Advances in Feedforward Control

José Luis Guzmán Sánchez

Department of Informatics
Engineering and Systems Area
University of Almería (Spain)

joseluis.guzman@ual.es

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Outline

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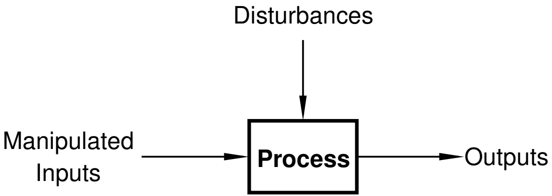
What are load disturbances?

- Typically low frequency input signals which affect the output of processes but that cannot be manipulated



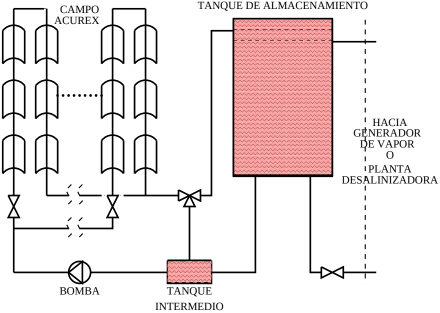
Introduction

- Most industrial processes are subject to disturbances and the nature and origin of disturbances may vary depending on the process and the operational environments.
- Effective disturbance effect reduction **is a key topic in process control**. In fact, disturbances together with process uncertainty, are one of the reasons for feedback control.

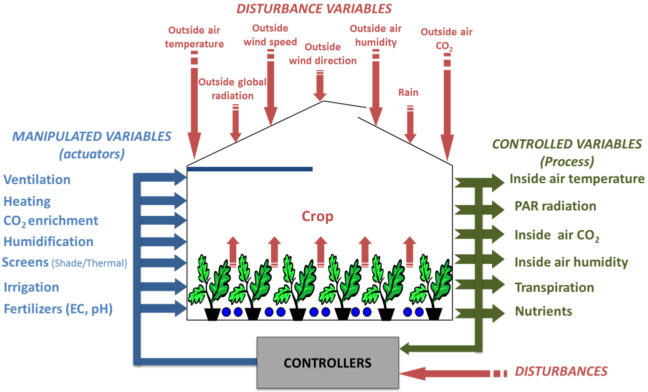
Introduction

Real plants at the Automatic Control research group in Almería

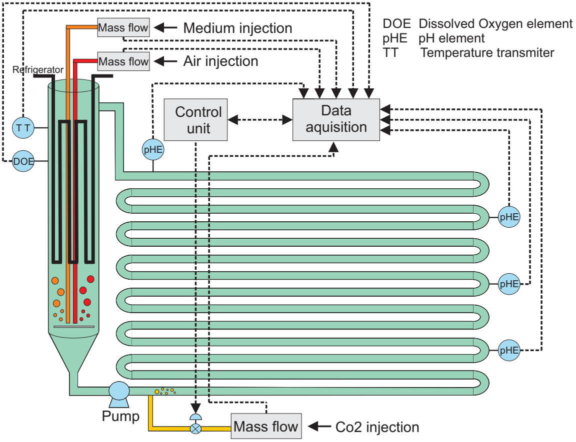
Energy production with solar plants



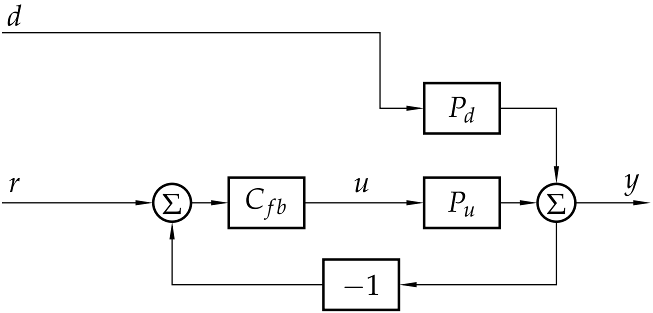
Crop production in greenhouses



Photobioreactors to microalgae production



Motivation: feedback controller



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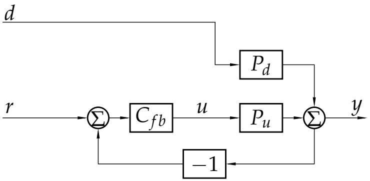
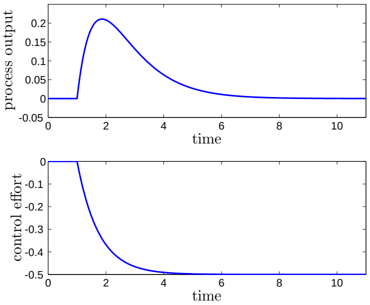
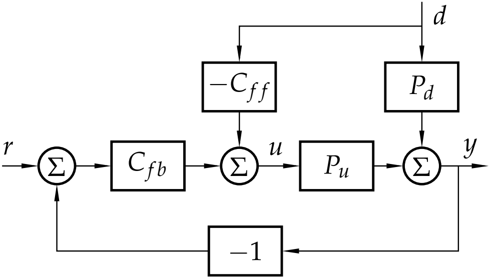




No reaction until there are discrepancies!

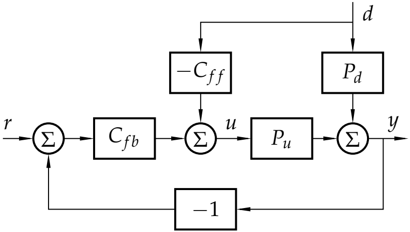
Motivation: feedforward compensator

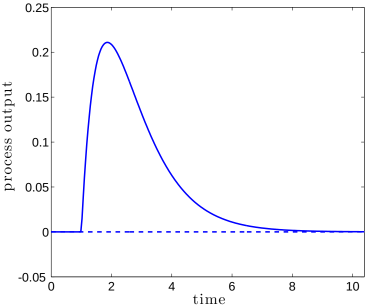


$$C_{ff} = \frac{P_d}{P_u}$$

$$Y = (P_d - P_u C_{ff})D$$

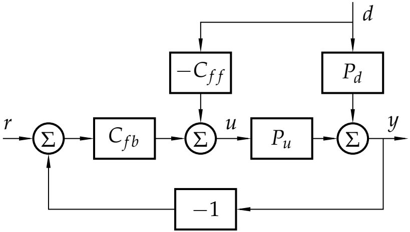
Motivation: feedforward compensator

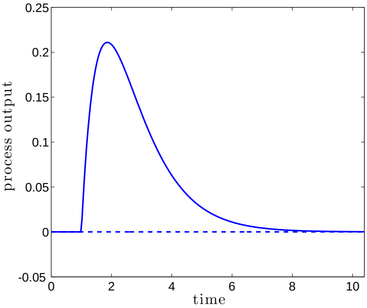




Ideal compensation: $C_{ff} = \dfrac{P_d}{P_u} = P_d\, P_u^{-1}$

Motivation: feedforward compensator





Ideal compensation: $C_{ff} = \dfrac{P_d}{P_u} = P_d \boxed{P_u^{-1}}$

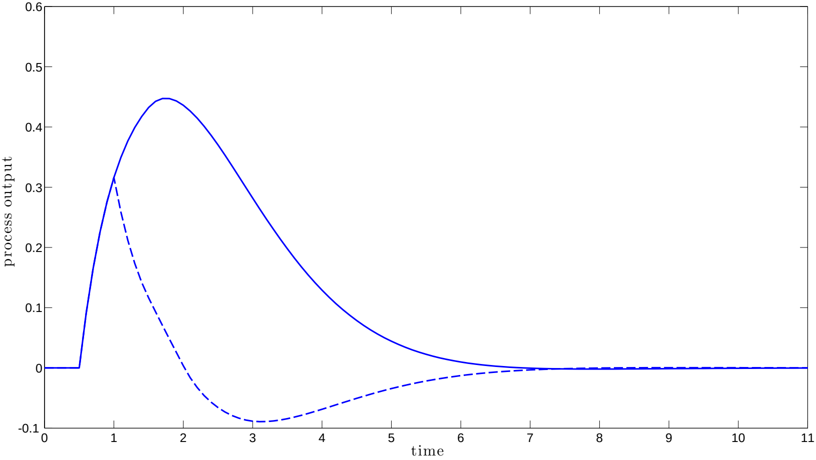
Perfect compensation is seldom realizable:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.
- Improper transfer functions.

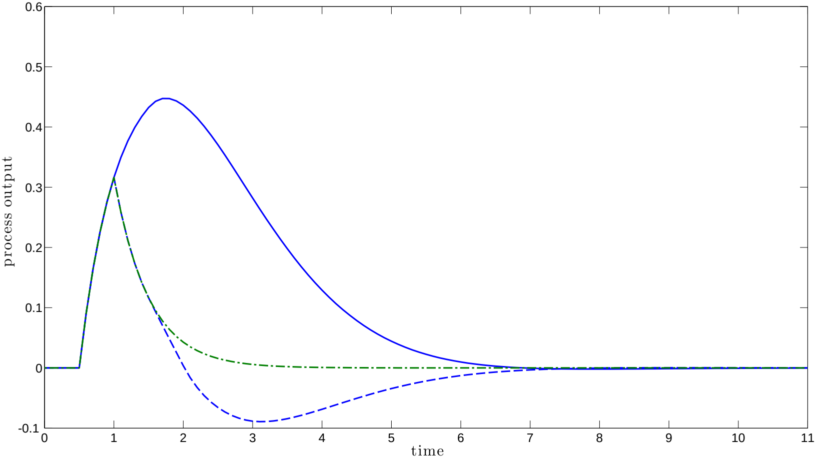
Classical solution

Ignore the non-realizable part of the compensator and implement the realizable one. In practice, static gain feedfoward compensators are quite common.

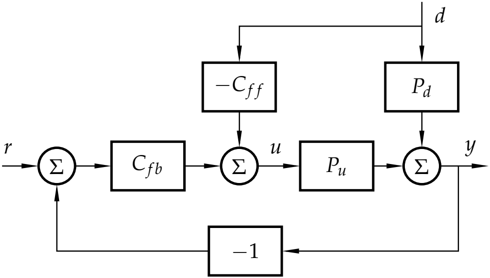
Motivation: non-ideal feedforward compensator



Motivation: non-ideal feedforward compensator



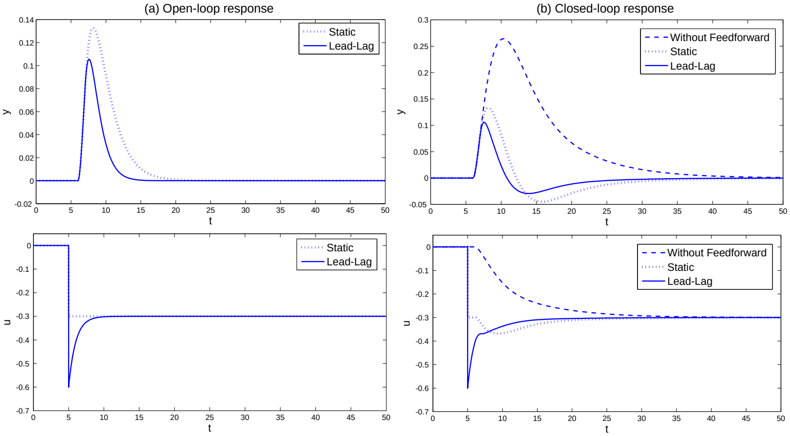
Motivation: residual term



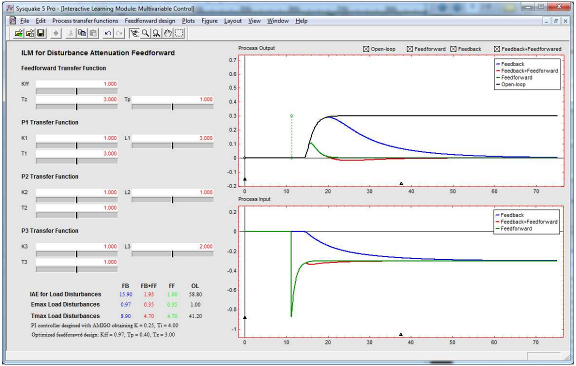
$$C_{ff} = \frac{P_d}{P_u}$$

$$Y = (P_d - P_u C_{ff})D$$

Motivation


(a) Open-loop response
Static
Lead-Lag
y
t
u
(b) Closed-loop response
Without Feedforward
Static
Lead-Lag
y
t
u

Motivation



http://aer.ual.es/ilm/ http://fichas-interactivas.pearson.es/

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - C_{ff}P_u}{1+C_{fb}P_u}d$$

Other design strategies are required!

Motivation

An interaction between feedforward and feedback controllers arises

$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - \boxed{C_{ff}}P_u}{1+\boxed{C_{fb}}P_u}d$$

Other design strategies are required!

Motivation

Surprisingly there are very few studies in literature (we starting the project in 2010):

- D. Seborg, T. Edgar, D. Mellichamp, Process Dynamics and Control, Wiley, New York, 1989.
- F. G. Shinskey, Process Control Systems. Application Design Adjustment, McGraw-Hill, New York, 1996.
- C. Brosilow, B. Joseph, Techniques of Model-Based Control, Prentice-Hall, New Jersey, 2002.
- A. Isaksson, M. Molander, P. Modn, T. Matsko, K. Starr, Low-Order Feedforward Design Optimizing the Closed-Loop Response, Preprints, Control Systems, 2008, Vancouver, Canada.

Objectives

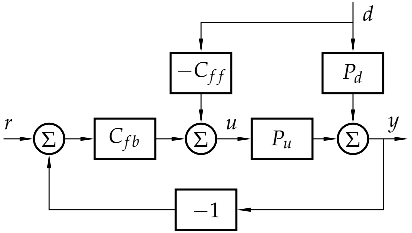
1. Study and development of a control methodology to improve disturbance compensation in industrial processes
2. Definition of nominal simple optimal tuning rules for designing feedforward compensators
3. Development of a robust methodology to cope with both reference tracking and disturbance rejection, using feedforward control structures
4. Integration of the obtained nominal and robust feedforward tuning rules into a general dead-time compensation solution
5. Propose performance indices for feedforward control

Outline

Feedforward control problem

- Feedforward control is an old topic in process control. In fact, its first application dates from 1925, where a feedforward compensator was used for drum level control of tanks connected in series.
- Many of the other early applications dealt with control of distillation columns.
- Since then, feedforward control has become a fundamental control technique for the compensation of measurable disturbances.
- *Nowadays, this mechanism is implemented in most distributed control systems to improve the control performance*.

The idea behind feedforward control from disturbances is to supply control actions before the disturbance affects the process output:



$$C_{ff} = \frac{P_d}{P_u}$$

In industry, PID control is commonly used as feedback controller and four structures of the feedforward compensator are widely considered:

$$C_{fb} = \kappa_{fb}\left(1 + \frac{1}{s\tau_i} + s\tau_d\right)$$

Static: $C_{ff} = \kappa_{ff}$

Static with delay: $C_{ff} = \kappa_{ff}e^{-sL_{ff}}$

Lead-lag: $C_{ff} = \kappa_{ff}\dfrac{1+s\beta_{ff}}{1+s\tau_{ff}}$

Lead-lag with delay: $C_{ff} = \kappa_{ff}\dfrac{1+s\beta_{ff}}{1+s\tau_{ff}}e^{-sL_{ff}}$

Then, if we consider that process transfer functions are modeled as first-order systems with time delay, i.e.

$$P_u = \frac{\kappa_u}{1+\tau_u} e^{-s\lambda_u}, \quad P_d = \frac{\kappa_d}{1+s\tau_d} e^{-s\lambda_d}$$

The following feedforward compensator can be considered:

$$\begin{array}{ll} \text{Static:} & C_{ff} = \dfrac{\kappa_d}{\kappa_u} \\ \text{Static with delay:} & C_{ff} = \dfrac{\kappa_d}{\kappa_u} e^{-s(\lambda_d-\lambda_u)} \\ \text{Lead-lag:} & C_{ff} = \dfrac{\kappa_d}{\kappa_u} \dfrac{1+s\tau_u}{1+s\tau_d} \\ \text{Lead-lag with delay:} & C_{ff} = \dfrac{\kappa_d}{\kappa_u} \dfrac{1+s\tau_u}{1+s\tau_d} e^{-s(\lambda_d-\lambda_u)} \end{array}$$

Feedforward control problem

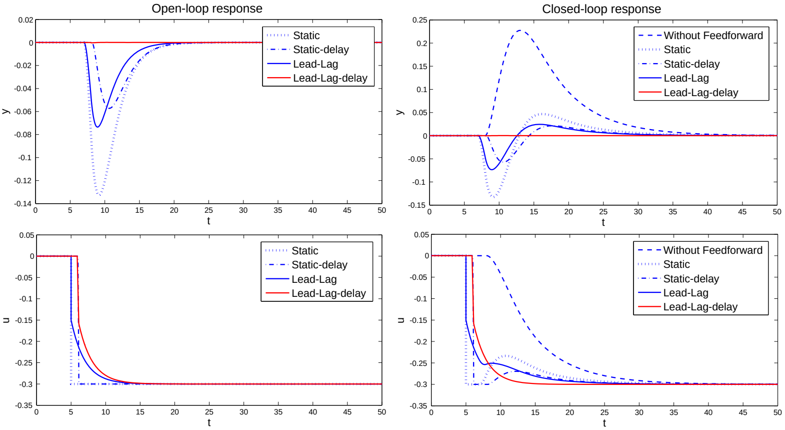
Lets consider the following example:

$$P_u(s) = \frac{1}{s+1}e^{-s}, \quad P_d(s) = \frac{1}{2s+1}e^{-2s}$$

Static:	$C_{ff} = 1$
Static with delay:	$C_{ff} = e^{-s}$
Lead-lag:	$C_{ff} = \dfrac{1+s}{1+2s}$
Lead-lag with delay:	$C_{ff} = \dfrac{1+s}{1+2s}e^{-s}$

C_{fb} is a PI controller tuned using the AMIGO rule, $\kappa_{fb} = 0.25$ and $\tau_i = 2.0$.

Feedforward control problem



Motivation

Then, lets consider a delay inversion problem, i.e., $\lambda_d < \lambda_u$. Then, the resulting feedforward compensators are given by:

$$C_{ff} = K_{ff} = \frac{\kappa_d}{\kappa_u}$$

$$C_{ff} = \frac{\kappa_d}{\kappa_u} \frac{\tau_u s + 1}{\tau_d s + 1}$$

Motivation

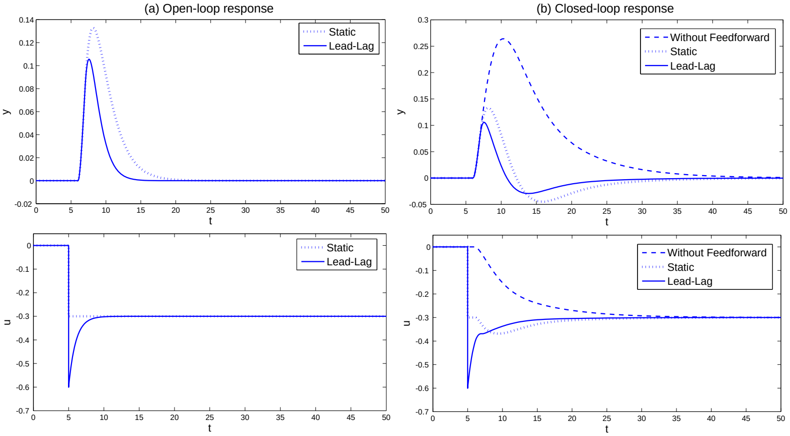
Example:

$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

$$C_{ff} = 1, \quad C_{ff} = \frac{2s+1}{s+1}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.32$ and $\tau_i = 2.85$.

Motivation


(a) Open-loop response
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Static
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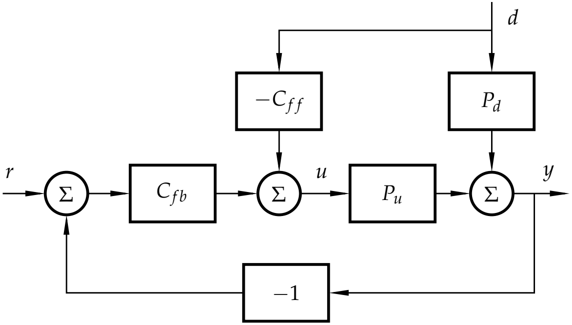
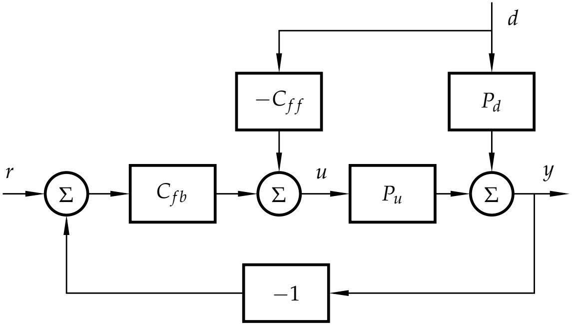


$$y = \frac{P_d - C_{ff}P_u}{1+L}d = \frac{P_d - C_{ff}P_u}{1+C_{fb}P_u}d$$



$$e = \frac{r}{1 + P_u C_{fb}}, \qquad e = \frac{r + P_d^*(e^{-\lambda_u s} - e^{-\lambda_d s})d}{1 + P_u C_{fb}},\ P_d = P_d^* e^{-\lambda_d}$$

Outline

Feedforward tuning rules

Cases to be evaluated in this talk:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.

