information is available about the disturbance signal. However, in many cases, disturbances can be measured and this information can be incorporated into the feedback loop to contribute to the disturbance rejection. This is the main idea of the feedforward control approach, as its name indicates. That is, the disturbance signal is measured and fed in advance (forward) into the loop before affecting the process output. Thus, feedforward control is proactive against load disturbances, while a feedback control scheme is a reactive since it acts once the process output has been modified by the disturbance signal (Liu, Tian, Xue, Zhang, & Chen, 2019).

Notice that in literature, the term feedforward control is used for two situations, the problem of setpoint tracking and the load disturbance rejection problem, respectively. In this paper, the second case is treated (Guzmán, Hägglund, & Visioli, 2012).

The first use of feedforward control for load disturbances dates from 1925 for drum level control in boilers using the three-element control configuration (Seborg, Edgar, & Mellichamp, 1989). For this application, the feedforward was almost a prerequisite to handle the shrink and swell effect of the process. Around this time, feedforward was also applied to control of distillation columns (Nisenfeld & Miyasak, 1973). Also in this case, the feedforward technique was used as a means to handle that difficult control problem. Feedforward was not treated as a general concept to improve feedback control by feeding information about disturbances forward to the controller.

Feedforward control was not widely used in process control until the 60'ies, when Greg Shinskey wrote the pioneering paper (Shinskey, 1963). The technique was also presented in his book (Shinskey, 1967), with the latest edition (Shinskey, 1996). Since then, the advantages of feedforward control arose and is nowadays part of most basic control courses and textbooks, and implemented in most industrial distributed control systems and particularly as complement to PID control. The increased use of feedforward control is of course enabled by the technical development of control equipment during the last century.

The design of the feedforward compensator to deal with measurable disturbances is in principle very simple and is based in open-loop dynamics of the process. The ideal compensator is formed as the dynamics between the load disturbance and the process output divided by the dynamics between the control signal and the process output, with reversed sign. If this feedforward compensator is used, the effects of the load disturbance are completely eliminated from the process output. This is the basic idea that is taught in most undergraduate and postgraduate courses. Feedforward control is normally introduced during a few teaching hours as a control approach to completely remove the disturbance effect.

However, it is not always possible to implement and apply the ideal feedforward design, since it results from a division between two transfer functions. Thus, the compensator may be non-causal (having a negative delay), non-proper (having more zeros than poles), or unstable. Perfect cancellation of the disturbance is not possible in these cases, and there is a need for tuning rules to determine the feedforward compensator. This situation is rarely studied in control courses, and when non-causal problems appear in industry, only static feedforward compensators are usually implemented to treat these problems.

Although the realization problems, caused by the fact that the compensator includes the inversion of a process transfer function, seem obvious, it is surprising that until 2011 there were no design rules presented for feedforward compensators to account for this situation. Only a few textbooks and research papers mentioned the inversion problems.

In (Shinskey, 1996), a design procedure for a lead–lag compensator was proposed, but without treating the inversion problem. The static gain of the compensator is first determined from pure static models, where the gain is chosen so that a step change in the load is eliminated in steady state, without any action from the feedback controller. The time constants of a lead-lag filter are then determined with the goal to reach an Integral Error (IE), IE = 0, with minimized the Integral Absolute Error (IAE).

Another design procedure was presented in (Seborg et al., 1989), where the feedforward gain is determined in the same way as in (Shinskey, 1996). The difference is the way the time constants in the lead-lag filter are determined. (Seborg et al., 1989) suggested a manual tuning procedure based on repeated step changes of the load. (Coughanowr, 1991) presented a tuning procedure that was based on a training film from Foxboro, produced in 1978. The tuning procedure is made in the same way as in (Shinskey, 1996), but with different time constants of the lead–lag filter.

All procedures mentioned so far were based on an open-loop design, i.e. the feedback controller is not taken into account when the feedforward compensator is designed. This drawback was noticed in (Brosilow & Joseph, 2002), and the suggested solution to the problem was to add another feedforward component to the control structure, so that a load change not only affects the controller output, but also its input. As a result, a new non-interacting control scheme was proposed. It is described in Section 5. (Isaksson, Molander, Modén, Matsko, & Starr, 2008) also pointed out that the feedback controller should be taken into account when designing the feedforward compensator. The paper presented an advanced design procedure based on repeated solutions to least-squares problems.

This lack of tuning rules motivated the starting of a new research line on this topic, and in the last decade several tuning rules for feedforward control have been proposed. In (Guzmán & Hägglund, 2011), the first tuning rules for feedforward compensators were developed to deal with the time dealy inversion problems. The rules allow to design the feedforward parameters directly from the process models and the feedback controller parameters. The proposed design takes the feedback controller into account in the design process, and provides a load disturbance response that has a minumum IAE value without overshoot.

Afterwards, these tuning rules were adapted to the non-interacting control scheme in (Rodríguez, Guzmán, Berenguel, & Hägglund, 2013). The rules are therefore based on the open-loop dynamics and the controller is not taken into account. The new rules allow to tune the response looking for non-overshoots, IAE minimization, or ISE minimization. Moreover, a hybrid tuning rule was proposed where a faster response without oscillations was obtained. In parallel, an analytic solution to design a feedforward which minimizes the ISE was developed in (Hast & Hägglund, 2014), also for the non-interacting control scheme. In that work, the feedforward compensator is low-pass filtered to assure that the controller attenuates high-frequency noise.