Outline

Nominal feedforward design: non-realizable delay

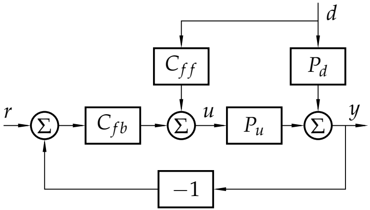
Objective

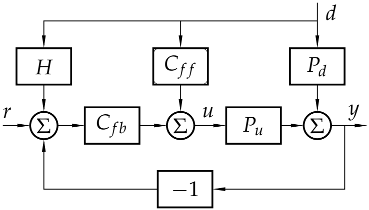
To improve the final disturbance response of the closed-loop system when delay inversion is not realizable ($\lambda_u > \lambda_d$)

Methodology

- Adapt the open-loop tuning rules to closed-loop design
- Obtain optimal open-loop tuning rules
- Design a switching controller to improve the results

Two approaches:



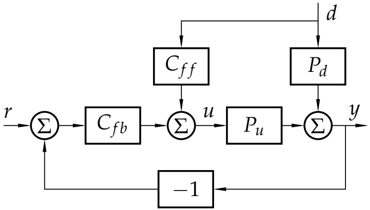


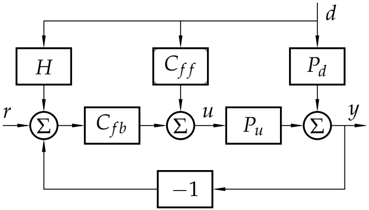
$$P_k(s) = \frac{\kappa_k}{\tau_k s + 1} e^{-\lambda_k s} \qquad k \in [u, d] \qquad \lambda_u > \lambda_d$$

$$C_{fb}(s) = \kappa_{fb} \frac{\tau_i s + 1}{\tau_i s} \qquad C_{ff}(s) = \kappa_{ff} \frac{\beta_{ff} s + 1}{\tau_{ff} s + 1}$$

Nominal feedforward design: non-realizable delay

Two approaches:





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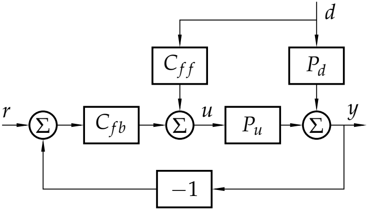
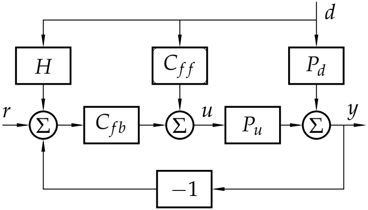
Two approaches:





$$P_k(s) = \frac{\kappa_k}{\tau_k s + 1} e^{-\lambda_k s} \qquad k \in [u, d] \qquad \lambda_u > \lambda_d$$

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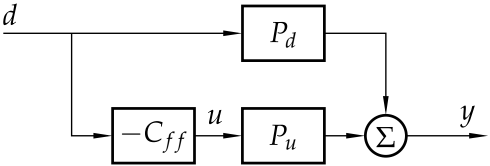
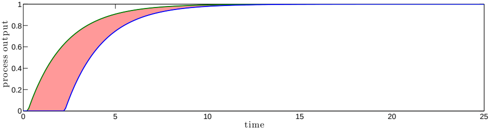
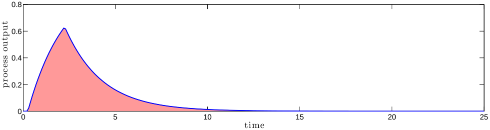
Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d \qquad C_{ff} = \frac{\kappa_d}{\kappa_u} \cdot \frac{\tau_u s + 1}{\tau_d s + 1}$$

Delay inversion: open-loop compensation





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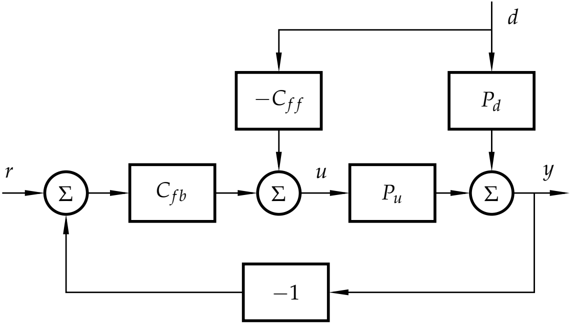
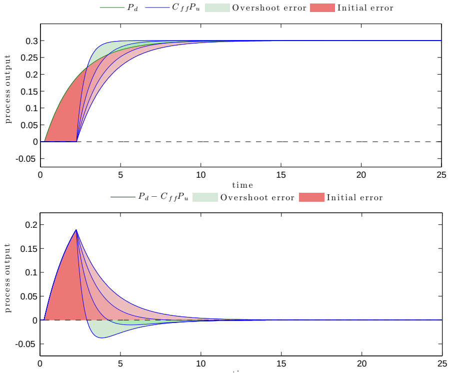
Nominal feedforward design: non-realizable delay



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right)d + u_{fb}P_u \qquad C_{ff} = \frac{\kappa_d}{\kappa_u}\cdot\frac{\tau_u s + 1}{\tau_d s + 1}$$

Delay inversion: open-loop compensation



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right)d + u_{fb}P_u$$

First approach

To deal with the non-realizable delay case, the first approach was to work with the following:

- Use the classical feedforward control scheme.
- Remove the overshoot observed in the response.
- Proposed a tuning rule to minimize Integral Absolute Error (IAE).
- The rules should be simple and based on the feedback and model parameters.

Nominal feedforward design: non-realizable delay

To remove the overshoot, the feedback control action is taken into account to calculate the feedforward gain, κ_{ff}.

$$\Delta u = \frac{\kappa_{fb}}{\tau_i}\int e dt = \frac{\kappa_{fb}}{\tau_i} IE \cdot d$$

So, in the new rule, the goal is to take the control signal to the correct stationary level $-\Delta u$ in order to take the feedback control signal into account and reduce the overshoot. The gain is therefore reduced to

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

Closed-loop design

Nominal feedforward design: non-realizable delay

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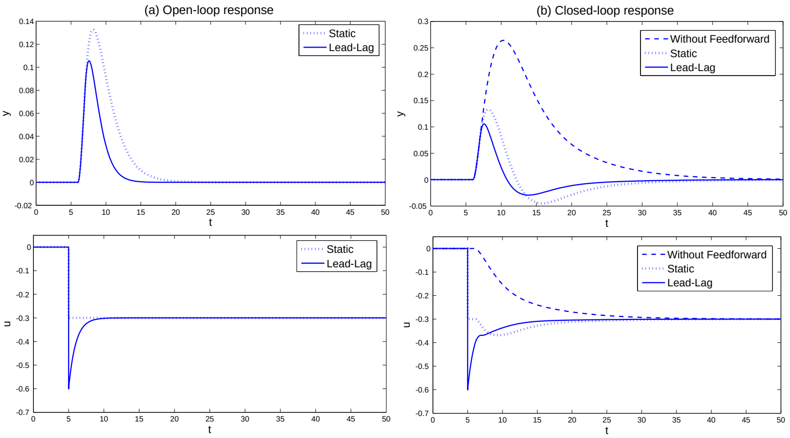
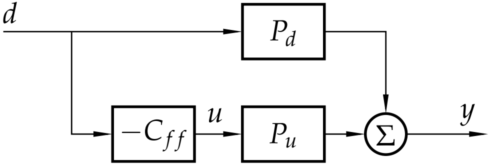


Nominal feedforward design: non-realizable delay

Delay inversion: open-loop compensation



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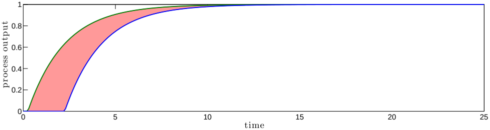
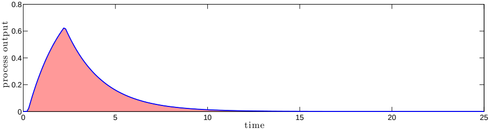
Delay inversion: open-loop compensation





$$y = P_{ff} = \left(P_d - C_{ff}P_u\right)d$$

Nominal feedforward design: non-realizable delay

IE estimation:

$$Y = (P_d - P_u C_{ff})D = P_d D - P_u C_{ff} D$$

$$y(t) - y_{sp} = \begin{cases} k_d \left(1 - e^{-\frac{t}{\tau_d}}\right) d & 0 \le t \le \lambda_b \\ k_d \left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-\lambda_b}{T_b}}\right)\right) d & \lambda_b < t \end{cases}$$

$$\lambda_b = \max(0, \lambda_u - \lambda_d), \quad T_b = \tau_u + \tau_{ff} - \beta_{ff}$$

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Nominal feedforward design: non-realizable delay

IE estimation:

$$
\begin{aligned}
IE \cdot d &= \int_0^{\infty} (y(t) - y_{sp}) dt \\
&= k_d \int_0^{\lambda_b} \left(1 - e^{-\frac{t}{\tau_d}}\right) d\, dt + k_d \int_{\lambda_b}^{\infty} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) d\, dt \\
&= k_d \left[t + \tau_d e^{-\frac{t}{\tau_d}}\right]_0^{\lambda_b} d + k_d \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{\lambda_b}^{\infty} d \\
&= k_d \left(\lambda_b + \tau_d e^{-\frac{\lambda_b}{\tau_d}} - \tau_d - \tau_d e^{-\frac{\lambda_b}{\tau_d}} + T_b\right) d \\
&= k_d \left(\lambda_b - \tau_d + T_b\right) d
\end{aligned}
$$

IE estimation:

$$IE = \begin{cases} k_d(\tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d + \tau_u - \tau_d + \tau_{ff} - \beta_{ff}) & \lambda_d < \lambda_u \end{cases}$$

$$\kappa_{ff} = \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE$$

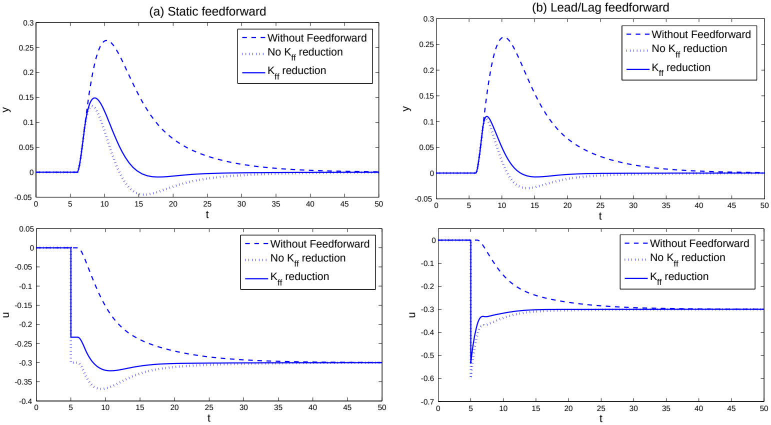
Nominal feedforward design: non-realizable delay

Lets consider the same previous example:

$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

$$C_{ff} = 1, \quad C_{ff} = \frac{2s+1}{s+1}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.32$ and $\tau_i = 2.85$.



The feedforward gain κ_{ff} has been reduced from 1 to 0.778 for the static feedforward and from 1 to 0.889 for the lead-lag filter.

Nominal feedforward design: non-realizable delay

Once the overshoot is reduced, the second goal is to design β_{ff} and τ_{ff} to minimize the IAE value. In this way, we keep $\beta_{ff} = \tau_u$ to cancel the pole of P_u and fix the pole of the compensator:

$$IAE = \int_0^\infty |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^\infty y(t)dt$$

where t_0 is the time when y crosses the setpoint, with $y_{sp} = 0$ and $d = 1$.

$$y(t) - y_{sp} = \begin{cases} k_d \left(1 - e^{-\frac{t}{\tau_d}}\right) d & 0 \le t \le \lambda_b \\ k_d \left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-\lambda_b}{T_b}}\right) \right) d & \lambda_b < t \end{cases}$$

$$IAE = \int_0^\infty |y(t)| dt = \int_0^{t_0} y(t) dt - \int_{t_0}^\infty y(t) dt$$

$$\frac{t_0}{\tau_d} = \frac{t_0 - \lambda_b}{T_b} \rightarrow t_0 = \frac{\tau_d \lambda_b}{\tau_d - T_b} = \frac{\tau_d}{\tau_u - \tau_{ff}} \lambda_b$$

$$T_b = \tau_u + \tau_{ff} - \beta_{ff}$$

Nominal feedforward design: non-realizable delay

$$
\begin{aligned}
IAE &= \int_0^{\lambda_b} \left(1 - e^{-\frac{t}{\tau_d}}\right) dt + \int_{\lambda_b}^{t_0} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) dt - \int_{t_0}^{\infty} \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-\lambda_b}{T_b}}\right) dt \\
&= \left[t + \tau_d e^{-\frac{t}{\tau_d}}\right]_0^{\lambda_b} + \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{\lambda_b}^{t_0} - \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-\lambda_b}{T_b}}\right]_{t_0}^{\infty} \\
&= \lambda_b - \tau_d + T_b + 2\tau_d e^{-\frac{t_0}{\tau_d}} - 2T_b e^{-\frac{t_0-\lambda_b}{T_b}} \\
&= \lambda_b - \tau_d + T_b + 2\tau_d e^{-\frac{\lambda_b}{\tau_d - T_b}} - 2T_b e^{-\frac{\lambda_b}{\tau_d - T_b}} \\
&= \lambda_b - \tau \left(1 - 2e^{-\frac{\lambda_b}{\tau}}\right)
\end{aligned}
$$

with $\tau = \tau_d - \tau_{ff}$.

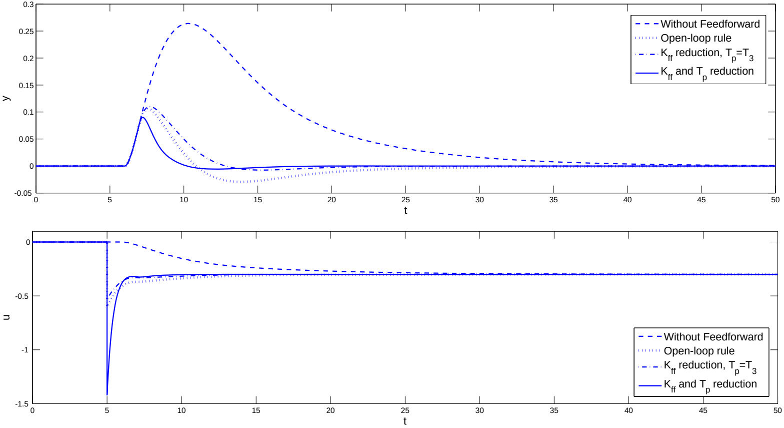
$$\frac{d}{d\tau}IAE = -1 + 2e^{-\frac{\lambda_b}{\tau}} + 2\frac{\lambda_b}{\tau}e^{-\frac{\lambda_b}{\tau}} = -1 + 2(1+x)e^{-x} = 0$$

where $x = \lambda_b/\tau$. A numerical solution of this equation gives $x \approx 1.7$, which gives

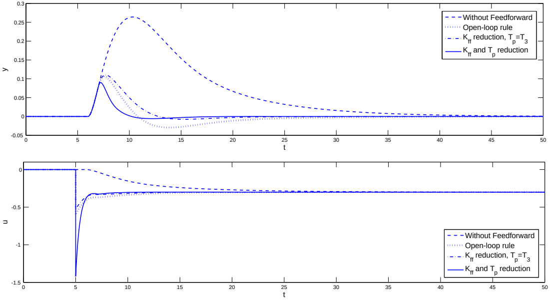
$$\tau_{ff} = T_b - \tau_d + \tau_u = \tau_d - \tau \approx \tau_d - \frac{\lambda_b}{1.7}$$

$$\tau_{ff} = \begin{cases} \tau_u & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \\ 0 & \lambda_u - \lambda_d > 1.7\tau_d \end{cases}$$

Gain and τ_{ff} reduction rule:



Gain and τ_{ff} reduction rule:



	No FF	Open-loop rule	κ_{ff} reduction	κ_{ff} &τ_{ff} reduction
IAE	9.03	1.76	1.37	0.59

First approach: Guideline summary

1. Set $\beta_{ff} = \tau_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_u & \lambda_u - \lambda_d \leq 0 \\ \tau_d - \dfrac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d < 1.7\tau_d \\ 0 & \lambda_u - \lambda_d > 1.7\tau_d \end{cases}$$

2. Calculate the compensator gain, κ_{ff}, as

$$\begin{aligned} \kappa_{ff} &= \frac{k_d}{k_u} - \frac{\kappa_{fb}}{\tau_i} IE \\ IE &= \begin{cases} k_d(\tau_d + \tau_{ff}) & \lambda_d \geq \lambda_u \\ k_d(\lambda_u - \lambda_d - \tau_d + \tau_{ff}) & \lambda_d < \lambda_u \end{cases} \end{aligned}$$

Nominal feedforward design: non-realizable delay

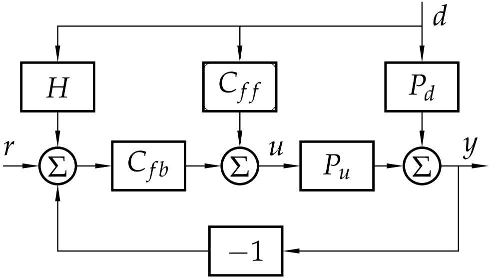
Second approach

To deal with the non-realizable delay case, the second approach was to work with the following:

- Use the non-interacting feedforward control scheme (feedback effect removed).
- Obtain a generalized tuning rule for τ_{ff} for moderate, aggressive and conservative responses.
- The rules should be simple and based on the feedback and model parameters.

Nominal feedforward design: non-realizable delay

Second approach: non-interacting structure

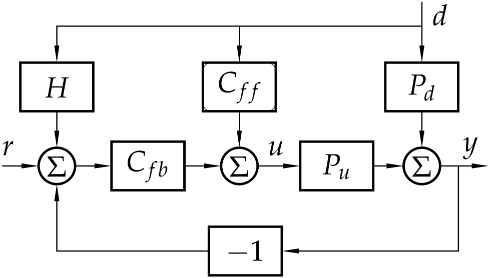


$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

Nominal feedforward design: non-realizable delay

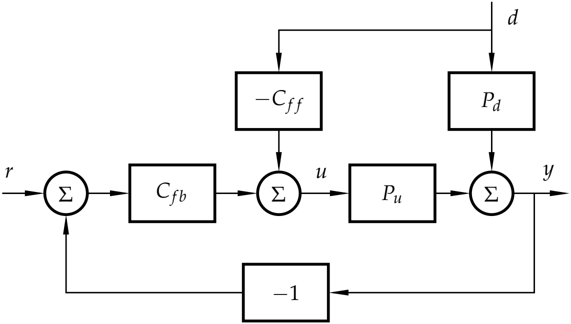
Second approach: non-interacting structure



$$y = \frac{P_{ff} + LH}{1+L} d = \left(P_{ff}\epsilon + H\eta\right) d \qquad H = P_{ff} = P_d - C_{ff}P_u$$

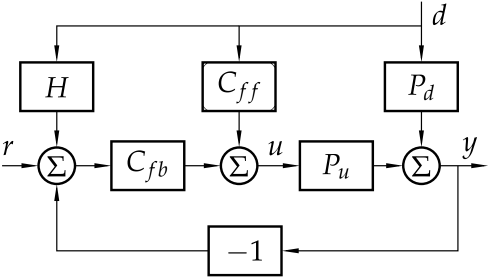
C. Brosilow and B. Joseph. Techniques of model-based control. Prentice Hall, New Jersey, 2012.

Feedforward control problem



$$e = \frac{r}{1 + P_u C_{fb}}, \qquad e = \frac{r + P_d^*(e^{-\lambda_u s} - e^{-\lambda_d s})d}{1 + P_u C_{fb}},\ P_d = P_d^* e^{-\lambda_d}$$

Second approach: non-interacting structure



$$e = \frac{r + (H - P_d + P_u C_{ff})d}{1 + P_u C_{fb}}, \quad H = P_{ff} = P_d - P_u C_{ff}$$

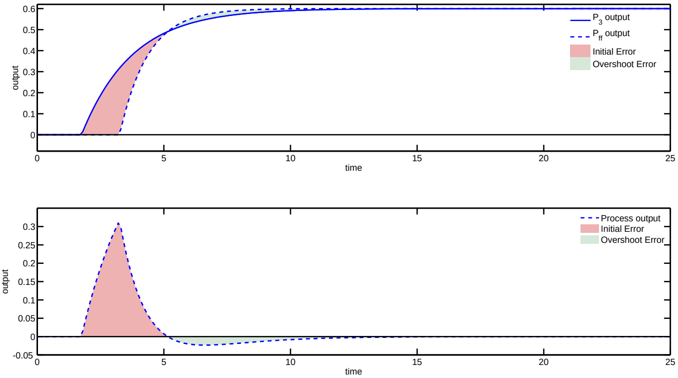
Nominal feedforward design: non-realizable delay

Second approach

The main idea of this second approach relies on analyzing the residual term appearing when perfect cancelation is not possible:

$$\frac{y}{d} = P_d - P_u C_{ff} = P_d - P_{ff}, \quad P_{ff} = P_u C_{ff}$$

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$



$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

From the previous analysis, it can be concluded that in order to totally remove the overshoot for the disturbance rejection problem by using a lead-lag filter, the settling times of both transfer functions must be the same:

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}, \quad \lambda_b = \lambda_d - \lambda_u$$

Nominal feedforward design: non-realizable delay

From the previous analysis, it can be concluded that in order to totally remove the overshoot for the disturbance rejection problem by using a lead-lag filter, the settling times of both transfer functions must be the same:

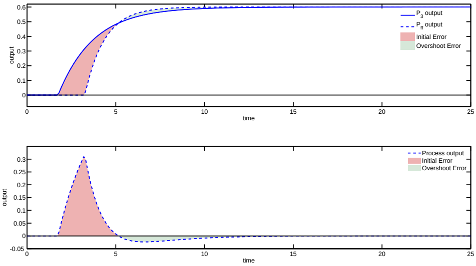
$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-\lambda_d s} - \frac{k_d}{\tau_{ff} s + 1} e^{-\lambda_u s}$$

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}, \quad \lambda_b = \lambda_d - \lambda_u$$

Nominal feedforward design: non-realizable delay

Notice that the new rule for τ_{ff} implies a natural limit on performance. If parameter τ_{ff} is chosen larger, performance will only get worse because of a late compensation. The only reasons why τ_{ff} should be even larger is to decrease the control signal peak:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{4}$$



Nominal feedforward design: non-realizable delay

So, considering the IAE rule obtained for the first approach, two tuning rules are available:

$$\tau_{ff} = \frac{4\tau_d + \lambda_d - \lambda_u}{4} = \tau_d - \frac{\lambda_b}{4}$$

$$\tau_{ff} = \tau_d - \frac{\lambda_u - \lambda_d}{1.7} = \tau_d - \frac{\lambda_b}{1.7}$$

And a third one (a more agreessive rule) can be calculated to minimize Integral Squared Error (ISE) instead of IAE such as proposed in the first approach.

Nominal feedforward design: non-realizable delay

ISE minimization:

$$\text{ISE} = \int_{\lambda_b}^{\infty} \left(e^{-\frac{(t-\lambda_b)}{\tau_{ff}}} - e^{-\frac{t}{\tau_d}} \right)^2 dt$$

$$= \int_{\lambda_b}^{\infty} \left(e^{-\frac{2(t-\lambda_b)}{\tau_{ff}}} - 2e^{-\frac{\tau_d(t-\lambda_b)+\tau_{ff}t}{\tau_d\tau_{ff}}} + e^{-\frac{2t}{\tau_d}} \right) dt$$

$$= -\frac{\tau_{ff}}{2} \left[e^{-\frac{2(t-\lambda_b)}{\tau_{ff}}} \right]_{\lambda_b}^{\infty} + 2\frac{\tau_d\tau_{ff}}{\tau_d+\tau_{ff}} \left[e^{-\frac{\tau_d(t-\lambda_b)+\tau_{ff}t}{\tau_d\tau_{ff}}} \right]_{\lambda_b}^{\infty} - \frac{\tau_d}{2} \left[e^{-\frac{2t}{\tau_d}} \right]_{\lambda_b}^{\infty}$$

$$= \frac{\tau_{ff}}{2} - 2\tau_d \frac{\tau_{ff}}{\tau_d+\tau_{ff}} e^{-\frac{\lambda_b}{\tau_d}} + \frac{\tau_d}{2} e^{-\frac{2\lambda_b}{\tau_d}}$$

Nominal feedforward design: non-realizable delay

ISE minimization:

$$\frac{d\,\text{ISE}}{d\,\tau_{ff}} = \frac{1}{2} - 2\tau_d e^{-\frac{\lambda_b}{\tau_d}} \left(\frac{1}{\tau_d + \tau_{ff}} + \frac{-\tau_{ff}}{(\tau_d + \tau_{ff})^2} \right) = \frac{1}{2} - \frac{2\tau_d^2}{(\tau_d + \tau_{ff})^2} e^{-\frac{\lambda_b}{\tau_d}} = 0$$

$$\tau_{ff}^2 + 2\tau_d \tau_{ff} + \tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}}) = 0$$

$$\tau_{ff} = \frac{-2\tau_d + \sqrt{4\tau_d^2 - 4\tau_d^2 (1 - 4e^{-\frac{\lambda_b}{\tau_d}})}}{2} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1 \right)$$

Nominal feedforward design: non-realizable delay

Thus, three tuning rules are available:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{4}$$
$$\tau_{ff} = \tau_d - \frac{\lambda_b}{1.7}$$
$$\tau_{ff} = \tau_d \left(2\sqrt{e^{-\frac{\lambda_b}{\tau_d}}} - 1\right)$$

which can be generalized as:

$$\tau_{ff} = \tau_d - \frac{\lambda_b}{\alpha}$$

Nominal feedforward design: non-realizable delay

Second approach: Guideline summary

1. Set $\beta_{ff} = \tau_u$, $\kappa_{ff} = k_d/k_u$ and calculate τ_{ff} as:

$$\tau_{ff} = \begin{cases} \tau_d & \lambda_b \leq 0 \\ \tau_d - \frac{\lambda_b}{\alpha} & 0 < \lambda_b < 4\tau_d \\ 0 & \lambda_b \geq 4\tau_d \end{cases}$$

2. Determine τ_{ff} with $\lambda_b/\tau_d < \alpha < \infty$ using:

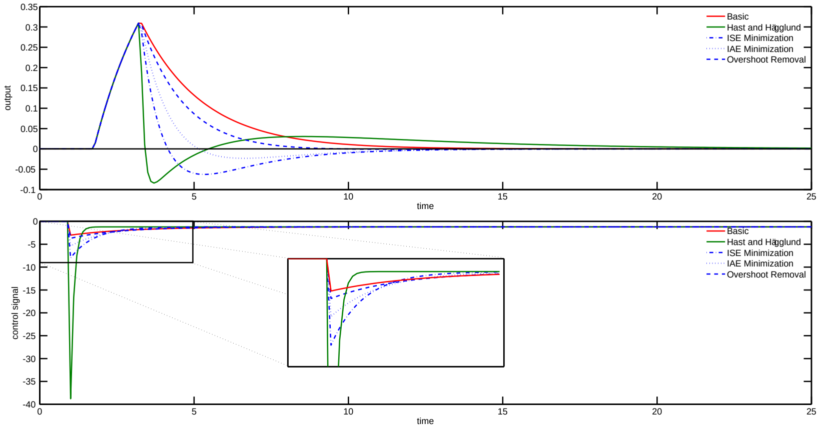
$$\alpha = \begin{cases} \dfrac{\lambda_b}{2\tau_d\left(1-\sqrt{e^{-\lambda_b/\tau_d}}\right)} & \text{aggressive (ISE minimization)} \\ 1.7 & \text{moderate (IAE minimization)} \\ 4 & \text{conservative (Overshoot removal)} \end{cases}$$

Example:

$$P_u(s) = \frac{0.5}{5s+1}e^{-2.25s}, \quad P_d(s) = \frac{1}{2s+1}e^{-0.75s}$$

The feedback controller is tuned using the AMIGO rule, which gives the parameters $\kappa_{fb} = 0.9$ and $\tau_i = 4.53$.

Nominal feedforward design: non-realizable delay



Nominal feedforward design: non-realizable delay

	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	**0.0739**	0.6423	38.7800	2.5710	0.8979
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	**0.5641**	5.3680	**0.9113**	**0.8315**
Overshoot Removal	0.1277	0.6833	**3.6920**	0.9323	0.8870

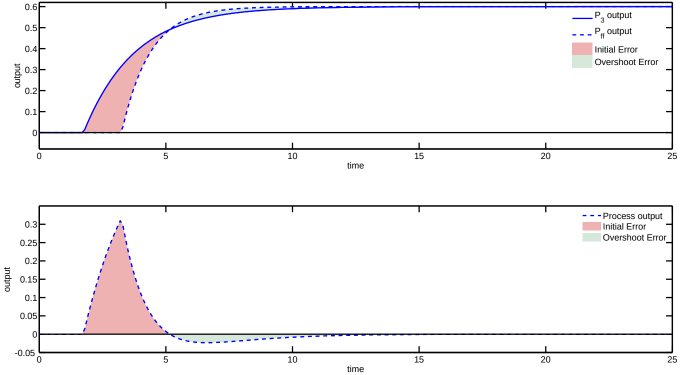
$$J_1(F,B) = \frac{1}{2}\left(\frac{\text{ISE}(F)}{\text{ISE}(B)} + \frac{\text{ISC}(F)}{\text{ISC}(B)}\right), \quad \text{ISC} = \int_0^\infty u(t)^2\,\mathrm{d}t$$

$$J_2(F,B) = \frac{1}{2}\left(\frac{\text{IAE}(F)}{\text{IAE}(B)} + \frac{\text{IAC}(F)}{\text{IAC}(B)}\right), \quad \text{IAC} = \int_0^\infty |u(t)|\,\mathrm{d}t$$

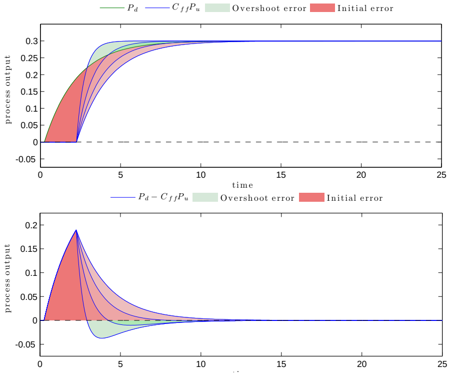
Second approach: A switching solution

It is clear that if the compensation is made too fast, the output will suffer a bigger overshoot error, while if it is too slow, the compensator will take too much time to reject the disturbance and it will have a bigger residual error. Therefore, a switching rule can be proposed in such a way that the feedforward compensator reacts fast before the outputs cross in order to decrease the residual error, and slower after this time to avoid the overshoot because of the residual error.

Second approach: A switching solution

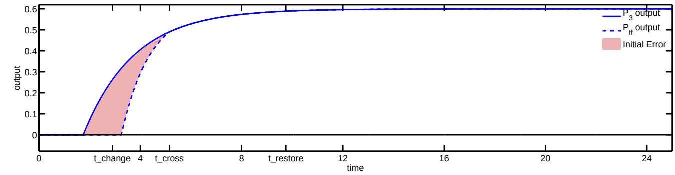


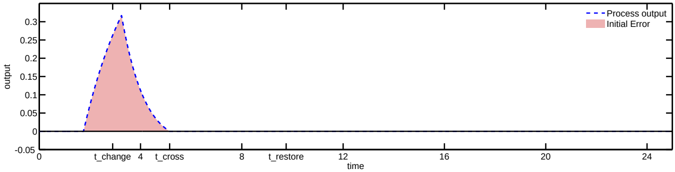
Second approach: A switching solution



$$y = P_{ff} = \left(P_d - C_{ff}P_u\right) d$$

Second approach: A switching solution





Nominal feedforward design: non-realizable delay

Second approach: A switching solution

The idea is to set τ_{ff} to a small value until the time when the responses of both transfer functions cross. After this time, the new value of τ_{ff} will be τ_d. Once the load disturbance is rejected, τ_{ff} will be set again to the small initial value in order to be ready for new coming disturbances.

Second approach: A switching solution

Thus, the first step is to calculate the time it takes since a step change in d appears at time instant t_d until the outputs of both transfer functions cross. This time, t_{cross}, corresponds to the point when the step responses of P_{ff} and P_d are equal:

$$\kappa_d d \left(e^{\frac{-(t_{cross}-t_d-\lambda_d)}{\tau_d}} - e^{\frac{-(t_{cross}-t_d-\lambda_u)}{\tau_{ff}}} \right) = 0$$

where it is straightforward to see that:

$$t_{cross} = \frac{\tau_d \lambda_u - \tau_{ff} \lambda_d}{\tau_d - \tau_{ff}} + t_d$$

Nominal feedforward design: non-realizable delay

Second approach: A switching solution

On the other hand, notice that the time event of the switching rule is really given at $t_{change} = t_{cross} - \lambda_u$.

Once the disturbance has been rejected, the feedforward switching controller should return to its original value in order to be ready for possible new coming load disturbances. This change must be done at a time instant, t_r, which can be proposed as the settling time of process P_d such as follows:

$$t_r = 4\tau_d + \lambda_d + t_d$$

Thus, τ_{ff} should be equal to τ_d when $t_d + t_{cross} - \lambda_u \leq t \leq t_d + t_r$ and it must be tuned for a faster response otherwise, specially for $t < t_{change}$.

Second approach: A switching solution

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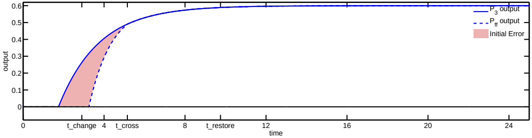
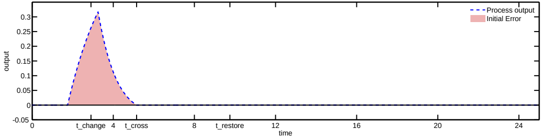
$$t_r = 4\tau_d + \lambda_d + t_d$$

Thus, τ_{ff} should be equal to τ_d when $t_d + t_{cross} - \lambda_u \leq t \leq t_d + t_r$ and it must be tuned for a faster response otherwise, specially for $t < t_{change}$.

Nominal feedforward design: non-realizable delay





$$t_{cross} = \frac{\tau_d \lambda_u - \tau_{ff} \lambda_d}{\tau_d - \tau_{ff}} + t_d \qquad t_{change} = t_{cross} - \lambda_u$$

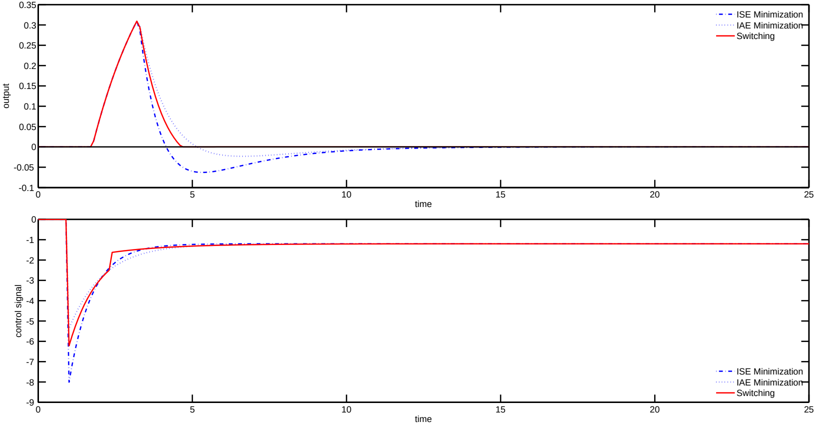
$$t_r = 4\tau_d + \lambda_d + t_d$$

Second approach: the switching solution guideline

1. Set τ_{ff} to a value as close to 0 as possible (tradeoff with the control signal peak).
2. Wait until a step load disturbance is detected at time instant t_d. Define t_{cross} and $t_{restore}$. Set $t_{change} = t_{cross} - \lambda_u$.
3. Using a non-interacting scheme, set C_{ff} and H as follows:

$$C_{ff}(s) = \begin{cases} \dfrac{\kappa_d}{\kappa_u}\dfrac{1+\tau_u s}{1+\tau_d s} & t_{change} \leq t \leq t_r \\ \dfrac{\kappa_d}{\kappa_u}\dfrac{1+\tau_u s}{1+\tau_{ff} s} & \text{otherwise} \end{cases}$$

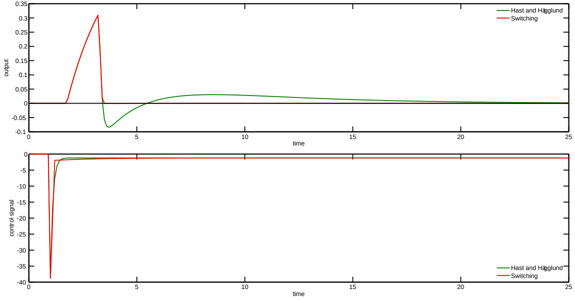
4. Go to step 2.


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
output
0
5
10
15
20
25
time
ISE Minimization
IAE Minimization
Switching
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
control signal
0
5
10
15
20
25
time
ISE Minimization
IAE Minimization
Switching

Nominal feedforward design: non-realizable delay

	ISE	IAE	u_{init}	J_1	J_2
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	0.5641	**5.3680**	0.9113	0.8315
Switching	**0.0889**	**0.4252**	6.2160	**0.9062**	**0.7527**

Nominal feedforward design: non-realizable delay



	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	0.0739	0.6423	38.78	**2.5710**	0.8979
Switching	**0.0630**	**0.2878**	38.78	2.6650	**0.7149**

Outline

Feedforward tuning rules: RH plane zeros

Right-half plane zeros

$$P_u(s) = \frac{k_u\left(-\beta_u s + 1\right)}{D_u^-(s)} e^{-\lambda_u s} \quad \beta_u > 0$$

$$P_d(s) = \frac{k_d}{D_d^-(s)} e^{-\lambda_d s}$$

such that $D_u^-(s) = 1 + \sum_{i=1}^{n_u} a_u[i]s^i$ and $D_d^-(s) = 1 + \sum_{i=1}^{n_d} a_d[i]s^i$ are polynomials with n_u and n_d degree, respectively, such that all their roots are located in the LHP (left-half plane). Moreover, $\lambda_u \leq \lambda_d$.

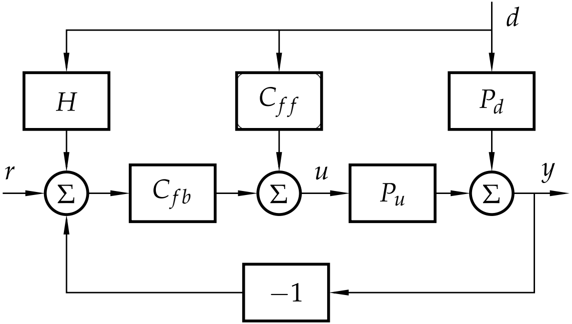
Feedforward tuning rules: RH plane zeros

Objective

To improve the final disturbance response of the closed-loop system when there are righ-half plane zeros in P_u

Methodology

- Decouple both reference tracking and disturbance rejection responses
- Shape the nominal disturbance rejection response as a critically damped system
- Obtain simple tuning rules for the time constant of the response



$$H(s) = P_d(s) - P_u(s)C_{ff}(s)$$

Feedforward tuning rules: RH plane zeros

$$\frac{y(s)}{d(s)} = e^{-\lambda_d s}\left(\frac{k_d}{D_d^-(s)} - C_{ff}(s)\frac{k_u\left(-\beta_u s+1\right)}{D_u^-(s)}e^{-(\lambda_u-\lambda_d)s}\right)$$

$$C_{ff}(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}e^{-(\lambda_d-\lambda_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-\lambda_d s}}{D_d^-(s)}\left(1-\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s+1\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}\right)$$

Feedforward tuning rules: RH plane zeros

$$\frac{y(s)}{d(s)} = e^{-\lambda_d s}\left(\frac{k_d}{D_d^-(s)} - C_{ff}(s)\frac{k_u\left(-\beta_u s+1\right)}{D_u^-(s)}e^{-(\lambda_u-\lambda_d)s}\right)$$

$$C_{ff}(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}e^{-(\lambda_d-\lambda_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-\lambda_d s}}{D_d^-(s)}\left(1-\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s+1\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}\right)$$

Feedforward tuning rules: RH plane zeros

$$\frac{y(s)}{d(s)} = e^{-\lambda_d s}\left(\frac{k_d}{D_d^-(s)} - C_{ff}(s)\frac{k_u\left(-\beta_u s+1\right)}{D_u^-(s)}e^{-(\lambda_u-\lambda_d)s}\right)$$

$$C_{ff}(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}e^{-(\lambda_d-\lambda_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-\lambda_d s}}{D_d^-(s)}\left(1-\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s+1\right)}{\left(\tau_{ff}s+1\right)^{n_{ff}}}\right)$$

By using the binomial theorem, the previous expression results in:

$$\frac{y(s)}{d(s)} = \frac{k_d P_0 s}{\left(\tau_{ff} s + 1\right)^{n_u}} \cdot \frac{P(s)}{D_d^-(s)} e^{-\lambda_d s}$$

with

$$P(s) = P_0^{-1} \left(\beta_u \sum_{i=1}^{n_d} \beta_{ff}[i] s^i - \sum_{i=1}^{n_d-1} \beta_{ff}[i+1] s^i + \right.$$

$$\left. + \sum_{i=1}^{n_u-1} \frac{n_u!}{(i+1)!\,(n_u - i - 1)!} \tau_{ff}^{i+1} s^i \right) + 1$$

$$P_0 = n_u \tau_{ff} + \beta_u - \beta_{ff}[1]$$

After solving $\beta_{ff}[i]$ coefficients and cancelling $D_d^-(s)$, it is obtained that

$$G_d(s) = \frac{y(s)}{d(s)} = \frac{\kappa_{y/d}s}{\left(\tau_{ff}s+1\right)^{n_u}}e^{-\lambda_d s}$$

with

$$\kappa_{y/d} = k_d\frac{\beta_u^{n_d-n_u+1}\left(\beta_u+\tau_{ff}\right)^{n_u}}{\beta_u^{n_d}+\sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d-l}}$$

And where the unitary step response is given by

$$y(t) = \frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{(t-\lambda_d)}{\tau_{ff}}}$$

Feedforward tuning rules: RH plane zeros

After solving $\beta_{ff}[i]$ coefficients and cancelling $D_d^-(s)$, it is obtained that

$$G_d(s) = \frac{y(s)}{d(s)} = \frac{\kappa_{y/d}s}{\left(\tau_{ff}s+1\right)^{n_u}}e^{-\lambda_d s}$$

with

$$\kappa_{y/d} = k_d \frac{\beta_u^{n_d-n_u+1}\left(\beta_u+\tau_{ff}\right)^{n_u}}{\beta_u^{n_d}+\sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d-l}}$$

And where the unitary step response is given by

$$y(t) = \frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{\left(t-\lambda_d\right)}{\tau_{ff}}}$$

Feedforward tuning rules: RH plane zeros

Three different tuning rules are proposed for τ_{ff} looking for

- Obtaining a desired settling time.
- Minimize the H_∞ norm.
- Minimize the H_2 norm.