In (Veronesi, Guzmán, Visioli, & Hägglund, 2017), tuning rules for the static gain of the feedforward compensators were proposed by considering the closed-loop response and to deal with both overshoot reduction and IAE minimization solutions. Recently, this idea was generalized in (Rodríguez, Aranda-Escolástico, Guzmán, Berenguel, & Hägglund, 2020) for a wide range of processes and types of disturbance signals. The proposed rules are inspired by the IMC approach with the aim of minimising the ISE value. Other tuning rules dealing with realization problems because of non-minimum phase or non-proper transfer functions were presented in (Rodríguez, Guzmán, Berenguel, & Hägglund, 2014) and (Rodríguez, Guzmán, Berenguel, & Normey-Rico, 2014), respectively.

This tutorial paper presents an overview of most of these recent contributions to feedforward control as support to a single feedback control loop based on PID controllers. First, the load disturbance reject problem is introduced using pure feedback control with a PID controller. The limitations of this solution are presented and analyzed. Then, feedforward control is introduced and motivated, highlighting its main advantages. Afterwards, the inversion problems in the design of feedforward compensators will be presented, and solutions to this problem will be described. After that, the main tuning rules will be summarized and compared through several simulation examples. Finally, some challenges and opportunities about future works will be discussed.

2. The load disturbance rejection problem

This section summarizes the general control scheme considered in this paper as well as the controller and process transfer functions.

Figure 1 shows the classical feedback control scheme to deal with the rejection of load disturbances. The diagram consists of the basic feedback loop with feedback controller C, process P_u , and the signals setpoint r, control signal u, and process output y. A load disturbance d influences the feedback loop according to the figure, with transfer function P_d between load d and process output y.



Figure 1. Feedback control scheme to deal with load disturbances.

The classical regulation control problem is considered when the disturbances are not measurable. This case is traditionally represented by adding the load disturbance, d, to the control signal, u. However, notice that this situation is a particular case of the control scheme depicted in Figure 1, by considering that P_d is equal to P_u .

In this paper, we assume that the two process transfer functions are modeled as first-order systems with time delay, i.e.

$$P_{u} = \frac{K_{u}}{1 + sT_{u}}e^{-sL_{u}} \qquad P_{d} = \frac{K_{d}}{1 + sT_{d}}e^{-sL_{d}}$$
(1)

where K_u and K_d are the static gains, T_u and T_d the time constants, and L_u and L_d are the time delays.

There are, of course, processes that are not well described by these simple transfer functions, but for process control applications this structure is mostly good enough, and the structure has become the standard model in process control application. In section 5, other structures including non-minimum phase and integrating dynamics will be discussed. It is assumed that the feedback controller is a PID controller with transfer function

$$C = K \left(1 + \frac{1}{s\tau_i} + s\tau_d \right),\tag{2}$$

where K is the proportional gain, τ_i is the integral time, and τ_d the derivative time, and where $\tau_d = 0$ in case of PI control and $\tau_i = \infty$ for PD control. Note that (2) is only the basic structure. In the real implementation, features like filters, anti-windup, and limitations must be added. More complex controller structures can be used, and the results obtained in this paper can easily be extended to other feedback controller structures as well. For the sake of simplicity, PI control will mainly be used in this paper to show the different results in combination with the feedforward control approaches.

The closed-loop transfer function relating the load disturbance with the process output is given by:

$$G_{y/d} = \frac{P_d}{1 + CP_u} \tag{3}$$

and the closed-loop transfer function for the control effort is:

$$G_{u/d} = -\frac{CP_d}{1+CP_u} \tag{4}$$

Let's consider the Lambda tuning method to design the PI controller in order to simplify the analysis further. Thus, the PI controller parameters are given by:

$$K = \frac{T_u}{K_u(\lambda + L_u)}, \quad \tau_i = T_u \tag{5}$$

where λ is the desired closed-loop time constant.

An interesting analysis can be obtained by using the final value theorem and the initial value theorem. From (3), the transfer function between d and y becomes

$$G_{y/d} = \frac{P_d}{1 + CP_u} = \frac{\frac{K_d}{1 + sT_d}e^{-sL_d}}{1 + K\frac{1 + s\tau_i}{s\tau_i}K_u\frac{1}{1 + sT_u}e^{-sL_u}}$$

Using $\tau_i = T_u$ from the Lambda tuning rule, the expression can be simplified to

$$G_{y/d} = \frac{sT_u K_d e^{-sL_d}}{(1 + sT_d)(sT_u + KK_u e^{-sL_u})}$$

If the load disturbance is a unit step, the process output becomes

$$Y(s) = G_{y/d}D(s) = G_{y/d}\frac{1}{s}$$

Now, the final value theorem can be used to calculate the integral of y after the step

load disturbance, resulting in

$$IE = \int_0^\infty y(t)dt = \lim_{s \to 0} s \cdot \frac{1}{s} Y(s) = \lim_{s \to 0} \frac{T_u K_d e^{-sL_d}}{(1+sT_d)(sT_u + KK_u e^{-sL_u})} = \frac{K_d T_u}{KK_u}$$

Here, for simplicity, it is assumed that the setpoint is zero. Determining controller gain K from the Lambda tuning rule (5) finally gives

$$IE = \frac{K_d T_u K_u (\lambda + L_u)}{T_u K_u} = K_d (\lambda + L_u)$$
(6)

Since the controller is tuned using the Lambda tuning rule, the response is overdamped as long as λ is not too short. In this case, IE is equal to IAE which is one of the most common measures of process control performance. Equation (6) shows that the IE value is proportional to the gain of P_d and that it increases when the delay of the process P_u and the desired time constant of the closed-loop system, λ , increases.

From (4) the transfer function between d and u becomes

$$G_{u/d} = -\frac{CP_d}{1+CP_u} = -\frac{K\frac{1+s\tau_i}{s\tau_i}K_d\frac{1}{1+sT_d}e^{-sL_d}}{1+K\frac{1+s\tau_i}{s\tau_i}K_u\frac{1}{1+sT_u}e^{-sL_u}}$$

Using $\tau_i = T_u$ from the Lambda tuning rule (5), the expression can be simplified to

$$G_{u/d} = -\frac{KK_d(1+sT_u)e^{-sL_d}}{(1+sT_d)(sT_u+KK_ue^{-sL_u})}$$

If the load disturbance is a unit step, the control signal becomes

$$U(s) = G_{u/d}D(s) = G_{u/d}\frac{1}{s}$$

Now, the initial value theorem can be used to calculate the initial derivative of u after the step load disturbance. Whithout loss of generality, it can be assumed that $L_d = 0$.

$$\lim_{t \to 0} \dot{u}(t) = \lim_{s \to \infty} s \cdot sU(s) = \lim_{s \to \infty} -\frac{sKK_d(1 + sT_u)}{(1 + sT_d)(sT_u + KK_u e^{-sL_u})} = -\frac{KK_d}{T_d}$$

Determining controller gain K from the Lambda tuning rule (5) finally gives

$$\lim_{t \to 0} \dot{u}(t) = -\frac{K_d T_u}{K_u T_d (\lambda + L_u)} \tag{7}$$

The initial derivative of the control signal is a measure of the control signal activity. Comparing Equations (6) and (7), it is seen that the factor $(\lambda + L_u)$ appears in both equations, in the numerator in (6) and in the denominator in (7). Thus, there is a tradeoff between performance and control signal activity when it comes to load disturbance rejection using feedback control.

The analysis in this section has demonstrated that load disturbance rejection using feedack control is a trade-off between rejection efficiency and control signal effort. However, one should keep in mind that this trade-off can not be made without taking other aspects like robustness and stability into account. These aspects limit the possibility to reject load disturbances effectively.

Nevertheless, if the load disturbances are measurable, feedforward control can be used to solve this problem and improve the load disturbance rejection further without influencing robustness and stability. This is discussed in the next section.

3. Feedforward control

Feedforward control can be considered as the classical solution to deal with the measurable disturbance rejection problem and treat the limitations of feedback control described above. The feedforward control scheme is presented in Figure 2. A feedforward compensator C_{ff} is connected in open-loop to counteract the effect of the measurable disturbance.



Figure 2. Classical feedforward control scheme to deal with measurable disturbances.

According to this scheme, the transfer function between disturbance d and process output y becomes

$$G_{y/d} = \frac{P_d - P_u C_{ff}}{1 + C P_u} \tag{8}$$

where the feedforward compensator should be calculated in the following way in order to remove the disturbance effect:

$$C_{ff} = \frac{P_d}{P_u} \tag{9}$$

When (9) is realizable and is substituted in (8), the disturbance signal is completely removed before affecting the process output. As P_u and P_d are represented by low-order

models, then (9) is usually given by a classical lead-lag filter:

$$C_{ff} = k_{ff} \frac{sT_z + 1}{sT_p + 1} e^{-sL_{ff}}$$
(10)

Notice that in process control plants, more complex structures are seldom used. Using the transfer functions (1) in (10), the feedforward compensator becomes:

$$C_{ff} = \frac{K_d}{K_u} \frac{sT_u + 1}{sT_d + 1} e^{-s(L_d - L_u)}$$
(11)

4. Inversion problems

From (11), it can be seen that when $L_d < L_u$ the compensator is not realizable and thus perfect cancellation is not be possible. When this situation arises, the performance of the feedforward control approach will be considerably affected and additional analysis is required.

Notice that realization problems can appear because of some other reasons such as non-minimum phase behavior, improper transfer function or integrator poles. However, in this work, the delay inversion problem will be used as a guide case to illustrate all the discussions and motivations around the realization problems. However, in Section 6, solutions for some other realizable problems will be also presented.

To illustrate the delay inversion problem, consider the following example:

$$P_u = \frac{1}{1+2s}e^{-2s} \qquad P_d = \frac{1}{1+s}e^{-s}$$
(12)

The PI controller parameters are set to K = 0.3431 and $\tau_i = 2$ according to the λ method and considering $\lambda = (0.5 + \sqrt{2})L_u = 3.83$ to avoid oscillatory responses (Guzmán, Hägglund, Veronesi, & Visioli, 2015). From (11), the feedforward compensator becomes

$$C_{ff} = \frac{2s+1}{s+1}$$
(13)

where the delays in P_u and P_d have been neglected since they would had provided a non-causal compensator.

The proposed example has been simulated for the open-loop (when the feedback controller is disconnected) and closed-loop cases. Figure 3 shows the simulation results. As observed, the disturbance cannot be completely rejected because of the delay inversion problem. The reason is that the feedforward contribution can only act on the system once the disturbance has affected the process output, that is, after $L_u - L_d = 1$ second. It is interesting to see that the closed-loop case provides the worst response with a significant overshoot (Guzmán & Hägglund, 2011). This is not surprising. The feedforward compensation is designed for the situation when a complete elimination of the disturbance response is possible, i.e. when no feedback action from controller C is made. Now, both the feedforward compensator and the feedback controller reacts on the disturbance, resulting in an overcompensation and therefore a significant overshoot.