Settling time rule

$$y(t) = \frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{\left(t-\lambda_d\right)}{\tau_{ff}}}$$

The settling time is defined as the time that the system takes to reach around 5% of its maximum value

$$y(t_{5\%}) = 0.05M_{peak}$$

$$\frac{dy(t)}{dt} = 0 \Rightarrow t_{peak} \Rightarrow M_{peak} \Rightarrow t_{5\%}$$

Settling time rule

$$\frac{dy(t)}{dt} = \frac{\kappa_{y/d} e^{-\frac{(t-\lambda_d)}{\tau_{ff}}}}{(n_u - 1)!\tau_{ff}^{n}} \left((n_u - 1)(t - \lambda_d)^{n_u - 2} - \frac{(t - \lambda_d)^{n_u - 1}}{\tau_{ff}} \right)$$

$$e^{-\frac{(t_{peak}-\lambda_d)}{\tau_{ff}}} \left(t_{peak} - \lambda_d\right)^{n_u - 2} \left(\tau_{ff}\left(n_u - 1\right) - \left(t_{peak} - \lambda_d\right)\right) = 0$$

$$t_{peak} = \lambda_d + \tau_{ff}\left(n_u - 1\right).$$

Settling time rule

Thus, the maximum peak M_{peak} is given by

$$M_{peak} = y(t_{peak}) = \frac{\kappa_{y/d}}{\tau_{ff}} \cdot \frac{e^{1-n_u}\left(n_u - 1\right)^{n_u - 1}}{(n_u - 1)!}.$$

If this expression is used in

$$y(t_{5\%}) = 0.05 M_{peak}$$

with $t = t_{5\%}$, the following equation is obtained

$$\frac{\kappa_{y/d}\left(t_{5\%} - \lambda_d\right)^{n_u - 1}}{\tau_{ff}^{n_u}\left(n_u - 1\right)!} e^{-\frac{t_{5\%} - \lambda_d}{\tau_{ff}}} = 0.05 \frac{\kappa_{y/d}}{\tau_{ff}} \cdot \frac{e^{1-n_u}\left(n_u - 1\right)^{n_u - 1}}{(n_u - 1)!}.$$

Settling time rule

$$t_{5\%} = \lambda_d + x\tau_{ff}, \quad 0.05 - \frac{x^{n_u-1}}{(n_u-1)^{n_u-1}} e^{-x+n_u-1} = 0$$

$$\tau_{ff} = \frac{(t_{5\%} - \lambda_d)}{x}$$

For $n_u = 1$, the following solution is obtained

$$\tau_{ff} \approx \frac{t_{5\%-\lambda_d}}{3}$$

Feedforward tuning rules: RH plane zeros

Settling time rule: Example

$$P_u(s) = \frac{-0.8s+1}{s^2+s+1}, \quad P_d(s) = \frac{0.45}{0.75s+1}$$

$$C_{ff}(s) = 0.45\frac{s^2+s+1}{0.75s+1} \cdot \frac{\beta_{ff}[1]s+1}{(\tau_{ff}s+1)^2}$$

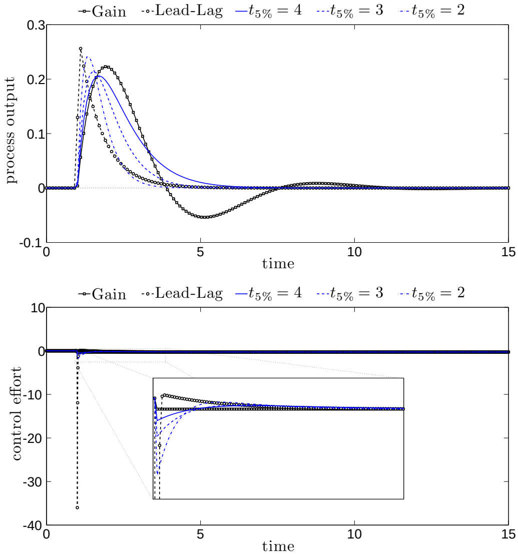
To cancel the stable pole of $P_d(s)$, it is necessary to set

$$\beta_{ff}[1] = -0.6452\tau_{ff}^2 + 0.9677\tau_{ff} + 0.3871$$

Then, τ_{ff} is selected according to the desired settling time

$$\tau_{ff} \approx \frac{t_{5\%}}{5.74}$$

Feedforward tuning rules: RH plane zeros



Feedforward tuning rules: RH plane zeros

Feedforward controller	$\beta_{ff}[1]$	τ_{ff}
$t_{5\%} = 4$	0.75	0.70
$t_{5\%} = 3$	0.72	0.52
$t_{5\%} = 2$	0.65	0.35

Feedforward tuning rules: RH plane zeros

H_∞***-norm rule***

$$y(t) = \frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{\left(t-\lambda_d\right)}{\tau_{ff}}}$$

An H_∞ optimal feedforward compensator to minimize the maximum value of the disturbance response can be found by minimizing the absolute value of the maximum peak:

$$\frac{d\left\|y(t)\right\|_\infty}{d\tau_{ff}} = \frac{d|M_{peak}|}{d\tau_{ff}} = 0$$

Feedforward tuning rules: RH plane zeros

H_∞-norm rule

$$\frac{d\left\|y(t)\right\|_\infty}{d\tau_{ff}} = c_1\left(\frac{n_u\left(\beta_u+\tau_{ff}\right)^{n_u-1}}{\tau_{ff}} - \frac{\left(\beta_u+\tau_{ff}\right)^{n_u}}{\tau_{ff}^2}\right)$$

$$c_1 = \frac{\left|\kappa_d\right|\beta_u^{n_d-n_u+1}\left(n_u-1\right)^{n_u-1}}{\left(\beta_u^{n_d}+\sum_{l=1}^{n_d}a_d[l]\beta_u^{n_d-l}\right)\left(n_u-1\right)!}.$$

$$\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-\left(\beta_u+\tau_{ff}\right)\right)=0 \Rightarrow \tau_{ff}=\frac{\beta_u}{n_u-1}$$

Feedforward tuning rules: RH plane zeros

H_∞*-norm rule*

$$\frac{d\left\|y(t)\right\|_\infty}{d\tau_{ff}} = c_1\left(\frac{n_u\left(\beta_u+\tau_{ff}\right)^{n_u-1}}{\tau_{ff}} - \frac{\left(\beta_u+\tau_{ff}\right)^{n_u}}{\tau_{ff}^2}\right)$$

$$c_1 = \frac{\left|\kappa_d\right|\beta_u^{n_d-n_u+1}\left(n_u-1\right)^{n_u-1}}{\left(\beta_u^{n_d}+\sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d-l}\right)\left(n_u-1\right)!}.$$

$$\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-\left(\beta_u+\tau_{ff}\right)\right)=0 \Rightarrow \tau_{ff}=\frac{\beta_u}{n_u-1}$$

Feedforward tuning rules: RH plane zeros

H_∞***-norm rule***

$$\frac{d\left\|y(t)\right\|_\infty}{d\tau_{ff}} = c_1\left(\frac{n_u\left(\beta_u+\tau_{ff}\right)^{n_u-1}}{\tau_{ff}} - \frac{\left(\beta_u+\tau_{ff}\right)^{n_u}}{\tau_{ff}^2}\right)$$

$$c_1 = \frac{\left|\kappa_d\right|\beta_u^{n_d-n_u+1}\left(n_u-1\right)^{n_u-1}}{\left(\beta_u^{n_d}+\sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d-l}\right)\left(n_u-1\right)!}.$$

$$\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-\left(\beta_u+\tau_{ff}\right)\right)=0 \Rightarrow \boxed{\tau_{ff}=\frac{\beta_u}{n_u-1}}$$

H_2-norm rule

$$y(t) = \frac{\kappa_{y/d} \left(t - \lambda_d\right)^{n_u - 1}}{\tau_{ff}^{n_u} \left(n_u - 1\right)!} e^{-\frac{\left(t - \lambda_d\right)}{\tau_{ff}}}$$

An H_2 optimal feedforward compensator of the disturbance response can be found by minimizing the absolute value of the output:

$$\frac{d \left\| y(t) \right\|_2}{d\tau_{ff}} = 0$$

H_2-norm rule

$$\|y(t)\|_2 = \left(\int_{\lambda_d}^{\infty}\left|\frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{(t-\lambda_d)}{\tau_{ff}}}\right|^2 dt\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\int_0^{\infty}\xi^{2(n_u-1)}e^{-\frac{2\xi}{\tau_{ff}}}d\xi\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\left[\frac{-\left(2\left(n_u-1\right)\right)!\tau_{ff}^{2n_u-1}}{2^{2n_u-1}}e^{-\frac{2\xi}{\tau_{ff}}}\sum_{i=1}^{2(n_u-1)}\frac{\tau_{ff}^{2(n_u-1)-i}}{2^{2(n_u-1)-i}}\xi^i\right]_0^{\infty}\right)^{\frac{1}{2}}$$

$$\tau_{ff}^{-1.5}\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-0.5\left(\beta_u+\tau_{ff}\right)\right)=0 \Rightarrow \tau_{ff}=\frac{\beta_u}{2n_u-1}$$

Feedforward tuning rules: RH plane zeros

H_2*-norm rule*

$$\|y(t)\|_2 = \left(\int_{\lambda_d}^{\infty}\left|\frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{\left(t-\lambda_d\right)}{\tau_{ff}}}\right|^2 dt\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\int_0^{\infty}\xi^{2(n_u-1)}e^{-\frac{2\xi}{\tau_{ff}}}d\xi\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\left[\frac{-\left(2\left(n_u-1\right)\right)!\tau_{ff}^{2n_u-1}}{2^{2n_u-1}}e^{-\frac{2\xi}{\tau_{ff}}}\sum_{i=1}^{2(n_u-1)}\frac{\tau_{ff}^{2(n_u-1)-i}}{2^{2(n_u-1)-i}}\xi^i\right]_0^{\infty}\right)^{\frac{1}{2}}$$

$$\tau_{ff}^{-1.5}\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-0.5\left(\beta_u+\tau_{ff}\right)\right) = 0 \Rightarrow \tau_{ff} = \frac{\beta_u}{2n_u-1}$$

H_2-norm rule

$$\|y(t)\|_2 = \left(\int_{\lambda_d}^{\infty}\left|\frac{\kappa_{y/d}\left(t-\lambda_d\right)^{n_u-1}}{\tau_{ff}^{n_u}\left(n_u-1\right)!}e^{-\frac{(t-\lambda_d)}{\tau_{ff}}}\right|^2 dt\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\int_0^{\infty}\xi^{2(n_u-1)}e^{-\frac{2\xi}{\tau_{ff}}}d\xi\right)^{\frac{1}{2}}$$

$$= \frac{\left|\kappa_{y/d}\right|}{\tau_{ff}^{n_u}\left(n_u-1\right)!}\left(\left[\frac{-\left(2\left(n_u-1\right)\right)!\tau_{ff}^{2n_u-1}}{2^{2n_u-1}}e^{-\frac{2\xi}{\tau_{ff}}}\sum_{i=1}^{2(n_u-1)}\frac{\tau_{ff}^{2(n_u-1)-i}}{2^{2(n_u-1)-i}}\xi^i\right]_0^{\infty}\right)^{\frac{1}{2}}$$

$$\tau_{ff}^{-1.5}\left(\beta_u+\tau_{ff}\right)^{n_u-1}\left(n_u\tau_{ff}-0.5\left(\beta_u+\tau_{ff}\right)\right)=0 \Rightarrow \boxed{\tau_{ff}=\frac{\beta_u}{2n_u-1}}$$

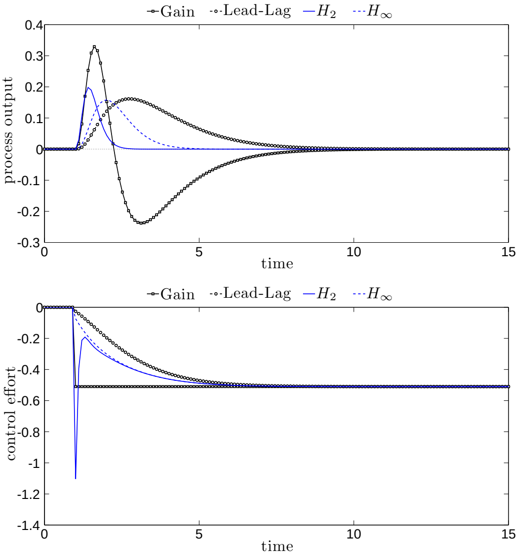
H_∞ ***and*** H_2 ***rules: Example***

$$P_u(s) = \frac{-s+1}{(0.25s+1)^4}, \quad P_d(s) = \frac{0.85}{(0.9s+1)^3}$$

$$C_{ff}(s) = 0.85\frac{(0.25s+1)^4}{(0.9s+1)^3} \cdot \frac{1+\sum_{i=1}^{3}\beta_{ff}[i]s^i}{(\tau_{ff}s+1)^4}$$

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	τ_{ff}
H_2	1.32	0.77	0.18	0.14
H_∞	1.87	1.30	0.32	0.33

Feedforward tuning rules: RH plane zeros



Feedforward tuning rules: RH plane zeros

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	$\|y(t)\|_\infty$
Gain	80.47	3.85	0.33
Lead-lag	51.51	2.39	0.16
H_2	**12.68**	**1.33**	0.20
H_∞	23.50	1.61	**0.16**

Feedforward tuning rules: RH plane zeros

1. Set τ_{ff} according to the desired specification:

$$\text{Settling time}: \quad \tau_{ff} = \left(t_{5\%} - \lambda_d\right)/x$$

$$H_\infty: \quad \tau_{ff} = \frac{\beta_u}{n_u - 1}$$

$$H_2: \quad \tau_{ff} = \frac{\beta_u}{2n_u - 1}.$$

2. Obtain the coefficients $\beta_{ff}[i]$ to cancel $D_d^-(s)$.
3. Define the feedforward compensator $F(s)$ as

$$F(s) = \frac{k_d}{k_u} \cdot \frac{D_u^-(s)}{D_d^-(s)} \cdot \frac{\left(1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i]s^i\right)}{\left(\tau_{ff}s + 1\right)^{n_{ff}}} e^{-(\lambda_d - \lambda_u)s}$$

4. Set $H(s) = P_{ff}(s) = P_d(s) - C_{ff}(s)P_u(s)$.

Outline

Integrating poles

$$P_u(s) = \frac{k_u}{D_u(s)s^{t_u}}$$

$$P_d(s) = \frac{k_d}{D_d^-(s)}$$

such that $D_u(s) = 1 + \sum_{i=1}^{n_u} a_u[i]s^i$ is a polynomial of degree n_u and $D_d^-(s) = 1 + \sum_{i=1}^{n_d} a_d[i]s^i$ is a polynomial of degree n_d with all its roots in the left half plane (LHP), and t_u is the type of process $P_u(s)$.

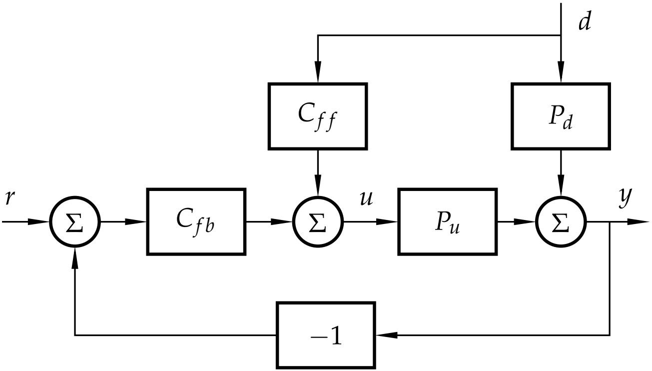
Nominal feedforward design: integrators

Objective

To improve the final disturbance response of the closed-loop system when there are integrating poles in P_u

Methodology

- Decouple both reference tracking and disturbance rejection responses
- Shape the nominal disturbance rejection response as a critically damped system
- Obtain simple tuning rules for the time constant of the response



d
C_{ff}
P_d
r
Σ
C_{fb}
Σ
u
P_u
Σ
y
-1

In this case, the feedback controller will be defined as follows

$$C_{fb}(s) = \kappa_{fb} \frac{N_{fb}(s)}{D_{fb}(s)s^{t_{fb}}}$$

such that t_{fb} is the type of $C_{fb}(s)$.

And the reference tracking response can be expressed as

$$\frac{y(s)}{r(s)} = \frac{N_{fb}(s)}{D_{cl}(s)}$$

where $D_{cl}(s)$ is a polynomial of degree n_{cl} that represents the closed-loop system dynamics.

Feedforward tuning rules: integrators

$$
\begin{aligned}
\frac{y(s)}{d(s)} &= \left(\frac{k_d}{D_d^-(s)} - C_{ff}(s)\frac{k_u}{D_u(s)} s^{-t_u} \right) \frac{D_u(s)s^{t_u} D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)} \\
&= \left(\frac{k_d d D_u(s)s^{t_u}}{D_d^-(s)} - C_{ff}(s)k_u \right) \frac{D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}
\end{aligned}
$$

$$
C_{ff}(s) = \frac{k_d}{k_u}\frac{1}{D_{fb}(s)D_d^-(s)}\frac{1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i}{(\tau_{ff}s+1)^{n_{ff}}}
$$

Feedforward tuning rules: integrators

$$\frac{y(s)}{d(s)} = \left(\frac{k_d}{D_d^-(s)} - C_{ff}(s)\frac{k_u}{D_u(s)}s^{-t_u}\right)\frac{D_u(s)s^{t_u}D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$
$$= \left(\frac{k_d d D_u(s)s^{t_u}}{D_d^-(s)} - C_{ff}(s)k_u\right)\frac{D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$

$$C_{ff}(s) = \frac{k_d}{k_u}\frac{1}{D_{fb}(s)D_d^-(s)}\frac{1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i}{\left(\tau_{ff}s+1\right)^{n_{ff}}}$$

By substituting the proposed compensator in the disturbance rejection response, it is obtained that

$$\frac{y(s)}{d(s)} = G_{y/d}(s) = \frac{-k_d d s^{t_{fb}}}{\left(\tau_{ff}s+1\right)^{n_{ff}}} \frac{P(s)}{D_{cl}(s)D_d^-(s)}$$

with

$$P(s) = 1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i]s^i - \left(\tau_{ff}s+1\right)^{n_{ff}} D_{fb}(s)D_u(s)s^{t_u}$$

The idea is to cancel all stable roots of $D_{cl}(s)$ and $D_d^-(s)$ with $\beta_{ff}[i]$ coefficients.

Feedforward tuning rules: integrators

So, the resulting response will not present any undesired dynamics or undershoot. This fact can be clearly observed by its consequent time response against unitary step

$$y(t) = \frac{-k_d t^{n_{ff}-1}}{\tau_{ff}^{n_{ff}} \left(n_{ff}-1\right)!} e^{-\frac{t}{\tau_{ff}}}$$

Three different tuning rules are proposed for τ_{ff} looking for

- Obtaining a desired settling time.
- Optimal solution for a tradeoff between maximum peak and settling time.