Figure 3. Example of delay inversion problem. Open-loop (C = 0) and closed-loop responses are compared for an unitary step disturbance signal at time instant t = 1.

The delay inversion problem can easily be analyzed in the open-loop case, which is given by:

$$G_{ol} = P_d - P_u C_{ff} \tag{14}$$

Figure 4 shows the simulation result for the previous example only for the openloop case. In the upper graphic, the two terms of (14) are simulated separately. As observed, the process output is affected by the load disturbance from the time instant t = 2 seconds through the P_d dynamics, and the feedforward action cannot act on the process output until after L_u seconds, which corresponds to the time instant t = 3seconds. Thus, the feedforward compensation arrives too late, after $L_u - L_d$ seconds, and the disturbance cannot be completely rejected.

The previous analysis opens a very interesting idea regarding feedforward tuning. As observed from Figure 4, the dotted line in the upper graphic is shaped by the feedforward compensator design, which indirectly determines the process output response given in the bottom graphic. In this case, the zero and pole of the compensator are set according to the classical tuning rule given in (11). However, if the feedforward pole, T_p , is tuned to be smaller or larger than T_d , the response of the process output can easily be made faster or slower as shown in Figure 5. Thus, this result shows that when non-realization problems arise, tuning rules for feedforward compensators can highly contribute to improve the load disturbance rejection.



Figure 4. Understanding the delay inversion problem. Analysis of the open-loop response for an unitary step disturbance signal at time instant t = 1.

Therefore, from the previous examples, two important conclusions can be drawn. First, it was noticed that when the disturbance effect cannot be completely rejected by the feedforward compensator, the closed-loop response is highly affected as shown in Figure 3. The feedback and feedforward compensators should be co-designed somehow to account for this problem. A second conclusion is derived from the results presented in Figure 5, where it was demonstrated that by proposing new tuning rules for the feedforward parameters, remarkable improvements of the process output performance can be obtsained.

5. Solutions for the realization problem and tuning rules for feedforward

This section introduces and summarizes different solutions and tuning rules for feedforward control when non-realizable problem appears. First, classical solutions are presented, and afterwards, several tuning rules to improve the control system performance according to classical indexes such as IAE, IE or process output overshoot are described. Most of the proposed tuning rules are presented for the delay inversion problem, which is the case where tuning methods are really necessary. Then, additional rules for the non-minimum and integrating cases are also introduced.



Figure 5. Effect of feedfoward compensator pole, T_p . Open-loop response for an unitary step disturbance signal at time instant t = 1.

5.1. Classical solutions

5.1.1. Static feedforward compensator

A static feedforward compensator is a solution widely used in industry, which is given by:

$$C_{ff} = \frac{K_d}{K_u} \tag{15}$$

The reason to use this simple solution is that drastic improvements can be obtained compared with pure feedback control by using just this simple compensator. Moreover, it can be used to account for any non-realizable problem. However, the resulting performance is also limited because of its simplicity.

5.1.2. Non-interactive control scheme

In (Brosilow & Joseph, 2002), a solution was proposed based on a modification of the classical feedforward control scheme, resulting in the control structure presented in Figure 6. This control approach is known as the non-interacting feedforward control scheme, where the main contribution is the use of a new block, H, which is determined as follows

$$H = P_d - P_u C_{ff},\tag{16}$$

and which allows to remove the disturbance influence from the feedback error.

Thus, this control structure allows to separate the design of feedforward and feedback controllers when the perfect compensator is not realizable. Therefore, it is possible to design the feedforward compensator by just considering the open-loop response from the disturbance avoiding the closed-loop effects observed in Figure 3.

Notice that this scheme can be combined with any feedforward structure, e.g. the static feedforward compensator described above.



Figure 6. Block diagram illustrating the non-interacting feedforward control scheme.

5.2. Tuning rules for delay inversion problem

The delay inversion problem within the feedforward control approach has been widely studied based on the motivation presented in Section 4. As a result, different tuning rules have been proposed to improve the system performance. The tuning rules are obtained for the two control schemes presented above, the classical control scheme and the non-interactive control approach. In the first case, the tuning rules must consider the overshoot effect when feedback and feedfoward actions are combined, such as presented in Figure 3, but with the non-interactive control scheme, open-loop tuning rules can be directly formulated.

5.2.1. Tuning rules for classical control scheme

For the classical control approach, three tuning rules are presented. The guidelines for all of them are given below, but more information ca be found in (Guzmán & Hägglund, 2011; Rodríguez et al., 2020; Veronesi et al., 2017).

5.2.1.1. Rule #1. IAE minimization and overshoot reduction: k_{ff} and T_p . The fist tuning rule consists in setting the feedforward gain and pole in order to reduce the overshoot in the process output and minimize the IAE value. It is summarized as follows (Guzmán & Hägglund, 2011):

(1) Set $T_z = T_u$ and $L_{ff} = \max(0, L_d - L_u)$.

(2) Calculate T_p as:

$$T_p = \begin{cases} T_d & L_u - L_d \le 0\\ T_d - \frac{L_u - L_d}{1.7} & 0 < L_u - L_d < 1.7 T_d\\ 0 & L_u - L_d > 1.7 T_d \end{cases}$$

(3) Calculate the compensator gain k_{ff} as:

$$k_{ff} = \frac{K_d}{K_u} - \frac{K}{\tau_i} IE$$

$$IE = \begin{cases} K_d \ (T_u - T_d + T_p - T_z) & L_d \ge L_u \\ K_d \ (L_u - L_d + T_u - T_d + T_p - T_z) & L_d < L_u \end{cases}$$

(4) End of design.