Feedforward tuning rules: integrators

Settling time rule

$$y(t) = \frac{-k_d t^{n_{ff}-1}}{\tau_{ff}^{n_{ff}} \left(n_{ff}-1\right)!} e^{-\frac{t}{\tau_{ff}}}$$

The settling time is defined as the time that the system takes to reach around 5% of its maximum value

$$y(t_{5\%}) = 0.05 M_{peak}$$

$$\frac{dy(t)}{dt} = 0 \Rightarrow t_{peak} \Rightarrow M_{peak} \Rightarrow t_{5\%}$$

Feedforward tuning rules: integrators

Settling time rule

$$t_{5\%} = \frac{x}{\tau_{ff}}, \quad 0.05 - \frac{x^{n_{ff}-1}}{\left(n_{ff}-1\right)^{n_{ff}-1}} e^{-x+n_{ff}-1} = 0$$

$$\tau_{ff} = \frac{t_{5\%}}{x}$$

For $n_u = 1$, the following solution is obtained

$$\tau_{ff} \approx \frac{t_{5\%}}{3}$$

Settling time rule: Example

$$P_u(s) = \frac{1}{s\,(0.25s+1)}$$

$$P_d(s) = \frac{0.5}{0.9s+1}$$

To obtain a reference tracking response with the closed-loop dynamics given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 2\frac{0.56s^2 + 1.5s + 1}{s\,(0.5s+1)}$$

Settling time rule: Example

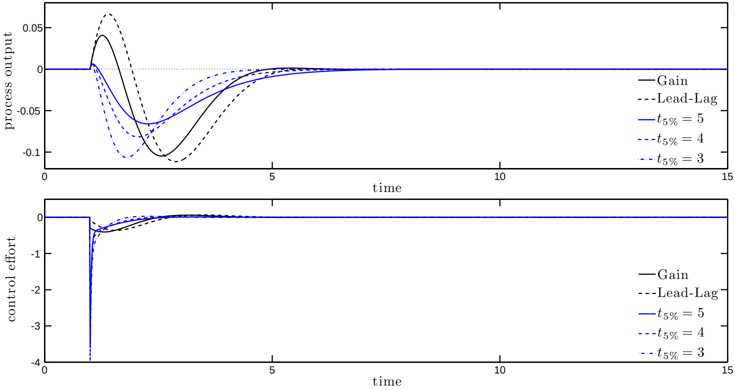
Then, the feedforward compensator is defined as

$$C_{ff}(s) = \frac{0.5}{(0.025s+1)\,(0.9s+1)\,(0.5s+1)} \frac{1+\sum_{i=1}^{6}\beta_{ff}[i]s^i}{\left(\tau_{ff}s+1\right)^3}$$

$$\tau_{ff} = 0.13t_{5\%}$$

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	τ_{ff}
$t_{5\%} = 5$	3.42	5.17	4.25	1.90	0.43	0.04	0.65
$t_{5\%} = 4$	3.42	4.78	3.50	1.38	0.27	0.02	0.52
$t_{5\%} = 3$	3.42	4.39	2.85	0.98	0.17	0.01	0.39

Settling time rule: Example



Settling time rule: Example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	18.57	1.16	-0.30
Lead-Lag	22.91	1.32	$-\mathbf{0.08}$
$t_{5\%} = 5$	15.14	$\mathbf{0.83}$	-3.47
$t_{5\%} = 4$	15.10	0.92	-3.60
$t_{5\%} = 3$	$\mathbf{15.05}$	1.06	-3.96

Optimal tuning rule

A tradeoff arises from the fact that by making τ_{ff} small, the settling time is reduced but the maximum peak is increased.

So, a cost function to find a tradeoff between settling time and maximum peak can be proposed as follows

$$J = \alpha t_{5\%} + (1-\alpha)\left|M_{peak}\right| \quad \alpha \in (0,1)$$

where α is a weighting parameter.

Optimal tuning rule

Then, substituting M_{peak} and $t_{5\%}$ equations previously calculated in J, when J is derivative with respect to τ_{ff} and is taken equal to zero

$$\frac{dJ}{d\tau_{ff}} = 0$$

the following solution is obtained

$$\tau_{ff} = \sqrt{|k_d| \frac{(1-\alpha)}{\alpha} \frac{e^{1-n_{ff}} \left(n_{ff}-1\right)^{n_{ff}-1}}{x \left(n_{ff}-1\right)!}}$$

α can be easily used as a tuning parameter to find a desired tradeoff between settling time and maximum peak values.

Optimal tuning rule: Example

$$P_u(s) = \frac{1}{s\left(s+1\right)}$$

$$P_d(s) = \frac{0.75}{\left(0.35s+1\right)^3}$$

To obtain a reference tracking response with the closed-loop dynamics given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 3.2\frac{0.75s^2 + 1.5s + 1}{s\left(0.2s+1\right)}$$

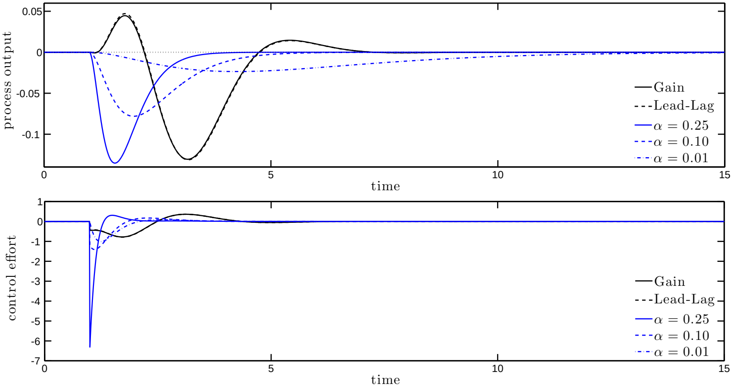
Optimal tuning rule: Example

Then, the feedforward compensator is defined as

$$C_{ff}(s) = \frac{0.75}{(0.35s+1)^3(0.2s+1)} \frac{1+\sum_{i=1}^{7}\beta_{ff}[i]s^i}{(\tau_{ff}s+1)^3}$$

Feedforward	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	$\beta_{ff}[7]$	τ_{ff}
$\alpha = 0.25$	3.55	5.05	3.54	1.39	0.32	0.04	0.01	0.28
$\alpha = 0.10$	3.55	5.67	4.75	2.17	0.53	0.06	0.01	0.49
$\alpha = 0.01$	3.55	9.06	15.95	15.52	6.89	6.88	0.01	1.62

Optimal tuning rule: Example



Optimal tuning rule: Example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	23.35	1.40	-0.45
Lead-Lag	23.60	1.41	-0.43
$\alpha = 0.25$	**14.06**	1.15	-6.31
$\alpha = 0.10$	**14.06**	0.87	-1.21
$\alpha = 0.01$	**14.06**	**0.48**	$\mathbf{-0.03}$

Feedforward tuning rules: integrators

1. Set τ_{ff} according to the desired specification:

$$\begin{aligned} \text{Settling time}: &\quad \tau_{ff} = t_{5\%}/x \\ \text{Optimal}: &\quad \text{tuning rule} \end{aligned}$$

2. Obtain the coefficients $\beta_{ff}[i]$ to cancel $D_d^-(s)D_{cl}(s)$.
3. Define the feedforward compensator as

$$C_{ff}(s) = \frac{k_d}{k_u}\frac{1}{D_{fb}(s)D_d^-(s)}\frac{1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i}{\left(\tau_{ff}s+1\right)^{n_{ff}}}$$

Outline

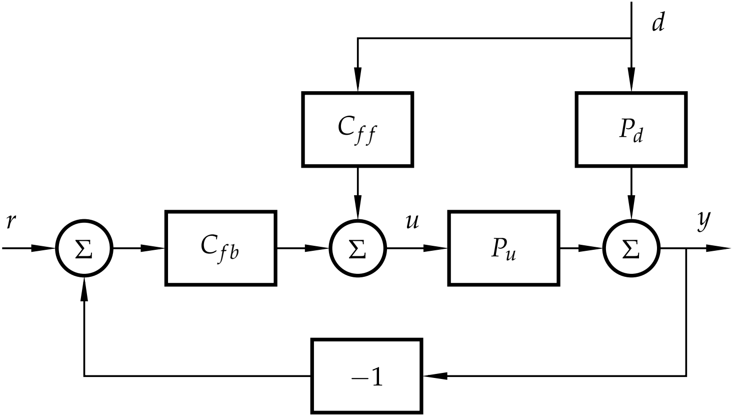
Robust disturbance compensation

Objective

To ensure a fast undershoot-free disturbance rejection even under the presence of uncertainty

Methodology

- Establish a robust disturbance rejection condition
- Propose an optimization procedure
- Suggest simple shapes for disturbance compensation


d
C_{ff}
P_d
r
Σ
C_{fb}
Σ
u
P_u
Σ
y
−1

Closed-loop relationships

$$\frac{y(s)}{r(s)} = \frac{L(s)}{1+L(s)} = \eta(s),$$

$$\frac{y(s)}{d(s)} = \frac{P_d(s) - C_{ff}(s)P_u(s)}{1+L(s)} = P_{ff}(s)\varepsilon(s),$$

$$L(s) = C_{fb}(s)P_u(s), \quad P_{ff}(s) = P_d(s) - C_{ff}(s)P_u(s)$$

Additive uncertainties are considered

$$P_k(s) = \overline{P}_k(s) + \Delta_k(s) \qquad k \in [u, k]$$

$$P_u(j\omega) = \overline{P}_u(j\omega) + \Delta_u(j\omega) \quad \forall\omega,$$
$$P_d(j\omega) = \overline{P}_d(j\omega) + \Delta_d(j\omega) \quad \forall\omega,$$

$$|\Delta_u(j\omega)| \leq \Delta_u^{max}(\omega) \quad \forall\omega,$$
$$|\Delta_d(j\omega)| \leq \Delta_d^{max}(\omega) \quad \forall\omega,$$

where $\Delta_u^{max}(\omega)$ and $\Delta_d^{max}(\omega)$ are the additive norm-bound uncertainties.

Robust feedforward and feedback tuning

Robust closed-loop relationships

$$G_{y/r} = \frac{y}{r} = \frac{\overline{L} + \Delta_L}{1 + \overline{L} + \Delta_L}$$

$$G_{y/d} = \frac{y}{d} = \frac{\overline{P}_{ff} + \Delta_{ff}}{1 + \overline{L} + \Delta_L}$$

where

$$\overline{L}(s) = C_{fb}(s)\overline{P}_u(s),$$

$$\overline{P}_{ff}(s) = \frac{\overline{y}_{ff}(s)}{d(s)} = \overline{P}_d(s) - C_{ff}(s)\overline{P}_u(s),$$

$$\Delta_L(s) = C_{fb}(s)\Delta_u(s),$$

$$\Delta_{ff}(s) = \Delta_d(s) - C_{ff}(s)\Delta_u(s).$$

Robust stability

The robust stability of the closed loop is determined by the robust stability of the feedback control system and the stability of the feedforward controller (as it acts on open loop).

The classical robust condition for a closed loop is obtained using Nyquist stability criterion

$$\left|C_{fb}(j\omega)\overline{\varepsilon}(j\omega)\right| \Delta_u^{max}(\omega) < 1 \quad \forall\omega,$$

Robust performance

It must be satisfied:

- Robust reference tracking
- Robust disturbance rejection

Robust performance: reference tracking

The problem for reference tracking remains the same as in a classical feedback scheme:

$$|\bar{\varepsilon}(j\omega)W_r(j\omega)| + \left|C_{fb}(j\omega)\bar{\varepsilon}(j\omega)\right| \Delta_u^{max}(\omega) < 1 \quad \forall\omega,$$

where W_r is a weighting function which determines the guaranteed performance.

Robust performance: disturbance rejection

Robust disturbance rejection performance depends on both controllers $C_{fb}(s)$ and $C_{ff}(s)$. A condition for robust disturbance rejection performance can be expressed as

$$\left|\frac{\overline{P}_{ff}(j\omega)+\Delta_{ff}(j\omega)}{1+\overline{L}(j\omega)+\Delta_L(j\omega)}\right| - |W_d(j\omega)| < |W_d(j\omega)|\,\psi(\omega) \quad \forall\omega$$

where $W_d(j\omega)$ is a weight that defines the desired disturbance rejection shape, and $\psi(\omega)$ is the tolerable degradation band over $W_d(j\omega)$.

Robust performance: disturbance rejection

So, a condition for robust disturbance rejection performance can be expressed as

$$\frac{\left|\overline{P}_{ff}(j\omega)\right| + \Delta_d^{max}(\omega) + \left|C_{ff}(j\omega)\right| \Delta_u^{max}(\omega)}{\left|1+\overline{L}(j\omega)\right| - \left|C_{fb}(j\omega)\right| \Delta_u^{max}(\omega)} \left|W_d(j\omega)\right|^{-1} < 1 + \psi(\omega), \quad \forall\omega.$$

where $W_d(j\omega)$ is a weight that defines the desired disturbance rejection shape, and $\psi(\omega)$ is the tolerable degradation band over $W_d(j\omega)$.

Constrained optimization problem

$$
\begin{aligned}
\min_{C_{fb},C_{ff}} \quad & \max_{\omega} \left(\theta_{rp}(\omega) + \theta_{dr}(\omega)\left|W_d^{-1}(s)\right|\right) \\
\text{subject to} \quad & \max_{\omega} \theta_{rp}(\omega) < 1 \\
& \max_{\omega} \theta_{dr}(\omega) < 0 \\
& \text{nominal stability}
\end{aligned}
$$

with

$$
\theta_{rp}(\omega) = |\overline{\varepsilon}(s)W_r(s)| + \left|C_{fb(s)}\overline{\varepsilon}(s)\right| \Delta_u^{max}(\omega)
$$

$$
\theta_{dr}(\omega) = \frac{\left|\overline{P}_{ff}(s)\right| + \Delta_d^{max}(\omega) + \left|C_{ff}(s)\right| \Delta_u^{max}(\omega)}{\left|1+\overline{L}(s)\right| - \left|C_{fb}(s)\right| \Delta_u^{max}(\omega)} \left|W_d(s)\right|^{-1} - (1+\psi(\omega)) .
$$

Constrained optimization problem

Note that $\theta_{dr}(\omega)$ is weighted by $\left|W_d^{-1}(s)\right|$ to scale the disturbance rejection shaping error at all the frequency range.

To efficiently solve this optimization problem, the following steps are executed:

1. *Define the controllers structure*. Both feedback and feedforward controllers must satisfy realizability constraints.
2. *Tune the optimization parameters*. A shaping procedure in time domain is proposed to determine the disturbance rejection weight.
3. *Choose an initial guess*. Initial parameter values are chosen using nominal conditions to guide the optimizer to a satisfying optimum.

Constrained optimization problem: controllers structure

The feedback controller is considered as

$$C_{fb}(s) = \frac{N_{fb}(s)}{D_{fb}(s)}$$

The feedforward compensator is defined as

$$C_{ff}(s) = \overline{\overline{C}}_{ff}(s)C'_{ff}(s)$$

Constrained optimization problem: controllers structure

The feedback controller is considered as

$$C_{fb}(s) = \frac{N_{fb}(s)}{D_{fb}(s)}$$

The feedforward compensator is defined as

$$C_{ff}(s) = \overline{C}_{ff}^{-}(s)C'_{ff}(s)$$

Constrained optimization problem: optimization parameters

There are three parameters for the optimization problem: $W_r(j\omega)$, $W_d(j\omega)$ and $\psi(\omega)$.

The reference tracking weight $W_r(s)$ can be selected following classical and well established recommendations that will not be discussed here for the sake of simplicity. $\varepsilon(s)$ is only a tolerance band that can be constant if the same error is admitted in all frequencies or can be defined as a function of ω to allow bigger errors in some frequency ranges.

The disturbance weight is the most difficult parameter to tune.

Constrained optimization problem: optimization parameters

In this case, a weighting methodology for $W_d(j\omega)$ in order to obtain an overshoot-free response based on time-domain specifications is proposed:

$$W_{td}(s) = \frac{y_{td}(s)}{d(s)} = \frac{\kappa_{td} s}{(\tau_{td} s + 1)^{n_{td}}} e^{-\lambda_{td} s} \quad n_{td} \in \mathbb{N}^+$$

where $\lambda_{td} = max(\lambda_u, \lambda_d)$ is a mandatory time delay, the zero at $s = 0$ gives the desired zero static gain (used to reject step disturbances) and κ_{td}, τ_{td}, n_{td} can be used to fix the other transient specifications of the response.

Constrained optimization problem: optimization parameters

However, since the effect of a time delay λ_{td} is not visible in the magnitude component, a H_2 optimization procedure is proposed in time domain using the following expression

$$\min_{C'_{ff}} \left\| \overline{y}_{ff}(t) - y_{td}(t) \right\|_2 \quad t_0 \leq t \leq t_f$$

where $\overline{y}_{ff}(t)$ and $y_{td}(t)$ are the step input responses for transfer functions $\overline{P}_{ff}(s) = y_{ff}(s)/d(s) = P_d(s) - C'_{ff}(s)P_u(s)$ and $W_{td}(s)$, respectively.

Constrained optimization problem: optimization parameters

$$\min_{C'_{ff}} \left\| \overline{y}_{ff}(t) - y_{td}(t) \right\|_2 \quad t_0 \leq t \leq t_f$$

The result of this procedure gives both, the optimal $C'_{ff}(s)$ and the consequent $\overline{P}_{ff}(s)$. Therefore, $\overline{P}_{ff}(s)$ can be used as an adequate weight $W_d(s)$ for the robust disturbance rejection response in the optimization problem. Notice also that $C'_{ff}(s)$ can be used as initial condition in the optimization process.

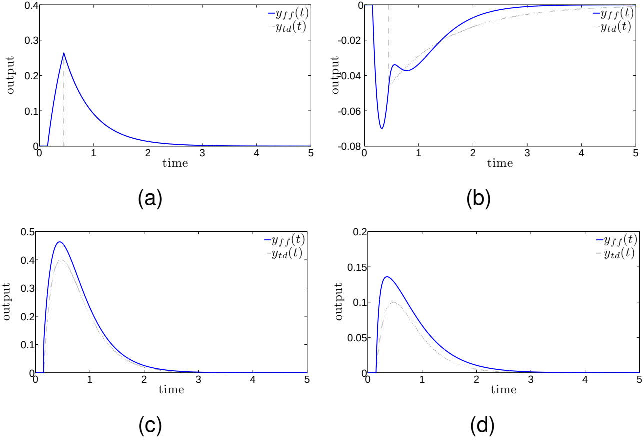
Design for typical cases

- **Case A. Non-realizable delay inversion**. This problem is originated when $\lambda_u > \lambda_d$. The desired settling time $t_{5\%}$ becomes a time domain design specification.
- **Case B. Non-minimum phase zeros**. The problem here is when $\overline{P}_u(s)$ has RHP zeros. Settling time $t_{5\%}$ or peak time t_{peak}, and peak value M_{peak} become two time domain design specifications.
- **Case C. Non-realizable delay inversion and non-minimum phase zeros**. Combination of the two previous cases results in another different problem. In this case a strictly proper weight with two time domain design specifications — settling or peak time, and maximum peak value — is required like in case B.