5.2.1.2. Rule #2. IAE minimization: k_{ff} . In this second tuning rule, only the feedforward compensator gain is calculated for IAE minimization. The rule is given by (Veronesi et al., 2017):

- (1) Set $T_z = T_u$ and $T_p = T_d$.
- (2) Calculate the compensator gain k_{ff} as:

$$k_{ff} = \frac{K_d \left(T_d + L_d\right)}{K_u \left(T_d + L_u\right)}$$

(3) End of design.

5.2.1.3. Rule #3. ISE minimization: k_{ff} . The third tuning rule is focused on minimizing the ISE value by setting the feedforward compensator gain in the following way (Rodríguez et al., 2020):

- (1) Set $T_z = T_u$ and $T_p = T_d$.
- (2) Calculate the compensator gain k_{ff} as:

$$k_{ff} = \frac{K_d}{K_u} \ e^{-\frac{L_u - L_d}{\lambda + T_d}}$$

where λ is the closed-loop time constant given for tuning the PI controller with the Lambda method.

(3) End of design.

5.2.2. Tuning rules for non-interacting control scheme

In the case of the non-interactive control scheme, two tuning rules are presented to design the feedforward compensator for the open-loop case. Detailed information can be found in (Hast & Hägglund, 2014; Rodríguez et al., 2013).

5.2.2.1. Rule #4. Overshoot removal, and IAE/ISE minimization: T_p . This tuning rule tries to generalize the results obtained in (Guzmán & Hägglund, 2011). Three different rules are given for the feedforward compensator time constant, T_p , depending on the desired performance objective: IAE minimization, ISE minimization, or overshoot removal, respectively. The guideline for this tuning rule is given as follows (Rodríguez et al., 2013):

- (1) Set $k_{ff} = K_d/K_u$, $T_z = T_u$ and $L_{ff} = \max(0, L_d L_u)$.
- (2) Calculate $L_b = L_u L_d$.
- (3) Calculate α depending on the desired behaviour:

$$\alpha = \begin{cases} \frac{L_b}{2T_d \left(1 - \sqrt{e^{-(L_b/T_d)}}\right)} & \text{aggressive (ISE minimization)} \\ 1.7 & \text{moderate (IAE minimization)} \\ 4 & \text{conservative (overshoot removal)} \end{cases}$$

(4) Set T_p according to:

$$T_{p} = \begin{cases} T_{d} & L_{b} \leq 0\\ T_{d} - \frac{L_{b}}{\alpha} & 0 < L_{b} < 4T_{d}\\ 0 & L_{b} \geq 4T_{d} \end{cases}$$

- If $T_p = 0$, select a value close to zero to obtain a realizable compensator.
- (5) Set H(s) with Equation (16) for the non-interacting scheme.
- (6) End of design.

5.2.2.2. Rule #5. ISE minimization: T_p and T_z . In this case, both the pole and zero of the feedforward compensator are tuned to minimize the ISE value. With this rule, additional considerations must be added to account for undesirable peaks in the control signal. The different steps are given in the following (Hast & Hägglund, 2014):

- (1) Calculate $L = L_u L_d$.
- (2) If L < 0, then set $L_{ff} = 0$.
- (3) Set $k_{ff} = K_d/K_u$
- (4) Calculate $a = T_u/T_d$ and $b = a(a+1)e^{L/T_d}$.
- (5) Calculate T_p as:

$$T_p = \begin{cases} \frac{3a - 1 - b + (a - 1)\sqrt{1 + 4b}}{b - 2} \tau_d & \text{If } b < 4a^2 - 2a \text{ or } b < a + \sqrt{a} \\ T_p = 0 & \text{Otherwise} \end{cases}$$

(6) Calculate T_z as:

$$T_z = (T_p + T_u) \left(1 - \frac{2\tau_u}{b(T_d + T_p)} \right)$$

(7) If $T_p = 0$, the feedforward compensator is augmented with a second-order low-

pass filter as follows:

$$C_{ff} = k_{ff} \frac{T_z \, s + 1}{(T_f \, s + 1)^2} \, e^{-L_{ff} \, s}$$

where T_f is the time constant of the filter. To obtain a control signal with a peak of Δ value (being $\Delta > 1$), T_f should be calculated with the following expression:

$$T_f = \frac{T_z}{1 + \frac{1}{W_0\left(\frac{e^{-1}}{\Delta - 1}\right)}}$$

where W_0 is the principal branch of the LambertW function (Hast & Hägglund, 2014).

- (8) Set H(s) with Equation (16) for the non-interacting scheme.
- (9) End of design.

5.3. Tuning rules for non-minimum phase and integrating behaviours

Apart from the delay inversion problem, other non-realizable situations can also appear, such as those because of non-minimum phase or integrating dynamics. Notice that other cases, like improper transfer functions can be solved by adding nondominant poles to the feedforward compensator. This section summarizes two tuning rules for right-half plane zeros (Rodríguez, Guzmán, Berenguel, & Hägglund, 2014) and integrating behaviours (Rodríguez et al., 2020), which are proposed for the noninteractive control scheme and the classical control approach, respectively.

5.3.1. Rule #6. Tuning rules for non-minimum phase dynamics

To consider the right-half plane zero case, the transfer function for P_u presented in (1) is modified as follows:

$$P_{u} = \frac{K_{u}(1+s\beta_{u})}{1+sT_{u}}e^{-sL_{u}}$$
(17)

with $\beta_u < 0$.

Moreover, the feedforward control structure is also modified to account for this particular case, resulting in the following transfer function:

$$C_{ff} = k_{ff} \frac{sT_z + 1}{sT_p + 1} \frac{s\beta_{ff} + 1}{sT_{ff} + 1} e^{-sL_{ff}}$$
(18)

where β_{ff} and T_{ff} are new tuning parameters.

The proposed tuning rule is derived for the non-interactive control scheme to reach a desired settling time, t_{st} , or minimize the ISE value. The final guideline is summarized as follows (Rodríguez, Guzmán, Berenguel, & Hägglund, 2014):

(1) Set $k_{ff} = K_d/K_u$, $T_p = T_d$, $T_z = T_u$ and $L_{ff} = \max(0, L_d - L_u)$.

(2) Set
$$\beta_{ff} = T_d$$
.

(3) Set T_{ff} according to the desired specification:

$$T_{ff} = \begin{cases} \frac{t_{st} - L_d}{3} & \text{To reach a settling time } t_{st} \\ \beta_u & \text{To minimize ISE} \end{cases}$$

- (4) Set H(s) with Equation (16) for the non-interacting scheme.
- (5) End of design.

5.3.2. Rule #7. Tuning rules for integrating dynamics

In this case, the transfer function P_u is also modified to include the integrating behavior in the following way:

$$P_u = \frac{K_u}{s(1+sT_u)}e^{-sL_u} \tag{19}$$

The tuning rule is proposed for the classical control scheme and the delay inversion problem is also considered. The rule minimizes the ISE value and the feedforward compensator is calculated according to the following structure:

$$C_{ff} = k_{ff} \frac{s(sT_z + 1)}{(sT_p + 1)^2} e^{-sL_{ff}}$$
(20)

Then, the tuning rule is summarized as follows (Rodríguez et al., 2020):

- (1) Set $T_z = T_u$ and $T_p = T_d/2$.
- (2) Calculate the compensator gain k_{ff} as:

$$k_{ff} = \frac{K_d}{K_u} \ e^{-\frac{L_u - L_d}{2\lambda + T_d}}$$

where λ is the closed-loop time constant given for tuning the PI controller with the Lambda method.

(3) End of design.

6. Examples

This section presents a simulation study to compare and discuss the different tuning rules described above, where the main advantages and disadvantages of them are highlighted. Graphical and numerical results are presented for all the simulations. In the case of the numerical values, IAE, ISE, maximum overshoot and maximum control signal peak are given as metrics to quantify the results. The case of using only feedback control without feedforward compensator is also included in the numerical results to show how much feedforward can contribute in the disturbance rejection problem.

6.1. Delay inversion case

In this first example, the same process transfer functions (12) as used above are considered. The same PI controller tuning is also used, with K = 0.3431 and $\tau_i = 2$.



Figure 7. Simulation example for the delay inversion case with the classical feedforward control scheme and for an unitary step disturbance signal at time instant t = 1. The classical static and lead-lag compensators are compared with the tuning rules #1, #2 and #3.

The simulation results are performed separately for the tuning rules using the classical control approach and those based on the non-interactive control scheme for a fair comparison.

Figure 7 shows the simulation results using the classical control scheme. The tuning rules #1, #2 and #3 are compared together with classical solutions based on static and lead-lag feedforward compensators. Table 1 shows the numerical results for these simulations. As observed, all the tuning rules provide better results than the classical feedforward design, since the large overshoot in the classical design is reduced considerably. Looking at the IAE and IE values, the classical design and rules #2 and #3 obtain about the same values, whereas rule #1 gives a significant reduction of these performance indices. One reason is that rule #1 has reduced time constant T_p in the compensator significantly, whereas rule #2 and #3 have kept the time constant used in the classical design. It is interesting to note that time constant T_z is the same in all tuning rules.

So, the rule #1 provides the best result from a performance point view obtaining the fastest response with only a small overshoot. However, it provides also the largest control signal peak. In this example, rule #2 is the only one that eliminates the overshoot completely, and gives the smallest control signal peak. Tuning rule #3 obtains an interesting tradeoff between performance and control effort. Thus, according to these results, three different options are available for the delay inversion case and using the classical control scheme: conservative response according to rule #2, fast but



Figure 8. Simulation example for the delay inversion case with the non-interacting feedforward control scheme and for an unitary step disturbance signal at time instant t = 1. The classical static and lead-lag compensators are compared with the tuning rules #4 and #5 to minize the ISE value.

aggressive result for rule #1, and balanced response for rule #3.

	Tuning rule					
Variable	FB^{a}	Static FF	Classical Lead-Lag	Rule #1	Rule $\#2$	Rule $\#3$
IAE ISE Overshoot (%) Control peak	5.82 3.55 0.00 1.00	2.87 0.83 19.38 1.32	$1.78 \\ 0.45 \\ 14.45 \\ 1.95$	$0.70 \\ 0.24 \\ 4.81 \\ 4.10$	$1.94 \\ 0.69 \\ 0.00 \\ 1.30$	$1.42 \\ 0.49 \\ 2.73 \\ 1.59$
$ \begin{array}{c} k_{ff} \\ T_z \\ T_p \end{array} $		1.00	$1.00 \\ 2.00 \\ 1.00$	$0.93 \\ 2.00 \\ 0.412$	$0.667 \\ 2.00 \\ 1.00$	$0.813 \\ 2.00 \\ 1.00$

Table 1. Numerical results for the simulations of Figure 7.

^aFB refers to the use of only the feedback controller and without feedforward compensator, which has been included for comparisons.