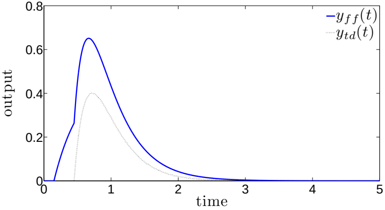
Robust feedforward and feedback tuning

Example	$\overline{P}_u(s)$	$\overline{P}_d(s)$	$\overline{C}_{ff}(s)$
1	$\frac{1}{s+1}e^{-0.45s}$	$\frac{0.5}{0.4s+1}e^{-0.15s}$	$\frac{0.5\,(s+1)}{0.4s+1}$
2	$\frac{1}{s+1}e^{-0.45s}$	$\frac{0.5\,(-0.3s+1)}{(0.4s+1)^2}e^{-0.15s}$	$\frac{0.5\,(s+1)}{(0.4s+1)^2}$
3	$\frac{-0.6s+1}{(s+1)^2}e^{-0.15s}$	$\frac{0.5}{0.4s+1}e^{-0.15s}$	$\frac{0.5\,(s+1)^2}{0.4s+1}$
4	$\frac{-0.6s+1}{(s+1)^2}e^{-0.15s}$	$\frac{0.5\,(-0.3s+1)}{(0.4s+1)^2}e^{-0.15s}$	$\frac{0.5\,(s+1)^2}{(0.4s+1)^2}$
5	$\frac{-0.6s+1}{(s+1)^2}e^{-0.45s}$	$\frac{0.5}{0.4s+1}e^{-0.15s}$	$\frac{0.5\,(s+1)^2}{0.4s+1}$
6	$\frac{-0.6s+1}{(s+1)^2}e^{-0.45s}$	$\frac{0.5\,(-0.3s+1)}{(0.4s+1)^2}e^{-0.15s}$	$\frac{0.5\,(s+1)^2}{(0.4s+1)^2}$

$W_d(j\omega)$ ***weighting methodology***



$W_d(j\omega)$ ***weighting methodology***



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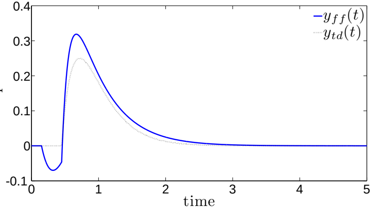

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$W_d(j\omega)$ ***weighting methodology***

Example	$t_{5\%}$	M_{peak}	$W_{td}(s)$	$C'_{ff}(s)$
1	2	$-$	$\frac{0.14s}{0.52s+1}e^{-0.45s}$	$\frac{0.45s+1}{0.52s+1}$
2	2	$-$	$\frac{-0.02s}{0.52s+1}e^{-0.45s}$	$\frac{0.08s+1}{0.01s+1}$
3	2	0.40	$\frac{0.35s}{(0.32s+1)^2}e^{-0.15s}$	$\frac{0.40s+1}{(0.26s+1)(0.01s+1)}$
4	2	0.10	$\frac{0.09s}{(0.32s+1)^2}e^{-0.15s}$	$\frac{0.09s+1}{0.05s+1}$
5	2	0.40	$\frac{0.29s}{(0.27s+1)^2}e^{-0.45s}$	$\frac{1}{0.18s+1}$
6	2	0.25	$\frac{0.18s}{(0.27s+1)^2}e^{-0.45s}$	$\frac{0.27s+1}{0.11s+1}$

Robust design parameters

Example	$W_r(s)$	$W_d(s)$
1	$\frac{15s}{10s+1}$	$\frac{0.5}{0.4s+1}e^{-0.15s}\left(1-\frac{0.45s+1}{0.52s+1}e^{-0.3s}\right)$
2	$\frac{15s}{10s+1}$	$\frac{0.5}{(0.4s+1)^2}e^{-0.15s}\left(-0.3s+1-\frac{0.08s+1}{0.01s+1}e^{-0.}\right.$
3	$\frac{15s}{10s+1}$	$\frac{0.5}{0.4s+1}e^{-0.15s}\left(1-\frac{-0.24s^2-0.2s+1}{0.0026s^2+0.27s+1}\right)$
4	$\frac{15s}{10s+1}$	$\frac{0.5}{(0.4s+1)^2}e^{-0.15s}\left(-0.3s+1-\frac{-0.054s^2-0.51}{0.05s+1}\right.$
5	$\frac{15s}{10s+1}$	$\frac{0.5}{0.4s+1}e^{-0.15s}\left(1-\frac{-0.6s+1}{0.18s+1}e^{-0.3s}\right)$
6	$\frac{15s}{10s+1}$	$\frac{0.5}{(0.4s+1)^2}e^{-0.15s}\left(-0.3s+1-\frac{-0.162s^2-0.33s+}{0.11s+1}\right.$

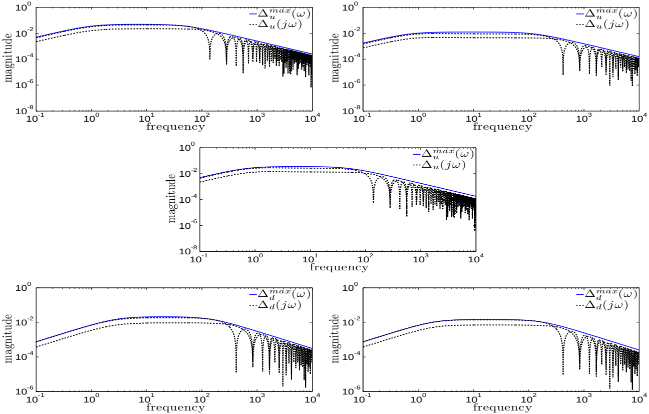
Additive norm-bound uncertainty

Assuming an uncertainty of $\pm 10\%$ in both λ_u and λ_d

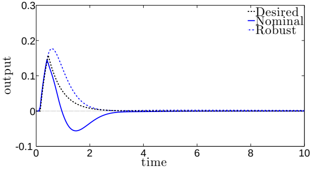
Example	$\Delta_u^{max}(s)$	$\Delta_d^{max}(s)$
1	$\frac{0.05s}{(s+1)(0.02s+1)}$	$\frac{0.0075s}{(0.35s+1)(0.007s+1)}$
2	$\frac{0.05s}{(s+1)(0.02s+1)}$	$\frac{0.0075s}{(0.5s+1)(0.006s+1)}$
3	$\frac{0.0175s}{(1.4s+1)(0.008s+1)}$	$\frac{0.0075s}{(0.35s+1)(0.007s+1)}$
4	$\frac{0.0175s}{(1.4s+1)(0.008s+1)}$	$\frac{0.0075s}{(0.5s+1)(0.006s+1)}$
5	$\frac{0.05s}{(1.4s+1)(0.02s+1)}$	$\frac{0.0075s}{(0.35s+1)(0.007s+1)}$
6	$\frac{0.05s}{(1.4s+1)(0.02s+1)}$	$\frac{0.0075s}{(0.5s+1)(0.006s+1)}$

Robust feedforward and feedback tuning

Additive norm-bound uncertainty



Process outputs of nominal and robust tuning



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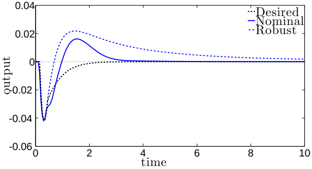

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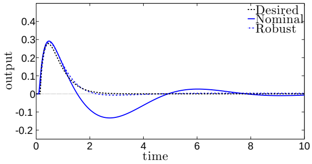

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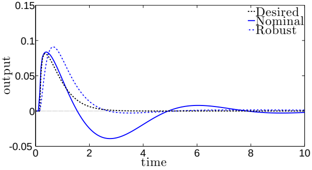

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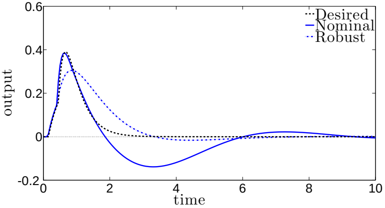
Process outputs of nominal and robust tuning



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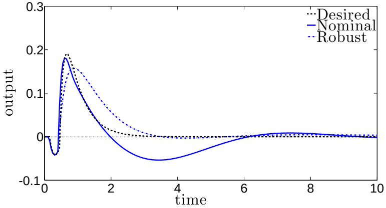

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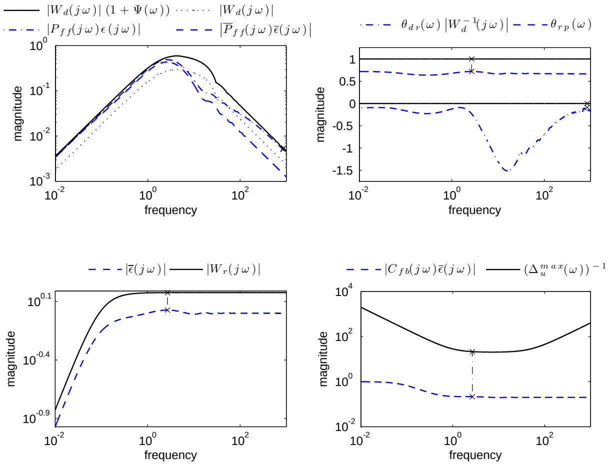
Numerical results

Example	Nominal			Robust		
	$\|e_d(t)\|_1$	$\|e_d(t)\|_2$	$\|e_d(t)\|_\infty$	$\|e_d(t)\|_1$	$\|e_d(t)\|_2$	$\|e_d(t)\|_\infty$
1	225.59	3.49	0.08	**153.10**	**2.56**	**0.07**
2	**56.10**	**0.85**	**0.02**	173.81	1.63	0.03
3	760.72	8.06	0.14	**115.02**	**1.17**	**0.03**
4	229.11	2.42	**0.04**	**111.96**	**1.57**	0.05
5	858.94	8.94	0.14	**478.58**	**6.15**	**0.13**
6	356.96	3.66	0.10	**236.31**	**3.02**	**0.07**

Example optimization result: non-realizable delay inversion



Outline

FSP with feedforward action

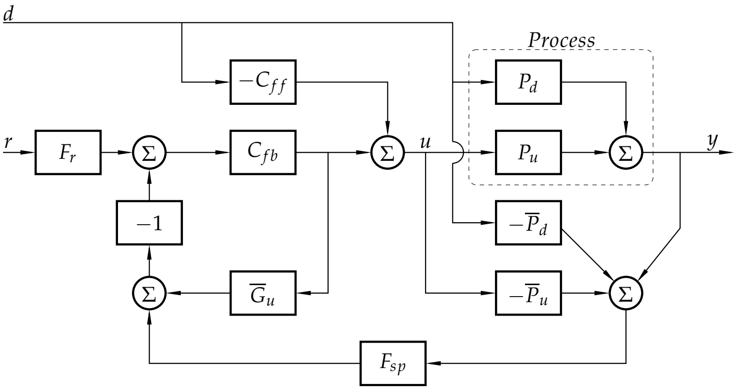
Objective

To obtain an optimal disturbance rejection for processes with large dead-times

Methodology

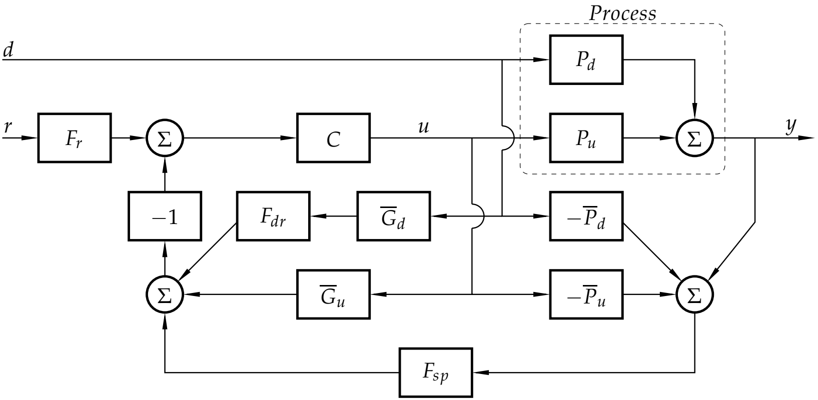
- Define a general structure for combined dead-time and feedforward compensation
- Decouple reference tracking, disturbance rejection and robustness tasks
- Propose simple tuning rules for fast overshoot-free disturbance rejection

Filtered Smith predictor

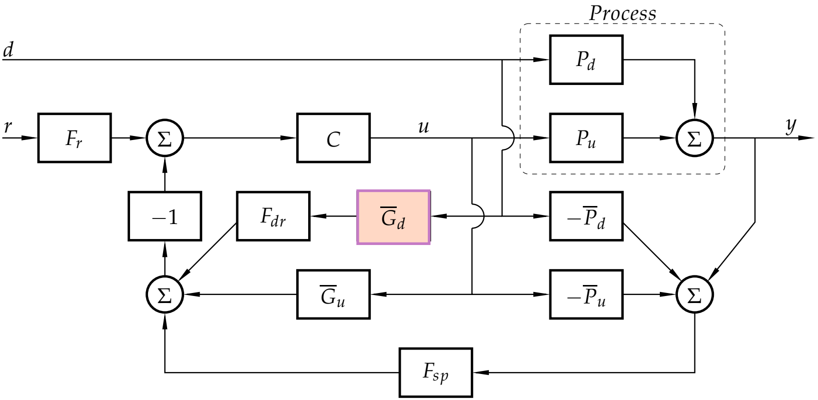


J. E. Normey-Rico and E. F. Camacho. Control of dead-time processes. Springer, London, 2007.

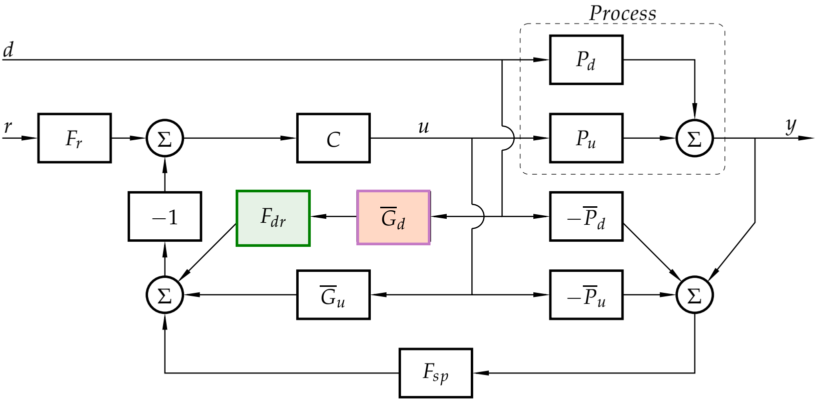
Proposed controller



Proposed controller



Proposed controller



Nominal closed-loop relationships

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{F_r(s)\overline{L}(s)}{1 + C(s)\overline{G}_u(s)}$$

$$G_{y/d}(s) = \frac{y(s)}{d(s)} = \overline{P}_d(s) - \frac{F_{dr}(s)\overline{G}_d(s)\overline{L}(s)}{1 + C(s)\overline{G}_u(s)}$$

Nominal closed-loop relationships

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{F_r(s)\overline{L}(s)}{1 + C(s)\boxed{\overline{G}_u(s)}}$$

$$G_{y/d}(s) = \frac{y(s)}{d(s)} = \overline{P}_d(s) - \frac{F_{dr}(s)\overline{G}_d(s)\overline{L}(s)}{1 + C(s)\boxed{\overline{G}_u(s)}}$$

Feedforward design for dead-time compensators

Nominal closed-loop relationships

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{F_r(s)\overline{L}(s)}{1 + C(s)\boxed{\overline{G}_u(s)}}$$

$$G_{y/d}(s) = \frac{y(s)}{d(s)} = \overline{P}_d(s) - \frac{F_{dr}(s)\boxed{\overline{G}_d(s)}\overline{L}(s)}{1 + C(s)\boxed{\overline{G}_u(s)}}$$

Robust stability

$$P_u(s) = \overline{P}_u(s)\left(1+\delta_u(s)\right)$$
$$P_d(s) = \overline{P}_d(s)\left(1+\delta_d(s)\right)$$

$$|\delta_u(j\omega)| \leq \delta_u^{max}(\omega) \quad \forall\omega > 0$$
$$|\delta_d(j\omega)| \leq \delta_d^{max}(\omega) \quad \forall\omega > 0$$

where $\delta_u^{max}(\omega)$, $\delta_d^{max}(\omega)$ are the multiplicative norm-bound uncertainties.

Robust stability

The characteristic equation for $P_u(s)$ is given by

$$1 + C(s)\overline{G}_u(s) + F_{sp}(s)\overline{L}(s)\delta_u(s) = 0.$$

Assuming that the *nominal system is stable*

$$\delta_u^{max}(\omega) < \mathrm{d}P_u(\omega) = \left|\frac{1 + C(j\omega)\overline{G}_u(j\omega)}{F_{sp}(j\omega)\overline{L}(j\omega)}\right| \quad \forall\omega > 0$$

J. E. Normey-Rico and E. F. Camacho. Unified approach for robust dead-time compensator design. Journal of Process Control, 19(1):38–47, 2009.

Robust stability

The characteristic equation for $P_u(s)$ is given by

$$1 + C(s)\overline{G}_u(s) + F_{sp}(s)\overline{L}(s)\delta_u(s) = 0.$$

Assuming that the *nominal system is stable*

$$\delta_u^{max}(\omega) < \mathrm{d}P_u(\omega) = \left| \frac{1 + C(j\omega)\overline{G}_u(j\omega)}{\boxed{F_{sp}(j\omega)}\overline{L}(j\omega)} \right| \quad \forall \omega > 0$$

J. E. Normey-Rico and E. F. Camacho. Unified approach for robust dead-time compensator design. Journal of Process Control, 19(1):38–47, 2009.

Tuning procedure

How to tune the proposed controller?

- Nominal reference tracking
- Nominal disturbance rejection
- Robustness

Tuning procedure: nominal reference tracking

$$D_{rt}(s) = N_u(s)N_c(s) + D_u(s)D_c(s)$$

$$F_r(s) = \frac{N_{rt}(s)}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)N_c(s) + D_u(s)D_c(s)$$

$$F_r(s) = \frac{N_{rt}(s)}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)\boxed{N_c(s)} + D_u(s)\boxed{D_c(s)}$$

$$F_r(s) = \frac{N_{rt}(s)}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)\boxed{N_c(s)} + D_u(s)\boxed{D_c(s)}$$

$$F_r(s) = \frac{N_{rt}(s)}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)\boxed{N_c(s)} + D_u(s)\boxed{D_c(s)}$$

$$F_r(s) = \frac{\boxed{N_{rt}(s)}}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)\boxed{N_c(s)} + D_u(s)\boxed{D_c(s)}$$

$$F_r(s) = \frac{\boxed{N_{rt}(s)}}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{N_u^+(s)N_{rt}(s)}{D_{rt}(s)}e^{-\lambda_u s}$$

Tuning procedure: nominal reference tracking

$$\boxed{D_{rt}(s)} = N_u(s)\boxed{N_c(s)} + D_u(s)\boxed{D_c(s)}$$

$$F_r(s) = \frac{\boxed{N_{rt}(s)}}{N_u^-(s)N_c(s)}$$

$$G_{y/r}(s) = \frac{y(s)}{r(s)} = \frac{\boxed{N_u^+(s)}N_{rt}(s)}{D_{rt}(s)}\boxed{e^{-\lambda_u s}}$$

Tuning procedure: nominal disturbance rejection

$$G_{y/d}(s) = \overline{P}_d(s) \cdot \left(1 - \frac{F_{dr}(s)N_c(s)N_u(s)}{D_{rt}(s)}e^{-(\lambda_u - \lambda_d)s}\right).$$

From previous equation, it can be seen that perfect disturbance rejection is accomplished for

$$F_{dr}(s) = \frac{D_{rt}(s)}{N_c(s)N_u(s)}e^{-(\lambda_d - \lambda_u)s}$$

Tuning procedure: nominal disturbance rejection

$$G_{y/d}(s) = \overline{P}_d(s) \cdot \left(1 - \frac{F_{dr}(s)N_c(s)N_u(s)}{D_{rt}(s)} e^{-(\lambda_u - \lambda_d)s}\right).$$

From previous equation, it can be seen that perfect disturbance rejection is accomplished for

$$F_{dr}(s) = \frac{D_{rt}(s)}{N_c(s)N_u(s)} e^{-(\lambda_d - \lambda_u)s}$$

Tuning procedure: nominal disturbance rejection

However, this expression may lead to an improper or even unstable transfer function and a more complicated design is required. Thus, the disturbance rejection filter can be chosen to cope with the commented problems and to decouple the reference and disturbance responses as

$$F_{dr}(s) = \frac{D_{rt}(s)}{N_c(s)N_u^-(s)} \cdot \frac{N_{dr}(s)}{D_{dr}(s)} e^{-\lambda_{dr}s}, \qquad (1)$$

where $N_{dr}(s)$ and $D_{dr}(s)$ are polynomials used to cancel undesired poles and to allocate a new set of them, respectively and $\lambda_{dr} = \max(0, \lambda_d - \lambda_u)$ is a dead time used to ensure that disturbance compensation is not made too early.

Tuning procedure: nominal disturbance rejection

With the proposed $F_{dr}(s)$, it is obtained

$$\begin{aligned} G_{y/d}(s) &= \overline{P}_d(s) \cdot \left(1 - \frac{N_u^+(s)N_{dr}(s)}{D_{dr}(s)} e^{-(\lambda_u - \lambda_d + \lambda_{dr})s}\right) \\ &= \frac{N_d(s)}{D_d(s)} \cdot \frac{QP_{dr}(s)}{D_{dr}(s)}, \end{aligned}$$

where $QP_{dr}(s)$ is a quasi-polynomial such that

$$QP_{dr}(s) = \left[D_{dr}(s) - N_u^+ N_{dr}(s) e^{-(\lambda_u - \lambda_d + \lambda_{dr})s}\right].$$

Tuning procedure: nominal disturbance rejection

$$G_{y/d}(s) = \frac{N_d(s)}{D_d(s)} \cdot \frac{QP_{dr}(s)}{D_{dr}(s)}$$

- $D_{dr}(s)$ should be designed to impose the main disturbance rejection dynamics:

$$D_{dr}(s) = (\tau_{dr}s + 1)^{n_{dr}}$$

- $N_{dr}(s)$ must be designed to eliminate the undesirable dynamics of $\overline{P}_d(s)$ (typically slow, integrating and unstable poles):

$$N_{dr}(s) = 1 + \sum_{i=1}^{m_{dr}} \beta_{dr}[i]s^i$$

Tuning procedure: robustness

$$F_{sp}(s) = \frac{D_{rt}(s)}{N_c(s)N_u^-(s)} \cdot \frac{N_{sp}(s)}{D_{sp}(s)}$$

$$\delta_u^{max}(\omega) < \left| \frac{D_{sp}(j\omega)}{N_{sp}(j\omega)N_u^+(j\omega)} \right| \quad \forall \omega > 0$$

Tuning procedure: robustness

$$F_{sp}(s) = \frac{D_{rt}(s)}{N_c(s)N_u^-(s)} \cdot \frac{N_{sp}(s)}{D_{sp}(s)}$$

$$\delta_u^{max}(\omega) < \left| \frac{D_{sp}(j\omega)}{N_{sp}(j\omega)N_u^+(j\omega)} \right| \quad \forall \omega > 0$$

Tuning guidline

1. Obtain process models $\overline{P}_u(s)$ and $\overline{P}_d(s)$.
2. Define the feedback controller $C(s)$ to set the desired reference tracking response denominator $D_{rt}(s)$.
3. Define the reference filter $F_r(s)$ to allocate the new set of zeros for the desired reference tracking response $N_{rt}(s)$.
4. Tune τ_{dr} and τ_{sp} to achieve, respectively, the desired speed of disturbance rejection response and robustness.
5. Compute the m_{dr} undesired poles of $\overline{P}_d(s)$, $s_d[i]$ $i = 1...m_{dr}$. Define $N_{dr}(s)$ as

$$N_{dr}(s) = 1 + \sum_{i=1}^{m_{dr}} \beta_{dr}[i]s^i$$

6. Set $n_{dr} = m_{dr} + degree\,(D_{rt}(s)) - degree\,(N_c(s)N_u^-(s))$ and define $D_{dr}(s)$ as

$$D_{dr}(s) = (\tau_{dr}s + 1)^{n_{dr}}$$

in order to have a proper compensator.

Tuning guidline

7. Set $\lambda_{dr} = \max\left(0, \lambda_d - \lambda_u\right)$ to ensure the fastest disturbance compensation as possible.
8. Compute the $\beta_{dr}[i]$ coefficients to impose that every $s_d[i], i = 1...m_{dr}$ is a root of the quasi-polynomial

$$QP_{dr}(s) = \left[D_{dr}(s) - N_{dr}(s)N_u^+(s)e^{-(\lambda_u - \lambda_d + \lambda_{dr})s}\right].$$

9. Compute the m_{sp} undesired poles of $\overline{P}_u(s)$, $s_u[i]$ $i = 1...m_{sp}$. Define $N_{sp}(s)$ as

$$N_{sp}(s) = 1 + \sum_{i=1}^{m_{sp}} \beta_{sp}[i]s^i$$

10. Set $n_{sp} = m_{sp} + degree\left(D_{rt}(s)\right) - degree\left(N_c(s)N_u^-(s)\right)$ and define $D_{sp}(s)$ as

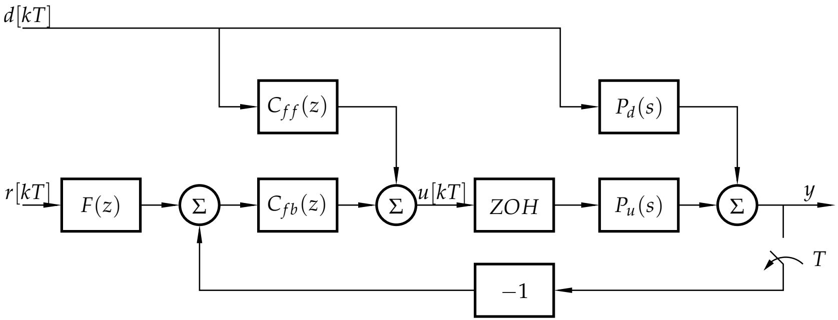
$$D_{sp}(s) = \left(\tau_{sp}s + 1\right)^{n_{sp}}$$

in order to have a proper compensator.

11. Compute the $\beta_{sp}[i]$ coefficients to impose that every $s_u[i], i = 1...m_{sp}$ is a root of the quasi-polynomial

$$QP_{sp}(s) = \left[D_{sp}(s) - N_{sp}(s)N_u^+(s)e^{-\lambda_u s}\right].$$

Discrete-time implementation



Discrete-time implementation

$$F(z) = \frac{F_r(z)}{F_{sp}(z)},$$

$$C_{fb}(z) = \frac{C(z)F_{sp}(z)}{1 + C(z)\left(\overline{G}_u(z) - F_{sp}(z)\overline{P}_u(z)\right)}$$

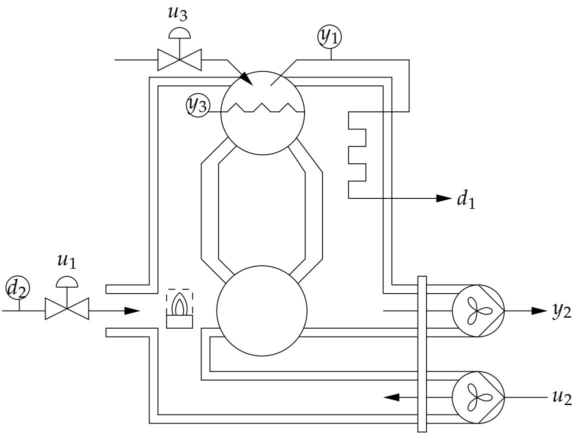
$$C_{ff}(z) = \frac{C(z)\left(F_{dr}(z)\overline{G}_d(z) - F_{sp}(z)\overline{P}_d(z)\right)}{1 + C(z)\left(\overline{G}_u(z) - F_{sp}(z)\overline{P}_u(z)\right)}$$

Case studies

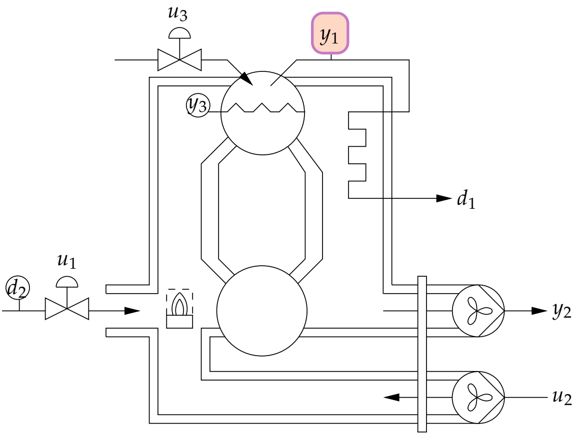
Some simulations are performed

- Steam pressure control in a boiler
- Concentration control in an unstable reactor
- Concentration control in a CSTR

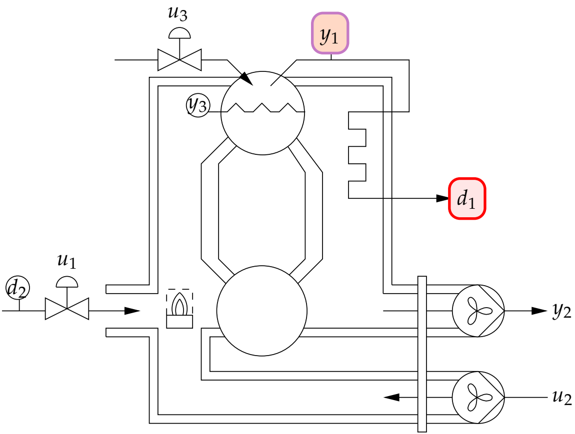
Results: boiler



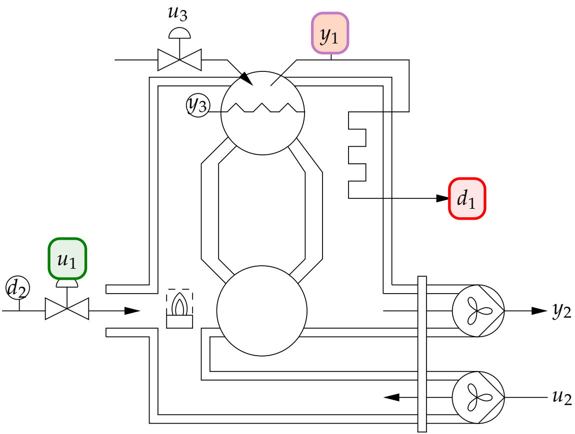
Results: boiler



Results: boiler



Results: boiler



Results: boiler

$$P_u(s) = \frac{y_1(s)}{u_1(s)} = \frac{0.355}{24.75s + 1} e^{-6.75s}$$

$$P_d(s) = \frac{y_1(s)}{d_1(s)} = \frac{-0.712}{195.8s + 1}$$

G. Pellegrinetti and J. Bentsman. Nonlinear control oriented boiler modeling – A benchmark problem for controller design. IEEE Transactions on Control Systems Technology, 4(1):57–64, 1996.

Results: boiler

The desired reference tracking was set as

$$G_{y/r}(s) = \frac{1}{(6.75s + 1)^2}$$

which results in

$$C(s) = 25.77 \cdot \frac{13.64s + 1}{13.64s}$$

$$F_r(s) = \frac{1}{13.64s + 1}.$$

Results: boiler

The feedforward controller is tuned using classic tuning rules

$$C_{ff}(s) = -\frac{0.712}{0.355} \cdot \frac{24.75s + 1}{195.8s + 1}.$$

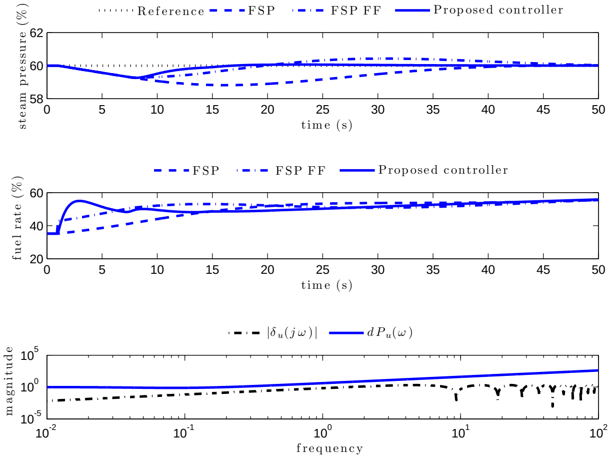
The slow disturbance pole is also cancellated and it is considered that $\tau_{dr} = 1.5$:

$$F_{dr}(s) = \frac{(6.75s + 1)^2}{13.64s + 1} \cdot \frac{8.8789s + 1}{(1.5s + 1)^2}.$$

The robustness filter is chosen to cancel the slow disturbance pole and $\tau_{sp} = 20$ is selected to obtain a faster response:

$$F_{sp}(s) = \frac{(6.75s + 1)^2}{13.64s + 1} \cdot \frac{24.8756s + 1}{(20s + 1)^2}.$$

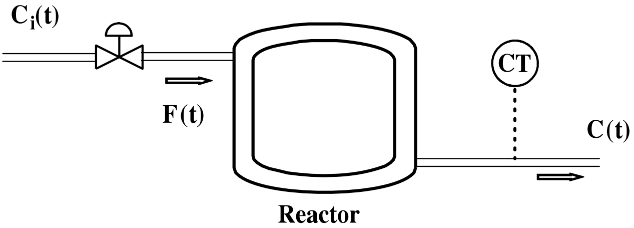
Results: boiler



Results: boiler

Controller	IAE	$ITAE$	ISE
FSP	26.55	495.92	23.26
FSP with open-loop feedforward	15.56	334.00	6.69
Proposed controller	**6.10**	**66.71**	**2.53**

Results: unstable reactor



Results: unstable reactor

$$P_u(s) = \frac{C(s)}{C_i(s)} = \frac{3.433}{103.1s - 1} \cdot e^{-20s}$$

$$P_d(s) = \frac{C(s)}{F(s)} = \frac{-206.9346}{103.1s - 1} \cdot e^{-10s}$$

Results: unstable reactor

The desired reference tracking was set as in the original paper:

$$C(s) = 3.29\frac{43.87s + 1}{43.87s}$$

$$F_r(s) = \frac{20s + 1}{43.87s + 1}$$

Results: unstable reactor

The feedforward controller is tuned using classic tuning rules

$$C_{ff}(s) = -\frac{206.9346}{3.433}$$

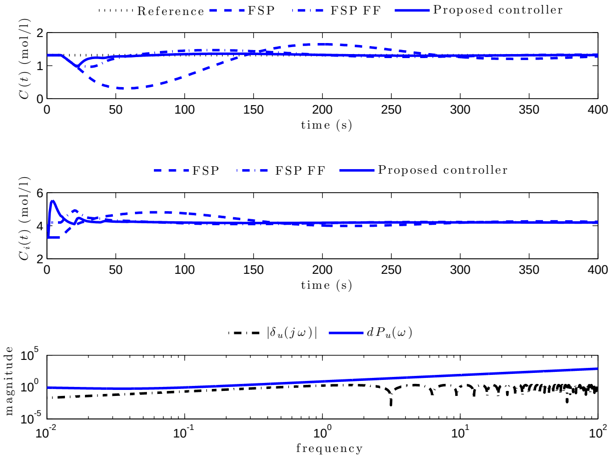
The slow disturbance pole is also cancellated and it is considered that $\tau_{dr} = 2.5$:

$$F_{dr}(s) = \frac{(20s+1)^2}{43.87s+1} \cdot \frac{13.7875s+1}{(2.5s+1)^2}$$

The robustness filter is chosen to cancel the slow disturbance pole and $\tau_{sp} = 26$ is selected:

$$F_{sp}(s) = \frac{(20s+1)^2}{43.87s+1} \cdot \frac{93.16s+1}{26s+1}$$

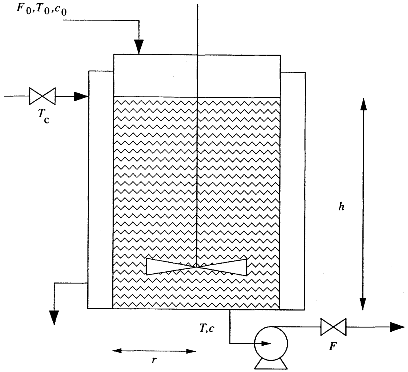
Results: unstable reactor



Results: unstable reactor

Controller	IAE	$ITAE$	ISE
FSP	119.36	14723.26	72.07
FSP with open-loop feedforward	28.41	3426.62	4.45
Proposed controller	**11.03**	**1118.02**	**1.17**

Results: Continuous Stirred Tank Reactor (CSTR)



Results: CSTR

$$P_u(s) = \frac{T(s)}{T_c(s)} = 1.6898 \cdot \frac{0.8491s + 1}{0.8286s^2 + 1.4555s + 1} \cdot e^{-s}$$

$$P_d(s) = \frac{T(s)}{F_0(s)} = -0.2339 \cdot \frac{(0.7363s + 1)(-0.2339s + 1)}{s(0.8286s^2 + 1.4555s + 1)} \cdot e^{-0.25s}$$

M. A. Henson and D. E. Seborg. Nonlinear Process Control. Prentice Hall PTR, Upper Saddle River, NJ, 1997.

Results: CSTR

The feedback controller is tuned to define the nominal reference tracking denominator as

$$D_{rt}(s) = s + 1$$

resulting in

$$C(s) = 0.5918 \cdot \frac{0.8286s^2 + 1.4555s + 1}{s\,(0.8491s + 1)}$$

Results: CSTR

The feedforward controller is tuned using classic tuning rules

$$C_{ff}(s) = -\frac{0.2339}{1.6898}$$

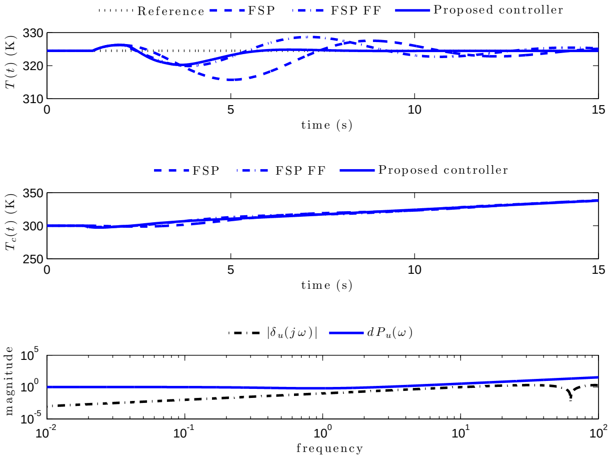
The slow disturbance pole is also cancellated and it is considered that $\tau_{dr} = 0.5$:

$$F_{dr}(s) = (s+1) \cdot \frac{1.75s+1}{(0.5s+1)^2}$$

The robustness filter is chosen to cancel the slow disturbance pole and $\tau_{sp} = 1$ is selected:

$$F_{sp}(s) = \frac{3s+1}{s+1}$$

Results: CSTR



Results: CSTR

Controller	IAE	$ITAE$	ISE
FSP	36.51	233.87	187.28
FSP with open-loop feedforward	25.55	174.13	72.33
Proposed controller	**10.12**	**38.93**	**27.86**

Outline

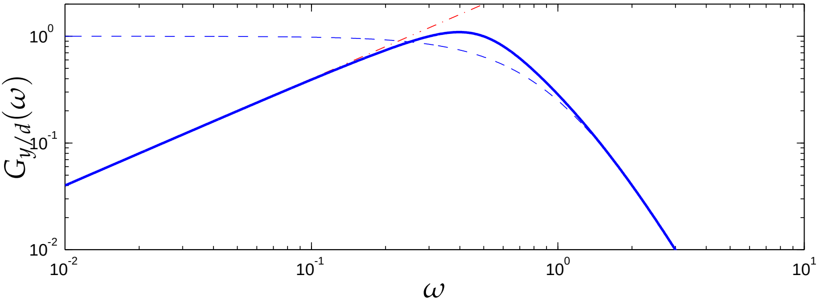
Performance indices for feedforward control

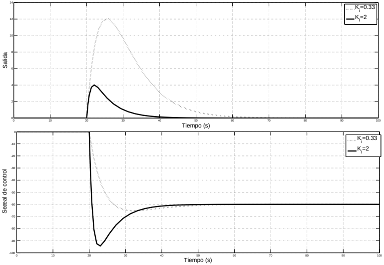
There exist metrics to evaluate feedback controllers for load disturbance rejection problem based on the controller parameters. For instance:

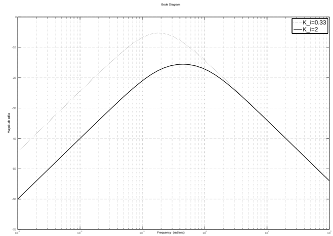
$$G_{y/d} = \frac{P_u(s)}{1 + C_{fb}(s)P_u(s)} = \frac{C_{fb}(s)P_u(s)}{1 + C_{fb}(s)P_u(s)}\frac{1}{C_{fb}(s)} \quad \omega \downarrow\downarrow$$

$$G_{y/d} \approx \frac{1}{C_{fb}(s)} \approx \frac{s}{\kappa_i}$$

Performance indices for feedforward control




Salida
Tiempo (s)
Seæal de control
Tiempo (s)
K_i=0.33
K_i=2


Bode Diagram
K_i=0.33
K_i=2

Performance indices for feedforward control

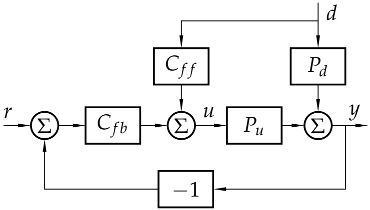
Objective

To proposed indices such that the advantage of using a feedforward compensator with respect to the use of a feedback controller only can be quantified.

Methodology

- Propose different indices
- Calculate the indices based on the process parameters

The two feedforward schemes are considered:



Performance indices for feedforward control

Assumptions:

$$P_u = \frac{\kappa_u}{1+\tau_u} e^{-s\lambda_u}, \quad P_d = \frac{\kappa_d}{1+s\tau_d} e^{-s\lambda_d}$$

Only, the non-inversion delay problem is analyzed:

$$\text{Lead-lag:} \quad C_{ff} = \frac{\kappa_d}{\kappa_u}\frac{1+s\tau_u}{1+s\tau_d}$$

Assumptions:

$$C_{fb} = \kappa_{fb}\left(1 + \frac{1}{s\tau_i}\right)$$

The lambda tuning rule is considered:

$$\kappa_{fb} = \frac{\tau_i}{\kappa_u(\lambda_u + \tau_{bc})}, \qquad \tau_i = \tau_u$$

where τ_{bc} is the closed-loop time constant.

Performance indices for feedforward control

The following index structure is proposed

$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}},$$

where IAE_{FB} is the integrated absolute value of the control error obtained when only feedback is used, and IAE_{FF} is the corresponding IAE value obtained when feedforward is added to the loop.

As long as the feedforward improves control, i.e. $IAE_{FF} < IAE_{FB}$, the index is in the region $0 < I_{FF/FB} < 1$.

Calculation of IAE_{fb}

In the feedback only case, the transfer function between disturbance d and process output y is

$$G_{y/d}(s) = \frac{P_d(s)}{1 + P_u(s)C_{fb}(s)} = \frac{\kappa_d \dfrac{e^{-s\lambda_d}}{1+s\tau_d}}{1 + \kappa_u \dfrac{e^{-s\lambda_u}}{1+s\tau_u}\kappa_{fb}\dfrac{1+s\tau_i}{s\tau_i}}$$

Assuming that $r = 0$ and d is a step with magnitude A_d and using the final value theorem, the Integrated Error (IE) value becomes (note that $e = -y$, with $r = 0$)

$$IE_{FB} = \int_0^\infty e(t)dt = \lim_{s\to 0} s \cdot \frac{1}{s}E(s) = \lim_{s\to 0} -G_{y/d}(s)\frac{A_d}{s} = -\frac{\tau_i \kappa_d}{\kappa_u \kappa_{fb}} A_d$$

Calculation of IAE_{fb}

The magnitude of the IE value can be set equal to the IAE value provided that the controller is tuned so that there are no oscillations:

$$IAE_{FB} = \frac{\tau_i \kappa_d}{\kappa_u \kappa_{fb}} A_d$$

Finally, considering the lambda tuning rule, it becomes

$$IAE_{FB} = \kappa_d A_d (\lambda_u + \tau_{bc})$$

Calculation of IAE_{FF} for classical FF scheme

In this case, the transfer function from the disturbance to the error is

$$G_{y/d}(s) = -\frac{P_d(s) + P_u(s)C_{ff}(s)}{1 + P_u(s)C_{fb}(s)} = \frac{\kappa_d \dfrac{e^{-s\lambda_d}}{1+s\tau_d} - \kappa_d \dfrac{e^{-s\lambda_u}}{1+s\tau_d}}{1 + \kappa_u \dfrac{e^{-s\lambda_u}}{1+s\tau_u} \kappa_{fb} \dfrac{1+s\tau_i}{s\tau_i}}$$

Considering the lambda tuning rule and that the delays are approximated as

$$e^{-\lambda_u s} \cong 1 - \lambda_u s, \qquad e^{-\lambda_d s} \cong 1 - \lambda_d s$$

It results in:

$$G_{y/d}(s) = -\frac{\kappa_d(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)s^2}{(1+\tau_d s)(1+\tau_{bc} s)}$$

Calculation of IAE_{FF} for classical FF scheme

The IE value for this case becomes

$$IE_{FF} = \int_0^{\infty} e(t)dt = \lim_{s \to 0} s \cdot \frac{1}{s} G_{y/d}(s) \frac{A_d}{s} = 0.$$

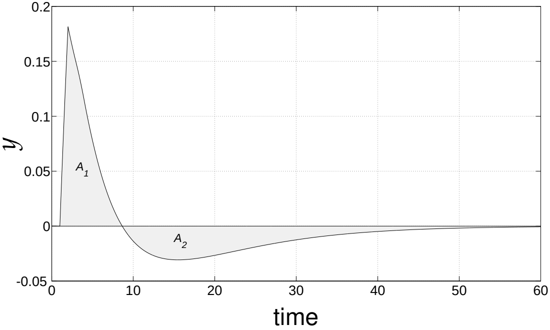
which demonstrates that zero steady-state error can be achieved by using feedforward control.