Now, the same example is used for the non-interactive control scheme, and the tuning rules #4 and #5 that are derived for this scheme are investigated. Figure 8 and Table 2 show the results. Again, classical static and lead-lag designs are included for analysis purposes. In this case, the version of rule #4 that is based on the ISE criteria is used, since this is the criteria also used in rule #5.

In Table 2 it can be seen that all tuning rules keep the gain used in the classical design, and that rule #5 is the only rule that adjusts time consant T_z . Time constant T_p is chosen differently in all rules. Tuning rule #5 provides a T_p value equal to zero,

which results in a non-realizable feedforward compensator. Thus, such as suggested above in the guideline for this tuning rule, a second-order low-pass filter must be added. In this case, the filter time constant was tuned as $T_f = 0.064$ in order to reach a control signal peak similar to the one obtained with the rule #4.

Both tuning rules #4 and #5 considerably improve the performance with respect to classical solutions. The two rules have about the same IAE and ISE values, and these values are significantly smaller than those given by the classical design. However, strong control signal peaks are required to reach those improvements, and rule #4 gives a significant overshoot. These problems can be solved by re-tuning the filter time constant in rule #5 for a smaller control signal peak, or using the other tuning options available for rule #4.



Figure 9. Simulation example for the delay inversion case with the non-interacting feedforward control scheme and for an unitary step disturbance signal at time instant t = 1. The classical lead-lag compensator is compared with the tuning rule #4 for tuning options: Overshoot removal, ISE minimization and IAE minimization.

Figure 9 shows a comparison between the classical lead-lag compensator with the tuning rule #4 for its three tuning options: overshoot removal, ISE minimization and IAE minimization. Numerical results are given in Table 2. Now, a tradeoff between performance and control effort can easily be selected ranging from the most conservative result with the overshoot removal option to the most aggressive response for the ISE minimization result. All three rules have the same gain and T_z as the classical design. The only parameter that differs between the tuning rules is time constant T_p .

Tables 1 and 2 show that the use of feedforward improves the load rejection sig-

Table 2. Numerical results for simulations of Figure 8 and 9.

	Tuning rule						
Variable	FB ^a	Static FF	Classical Lead-Lag	Rule #4 (IAE)	Rule #4 (ISE)	Rule #4 (OS)	Rule $\#5$
IAE	5.82	1.99	1.00	0.62	0.66	0.76	0.63
ISE	3.55	0.75	0.37	0.23	0.22	0.30	0.20
Overshoot (%)	0.00	0.00	0.00	5.77	14.5	0.00	2.2
Control peak	1.00	1.00	2.00	4.86	9.40	2.67	8.61
k _{f f}	_	1.00	1.00	1.00	1.00	1.00	1.00
T_z	-	_	2.00	2.00	2.00	2.00	1.51
T_p	-	_	1.00	0.412	0.213	0.75	0.00

 $^{\rm a}{\rm FB}$ refers to the use of only the feedback controller and without feedforward compensator, which has been included for comparisons.

nificantly compared with feedback only. The IAE value can be decreased by almost an order of magnitude. The tables also show that a significant improvement can be obtained using the new tuning rules compared to the classical tuning rules. Comparing the two tables, it can also be observed that better performance results can be obtained using the non-interactive control scheme, but also that larger control signal peaks are required to obtain these results. If a criteria for a tradeoff between performance and control effort is considered, rules #1 and #4 (OS) are probably the ones with best results for the classical and non-interactive control schemes, respectively.

6.2. Non-minimum phase case

For the non-minimum phase example, the transfer function P_u for the previous example was modified to include a positive zero as follows:

$$P_u = \frac{(-0.5s+1)}{2s+1}e^{-2s} \tag{21}$$

and the same P_d transfer function was used.

In this case, the Lambda method was used by considering $L_u \simeq L_u + \beta$, where β is the zero constant, $\beta = 0.5$ in this example. Thus, the PI controllers parameters become K = 0.2929 and $\tau_i = 2$.

Figure 10 and Table 3 show the simulation results for this example using the noninteractive control scheme. Classical static and lead-lag compensator are considered together with the tuning rule #6. The rule is used for the two options described above, to reach a desired settling time ($t_{st} = 3$ seconds in this example) and to minimize the ISE value.

Both tuning options improve the results of the classical solutions in terms of the IAE and ISE values, but with larger control signal peaks. There are no overshoots in the responses, but because of the inverse response bahaviour, there will be an increase in the load disturbance peak, and this peak increases when the feedforward gains increase. However, notice that the tuning option for fixing the settling time can be used to find a tradeoff between performance and control effort.



Figure 10. Simulation example for the non-minimum phase case with the non-interacting feedforward control scheme and for an unitary step disturbance signal at time instant t = 1. The classical static and lead-lag compensators are compared with the tuning rule #6 for the tuning options: settling time and ISE minimization.

Table 3. Numerical results for simulations of Figure 10.

	Tuning rule				
Variable	FB^{a}	Static FF	Classical Lead-Lag	Rule #6 ST	Rule #6 ISE
IAE ISE Overshoot (%) Control peak	$6.62 \\ 4.39 \\ 0.00 \\ 1.00$	$2.50 \\ 1.19 \\ 0.00 \\ 1.00$	$1.51 \\ 0.83 \\ 0.00 \\ 2.00$	$ \begin{array}{r} 1.20 \\ 0.78 \\ 0.00 \\ 3.00 \\ \end{array} $	$ 1.10 \\ 0.80 \\ 1.70 \\ 4.00 $

 $^{\rm a}{\rm FB}$ refers to the use of only the feedback controller and without feedforward compensator, which has been included for comparisons.

6.3. Integrating case

Finally, a process with integrating dynamics is analyzed. So, the transfer function P_u in Equation (12) is modified to add an integration in the following way:

$$P_u = \frac{1}{s(2s+1)}e^{-2s} \tag{22}$$

and transfer function P_d remains the same.

In this case, a PD controller was used following the lambda method approach, resulting in K = 0.1716 and $\tau_d = 2$. In this case, the classical static feedforward compensator cannot be evaluated since it would provide a stead-state error. The simulation results are provided in Figure 11 and Table 4, where the classical lead-lag compensator is compared with the tuning rule #7 using the classical control scheme. Notice that the classical lead-lag compensator would not be realizable, and it was calculated adding an extra pole of $T_d/20$.

As observed, the rule #7 slightly improves the performance of the classical control scheme but with a much smaller control signal peak. Thus, this rule provides a very promising tradeoff between performance and control effort.



Figure 11. Simulation example for the integrating case with the classical feedforward control scheme and for an unitary step disturbance signal at time instant t = 1. The classical lead-lag compensator is compared with the tuning rule #7.

	Tuning rule			
Variable	FB^{a}	Classical Lead-Lag	Rule $\#7$	
IAE	5.83	0.90	0.85	
ISE	3.57	0.25	0.30	
Overshoot (%)	0.00	15.15	8.36	
Control peak	1.56	39.98	7.13	

Table 4. Numerical results for simulations of Figure 11.

^aFB refers to the use of only the feedback controller and without feedforward compensator, which has been included for comparisons.

7. Challenges and opportunities

The results presented in this paper have highlighted the relevance and the importance of feedforward control for the load disturbance rejection problem. It was shown to be a very powerful complement to feedback control, and even when non-realization problems arise, simple tuning rules have been derived that improve the results of classical design methods considerably.

The presented design approaches are results of the research on this topic during the last decade, becoming a very interesting research field with remarkable practical capa-

bilities and where many challenges and opportunities are still open for investigation. Some of these are:

- Robustness. Most of the tuning rules summarized above are based on the process parameters and are derived for the nominal case. Thus, robustness analysis and robust design methodologies are required to account for this problem. In the literature, some preliminary works are available for the classical solutions (Adam & Marchetti, 2004; Hoyo, Moreno, Guzmán, & Hägglund, 2018; Rodríguez, Normey-Rico, Guzmán, & Berenguel, 2016; Vilanova, Arrieta, & Ponsa, 2009), but tuning rules with robust capabilities are still a challenging problem.
- Nonlinear or Adaptive solutions. All industrial processes are nonlinear systems. Typically, the processes are controlled around an operating point using linear models and linear control approaches. However, there can be situations where the system moves among different operating points and thus the proposed control algorithm should account for that. Therefore, nonlinear or adaptive solutions for the feedforward control problem is another topic with research opportunities.
- Extensions to multivariable systems. Interactions in multivariable processes can be considered as disturbances among the different control loops. In fact, feedforward-based solutions are commonly used in multivariable control as decoupling approaches. Thus, the extension or application of the proposed feedforward tuning rules to the decoupling control problem is another promising research topic.
- Detection and evaluation. To use feedforward for load disturbance rejection, relevant and measurable load disturbances useful for feedforward must be determined. This is often not a trivial task. Therefore, an interesting research topic is to derive methods for finding such signals in multivariable control systems. Related to this, it is also interesting to derive indices that quantify the improvements that can be obtained by using feedforward from different disturbance signals. Results on this are are given in (Guzmán et al., 2015).
- Practical implementation. Only a few of the new feedforward tuning rules have been evaluated in industrial facilities (García-Mañas, Guzmán, Rodríguez, Berenguel, & Hägglund, 2021; Montoya-Ríos, García-Mañas, Guzmán, & Rodríguez, 2020). So, the evaluation and experimental analysis of these solutions is another challenging topic to be considered.

8. Conclusions

Feedforward control started to appear in process control around hundred years ago, but in these early implementations the technique was not treated as a general concept to improve load disturbance rejection. This view emerged in the sixties, and since then the use of the feedforward technique has grown and is now a common component in process control instrumentation solutions.

There are realization problems associated with feedforward control, since it is desired to use process transfer function inverses in the compensators. In industry, this problem has traditionally been avoided by restricting the feedforward compensator to just static gains. However, significant improvements can be obtained by using more advanced compensators, e.g. lead-lag filters, but to obtain these improvements, tuning rules for the feedforward compensators that take the inversion problems into account are needed. It is surprising that such rules started to appear as late as ten years ago. This paper has provided a survey of the most common tuning rules that have appeared in the last decade, and a comparison of these methods has been made.

Two feedforward structures have been treated, the classical one and the noninteracting one, where the non-interacting structure enables a separation between the feedback and the feedforward actions. Seven tuning rules with different approaches and design goals have been presented and compared. They all have their advantages and disadvantages, and are useful for different purposes. It is shown that they can provide a significant improvement of the load disturbance rejection in terms of decreased IAE and ISE values. As for the pure feedback control case, there is a trade-off between performance and control signal effort, and the different tuning rules provide different solutions to this balance. A great advantage with feedforward control is that stability and robustness issues are not influenced by feedforward, and are therefore not part of the trade-off.

Feedforward control is an old control technique, but it is a young research field. It is our hope that this tutorial paper will provide an inspiration to further research in the area.

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