Calculation of IAE_{FF} ***for classical FF scheme***

Now, it is worth determining the expression of the error in the time domain when a step signal of amplitude A_d is applied as a disturbance. We have

$$e(t) = \begin{cases} \dfrac{\kappa_d A_d(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)}{\tau_{bc}\tau_d(\tau_{bc} - \tau_d)} \left(\tau_d e^{-t/\tau_{bc}} - \tau_{bc} e^{-t/\tau_d}\right) & \tau_{bc} \neq \tau_d \\ \dfrac{\kappa_d A_d(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)}{\tau_d^2} \left(1 - \dfrac{t}{\tau_d}\right) e^{-t/\tau_d} & \tau_{bc} = \tau_d \end{cases}$$

Calculation of IAE_{FF} for classical FF scheme



$$t_0 = \begin{cases} \frac{\tau_{bc}\tau_d}{\tau_{bc}-\tau_d}\log\left(\frac{\tau_{bc}}{\tau_d}\right) & \tau_{bc} \neq \tau_d \\ \tau_d & \tau_{bc} = \tau_d \end{cases}$$

Performance indices for feedforward control

Calculation of IAE_{FF} for classical FF scheme

We can therefore calculate the area of the first part of the transient as

$$A_1 = \int_0^{t_0} e(t)dt = \begin{cases} \dfrac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)\left(\dfrac{\tau_{bc}}{\tau_d}\right)^{-\dfrac{\tau_{bc}}{\tau_{bc}-\tau_d}} & \tau_{bc} \neq \tau_d \\ \dfrac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)e^{-1} & \tau_{bc} = \tau_d \end{cases}$$

Performance indices for feedforward control

According to

$$IE_{FF} = \int_0^{\infty} e(t)dt = \lim_{s \to 0} s \cdot \frac{1}{s} G_{y/d}(s) \frac{A_d}{s} = 0.$$

the area $|A_2|$ in the previous figure is equal to $|A_1|$, and the IAE value can finally be determined as

$$IAE_{FF} = 2|A_1| = \begin{cases} 2\dfrac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)\left(\dfrac{\tau_{bc}}{\tau_d}\right)^{-\dfrac{\tau_{bc}}{\tau_{bc} - \tau_d}} & \tau_{bc} \neq \tau_d \\ 2\dfrac{\kappa_d A_d}{\tau_d}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d)e^{-1} & \tau_{bc} = \tau_d \end{cases}$$

Calculation of IAE_{FF} for non-interacting FF scheme

In this case, the IAE_{FF} estimation can be obtained in a straightforward manner, as the effect from the feedback controller is removed.

The IAE result obtained in the non-invertible delay case can be reformulated as

$$
\begin{aligned}
IAE_{FF} &= \kappa_d A_d \left((\lambda_u - \lambda_d) - (\tau_d - \tau_u - \tau_u + \tau_u) \left(1 - 2e^{-\frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}} \right) \right) \\
&= \kappa_d A_d \left(1 - \frac{\tau_d - \tau_u - \tau_u + \tau_u}{\lambda_u - \lambda_d} \left(1 - 2e^{-\frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}} \right) \right) (\lambda_u - \lambda_d) \\
&= \kappa_d A_d \left(1 - \frac{1}{a} + \frac{2}{a} e^{-a} \right) (\lambda_u - \lambda_d) \\
&= \kappa_d A_d \alpha (\lambda_u - \lambda_d)
\end{aligned}
$$

where

$$
\alpha = 1 - \frac{1}{a} + \frac{2}{a} e^{-a}, \quad a = \frac{\lambda_u - \lambda_d}{\tau_d - \tau_u - \tau_u + \tau_u}
$$

Calculation of IAE_{FF} for non-interacting FF scheme

Here, when using classical feedforward design ($\tau_{ff} = \tau_d$, $\beta_{ff} = \tau_u$), it results that

$$IAE_{FF} = \kappa_d A_d(\lambda_u - \lambda_d) \quad \text{with } a = \infty \quad \text{and } \alpha = 1$$

However, if τ_u is tuned, for instance, to minimize IAE_{FF} using the following value

$$\tau_u = \begin{cases} \tau - \frac{\lambda_u - \lambda_d}{1.7} & 0 < \lambda_u - \lambda_d \leq 1.7\tau \\ 0 & \lambda_u - \lambda_d > 1.7\tau \end{cases}$$

The following values for a and α are obtained:

$$\begin{array}{lll} 0 \leq \lambda_u - \lambda_d \leq 1.7\tau_d: & a = 1.7 & \alpha \approx 0.63 \\ \lambda_u - \lambda_d > 1.7\tau_d: & a = \frac{\lambda_u - \lambda_d}{\tau_d} > 1.7 & 0.63 < \alpha < 1 \end{array}$$

Analysis and discussion on the indices

- Feedback control without feedforward:

$$IAE_{FB} = \kappa_d A_d(\lambda_u + \tau_{bc})$$

- Feedforward with classical control scheme and classical tuning:

$$IAE_{FF} = 2\frac{\kappa_d A_d}{\tau}(\lambda_u + \tau_{bc})(\lambda_u - \lambda_d) f(\tau_{bc}/\tau_d) \tag{2}$$

where

$$f(\tau_{bc}/\tau_d) = \begin{cases} \left(\dfrac{\tau_{bc}}{\tau_d}\right)^{-\dfrac{\tau_{bc}}{\tau_{bc}-\tau_d}} & \tau_{bc} \neq \tau_d \\ e^{-1} & \tau_{bc} = \tau_d \end{cases} \tag{3}$$

- Feedforward with non-interacting control scheme:

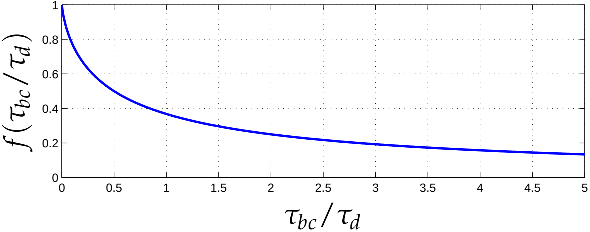
$$IAE_{FF} = \alpha\kappa_d A_d(\lambda_u - \lambda_d)$$

where α can vary based on the τ_{ff} value.

Performance indices for feedforward control

Analysis and discussion on the indices

Notice that the IAE_{FF} value corresponding to the classical scheme is quite complicated to analyze. To simplify the analysis, the function $f(\tau_{bc}/\tau_d)$ in (3) is shown



From this figure, one can see that the function is continuous, monotonically decreasing, and bounded to $0 \leq f(\tau_{bc}/\tau_d) \leq 1$.

Analysis and discussion on the indices

- All IAE_{FF} values are proportional to $(\lambda_u - \lambda_d)$. When $\lambda_u = \lambda_d$, we get $IAE_{FF} = 0$, which is correct since the feedforward action can eliminate the load disturbance response completely in this case.
- When $\lambda_u \gg \lambda_d$, the IAE_{FF} values become large. This is also correct, since the delay in the process prohibits the feedforward action from reducing the disturbance response in this case.

Analysis and discussion on the indices

The ratio between the IAE value of the classical scheme and the noninteracting scheme is

$$\frac{IAE_{\text{classical}}}{IAE_{\text{noninteracting}}} = \frac{2(\lambda_u + \tau_{bc}) f(\tau_{bc}/\tau_d)}{\tau_d \alpha}$$

Therefore, the classical scheme gives a smaller IAE value when

$$\tau_d > \frac{2(\lambda_u + \tau_{bc}) f(\tau_{bc}/\tau_d)}{\alpha}$$

Analysis and discussion on the indices

Since $0 < f(\tau_{bc}/\tau) \leq 1$ and $0.63 < \alpha \leq 1$, one can conclude that the classical scheme gives a better performance when τ_d is large compared to process deadtime λ_u or the desired closed-loop time constant τ_{bc}, i.e. when the load disturbance is varying slowly.

Index interpretation

For the classical feedforward control case, the index becomes

$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}} = 1 - \frac{2(\lambda_u - \lambda_d)}{\tau_d} f(\tau_{bc}/\tau_d)$$

For the noninteracting feedforward control scheme, the index is given by

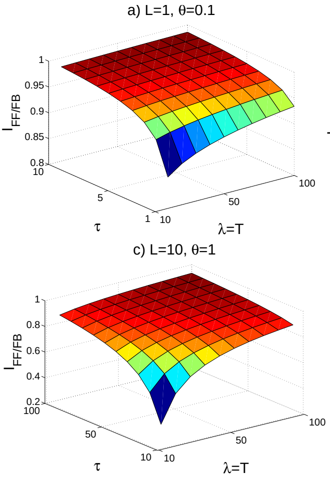
$$I_{FF/FB} = 1 - \frac{IAE_{FF}}{IAE_{FB}} = 1 - \frac{\alpha(\lambda_u - \lambda_d)}{\lambda_u + \tau_{bc}}$$

Performance indices for feedforward control

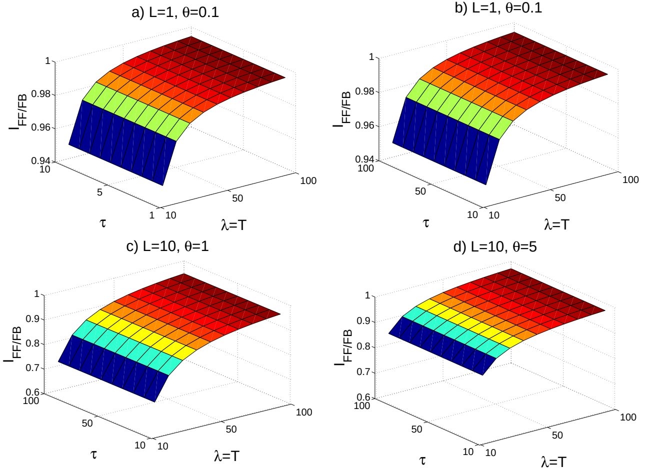
Index interpretation

- Increasing τ_{bc}, corresponding to a more conservative tuning, results in indices getting closer to one.
- In the classical scheme, $f(\tau_{bc}/\tau_d)$ decreases when τ_{bc} is increased.
- In the noninteracting scheme it is obvious that $I_{FF/FB}$ increases since τ_{bc} appears in the denominator of the second term.
- On the other hand, t can be observed that when $\lambda_u = \lambda_d$, all indices become $I_{FF/FB} = 1$, which means that the disturbance response can be eliminated completely by introducing feedforward.

Index interpretation: classical control scheme



Index interpretation: noon-interacting control scheme



Example 1

$$P_u(s) = \frac{e^{-2s}}{10s+1} \qquad P_d(s) = \frac{e^{-s}}{5s+1}$$

Using lambda tuning with $\tau_{bc} = \tau_u = 10$ gives the PI controller parameters $\kappa_{fb} = 0.83$ and $\tau_i = 10$.

The feedforward compensators are defined as

$$C_{ff}(s) = \frac{10s+1}{5s+1}$$

for the classical feedforward control scheme and as

$$C_{ff} = \frac{10s+1}{4.4s+1}$$

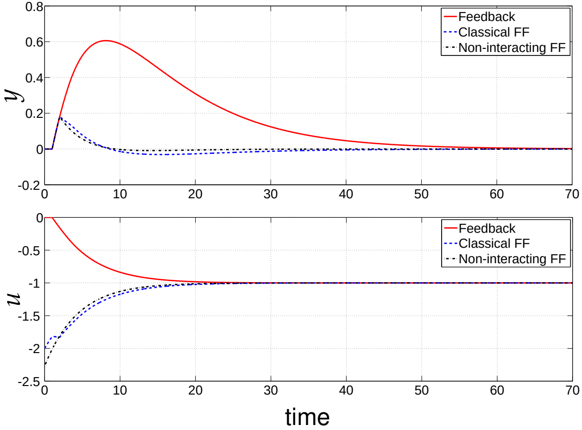
for the non-interacting feedforward control scheme (to minimize IAE).

Performance indices for feedforward control

Example 1

Control scheme	IAE^r	IAE^e	$I_{FF/FB}$
Feedback	11.99	12	–
Classical FF	1.21	1.2	0.9
Non-interacting FF	0.63	0.63	0.95

Example 1



0.8
0.6
0.4
0.2
0
-0.2
y
0
10
20
30
40
50
60
70
Feedback
Classical FF
Non-interacting FF
0
-0.5
-1
-1.5
-2
-2.5
u
time

Performance indices for feedforward control

Example 2

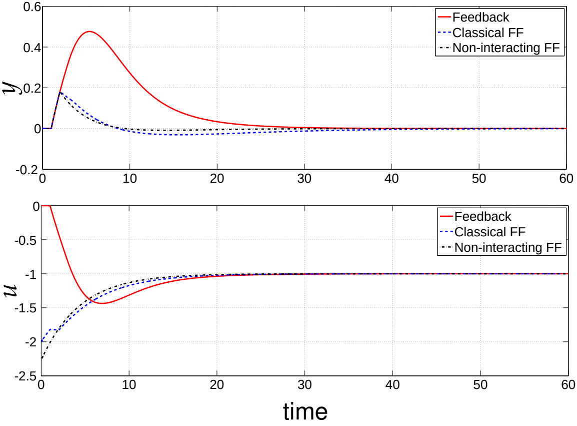
The differences between the pure feedback scheme and the feedforward schemes can be reduced by retuning the PI controller to obtain a more aggressive response. Lets retune the PI controller only for the case when pure feedback is used, by using $\tau_{bc} = 0.25\tau_u$.

Performance indices for feedforward control

Example 2

Control scheme	IAE^r	IAE^e	$I_{FF/FB}$
Feedback	4.5	4.5	–
Classical FF	1.21	1.2	0.73
Non-interacting FF	0.63	0.63	0.86

Example 2


0.6
0.4
0.2
0
-0.2
y
Feedback
Classical FF
Non-interacting FF
0
10
20
30
40
50
60
0
-0.5
-1
-1.5
-2
-2.5
u
time

Example 3

Assume that $\tau_{bc} = \tau_u = \lambda_u$. It means that we have a process model $P_u(s)$ where the delay is equal to the time constant and that the lambda tuning rule is used with $\tau_{bc} = \tau_u$. Two different values of the time constant $\tau_d = \eta\lambda_u$, where η = 1 or 10.

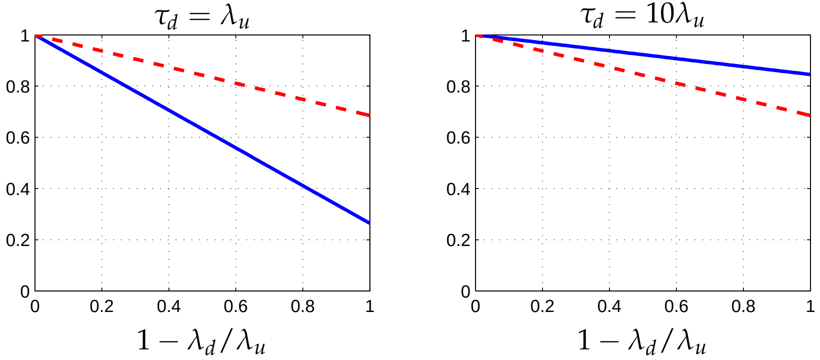
The index for the classical feedforward scheme becomes

$$I_{FF/FB} = 1 - \frac{2(\lambda_u - \lambda_d)}{\tau_d} f(1/\eta) = 1 - \frac{2}{\tau_d} f(1/\eta)\left(1 - \frac{\lambda_d}{\lambda_u}\right)$$

If instead the noninteracting scheme is used, the index is

$$I_{FF/FB} = 1 - \frac{\alpha(\lambda_u - \lambda_d)}{\lambda_u + \tau_{bc}} = 1 - \frac{\alpha}{2}\left(1 - \frac{\lambda_d}{\lambda_u}\right)$$

Example 3

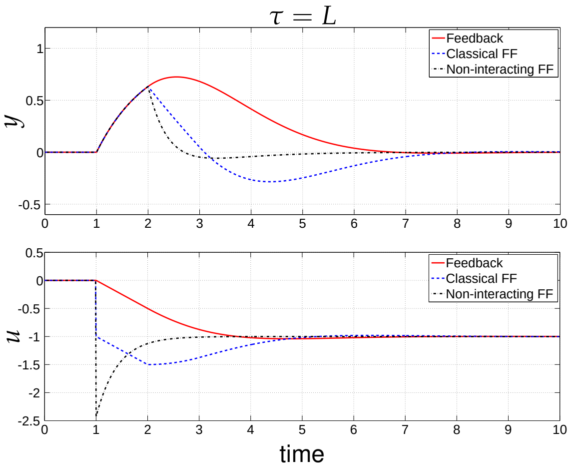


$\tau_d = \lambda_u$
$\tau_d = 10\lambda_u$
1
0.8
0.6
0.4
0.2
0
0.2
0.4
0.6
0.8
1
$1 - \lambda_d/\lambda_u$

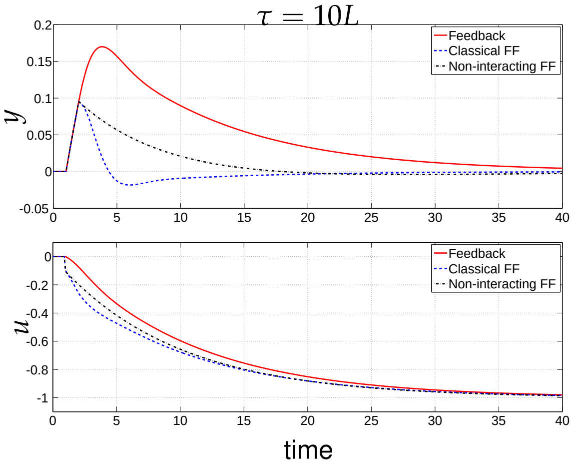
Example 3

τ_d	Control scheme	IAE^r	IAE^e	$I^r_{FF/FB}$	$I^e_{FF/FB}$
λ_u	Feedback	2.04	2.0		
	Classical FF	1.43	1.47	0.30	0.26
	Non-interacting FF	0.63	0.63	0.69	0.69
$10\lambda_u$	Feedback	2.00	2.0		
	Classical FF	0.34	0.31	0.83	0.85
	Non-interacting FF	0.63	0.63	0.69	0.69

Example 3


$\tau = L$
Feedback
Classical FF
Non-interacting FF
y
1
0.5
0
-0.5
0 1 2 3 4 5 6 7 8 9 10
Feedback
Classical FF
Non-interacting FF
u
0.5
0
-0.5
-1
-1.5
-2
-2.5
0 1 2 3 4 5 6 7 8 9 10
time

Example 3



Outline

Conclusions

- The motivation for feedforward tuning rules was introduced.
- The feedback effect on the feedforward design was analyzed.
- The different non-realizable situations were studied.
- The two available feedforward control schemes were used.
- Simple tuning rules based on the process and feedback controllers parameters were derived.
- Robust design should be used in processes with significant uncertainty.
- A general dead-time plus feedforward compensator can be used to efficiently decouple control tasks.
- Performance indices for feedforward control were proposed.

Conclusions

Future research

What else can be done?

- **Nominal tuning**. Unified methodology for low-order feedforward controllers tuning
- **Robust tuning**. Scale up to other feedforward structures
- **DTC with feedforward action**. Extension to MIMO processes
- **Experimental results**. Validate the theoretically claimed benefits
- **Distributed parameter systems**. Feedforward tuning rules to deal with resonance dynamics

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Collaborators

1. Carlos Rodríguez (UAL, Spain)
2. Manuel Berenguel (UAL, Spain)
3. Tore Hägllund (Lund, Sweden)
4. Julio Normey (Florianopolis, Brazil)
5. Antonio Visioli (Brescia, Italy)
6. Max Veronesi (Milano, Italy)

End of the presentation

Thank you for your attention