Advances in Feedforward Control

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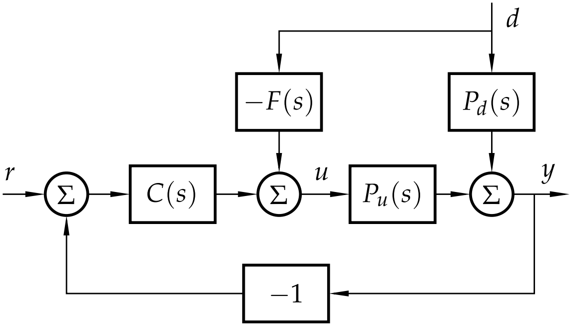
Outline

1. Feedforward control problem
2. Feedforward tuning rules
   - Non-realizable delay
   - Right-half plane zeros
   - Integrating behavior
3. Conclusions

Feedforward control problem



$$F(s) = \frac{P_d(s)}{P_u(s)}$$

$$Y = (P_d - P_u F)D$$

Feedforward control problem

Perfect compensation is seldom realizable:

- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.
- Improper transfer functions (high orders).

Classical solution

Ignore the non-realizable part of the compensator and implement the realizable one. In practice, static gain feedfoward compensators are quite common.

Feedforward control problem

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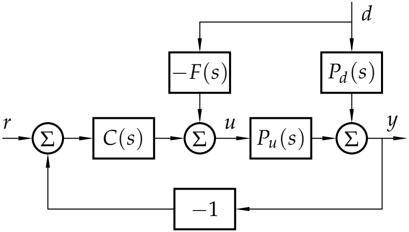
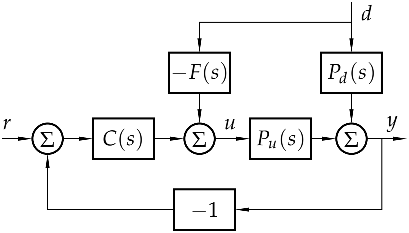
Feedforward control problem

Motivation

Feedforward compensators are tuned according to open-loop rules, $F(s) = P_d(s)/P_u(s)$, but when perfect compensation is not possible the feedback controller deteriorates the response.



Motivation



$$Y = (P_d - P_u F)D$$

Motivation

Lets consider the process and disturbance transfer functions are first-order systems with time delay:

$$P_u(s) = \frac{k_u}{\tau_u s + 1} e^{-sL_u}, \quad P_d(s) = \frac{k_d}{\tau_d s + 1} e^{-sL_d}$$

The feedback controller is a PID controller and the feedforward compensator is evaluated as a static controller and as a lead-lag filter:

$$F(s) = K_{ff}, \quad F(s) = K_{ff} \frac{T_z s + 1}{T_p s + 1} e^{-sL_{ff}}$$

Motivation

Then, lets consider a delay inversion problem, i.e., $L_d < L_u$. Then, the resulting feedforward compensators are given by:

$$F(s) = K_{ff} = \frac{k_d}{k_u}$$

$$F(s) = K_{ff} \frac{T_z s + 1}{T_p s + 1} e^{-sL_{ff}} = \frac{k_d}{k_u} \frac{\tau_u s + 1}{\tau_d s + 1}$$

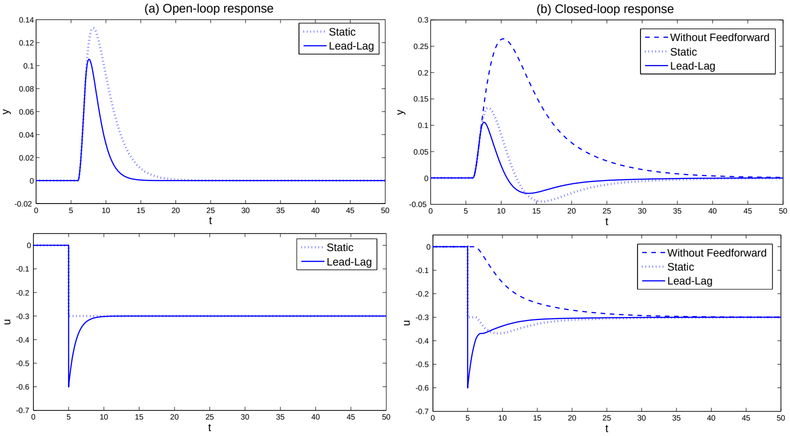
Motivation

Example:

$$P_u(s) = \frac{1}{2s+1}e^{-2s}, \quad P_d(s) = \frac{1}{s+1}e^{-s}$$

$$F(s) = 1, \quad F(s) = \frac{2s+1}{s+1}$$

Feedforward control problem



Feedforward control problem

Motivation

Conclusion

There is a need for tuning rules that take the feedback controller into account in the feedforward design.

Feedforward control problem

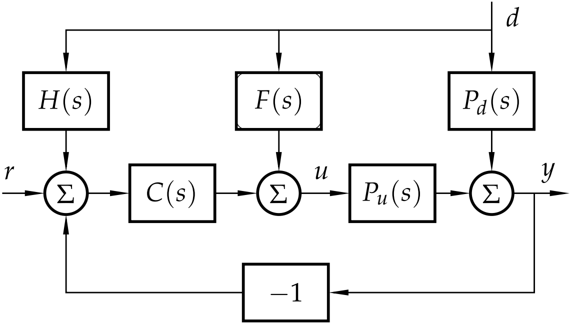
Motivation

There are not only a few tuning rules for feedforward control in literature:

- D. Seborg, T. Edgar, D. Mellichamp, Process Dynamics and Control, Wiley, New York, 1989.
- Shinskey, Process Control Systems. Application Design Adjustment, McGraw- Hill, New York, 1996.
- C. Brosilow, B. Joseph, Techniques of Model-Based Control, Prentice-Hall, New Jersey, 2002.
- A. Isaksson, M. Molander, P. Modn, T. Matsko, K. Starr, Low-Order Feedforward Design Optimizing the Closed-Loop Response, Preprints, Control Systems, 2008, Vancouver, Canada.

Feedforward control problem

Non-interacting control scheme or Brosilow scheme:



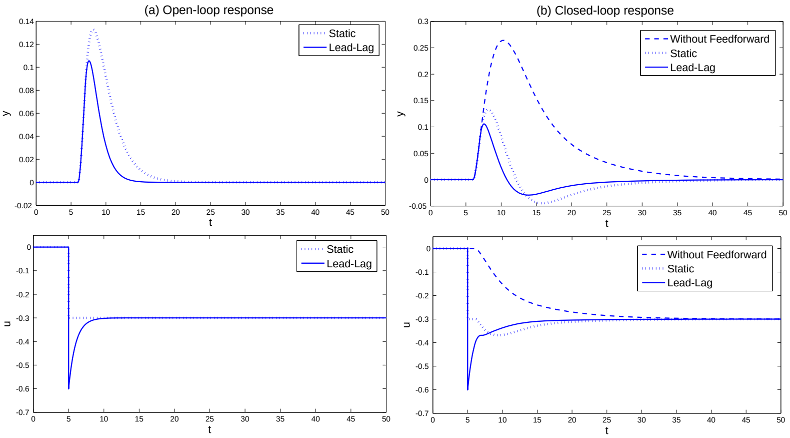
$$F(s) = \frac{P_d(s)}{P_u(s)}, \qquad H(s) = P_d(s) - P_u(s)F(s)$$

Feedforward tuning rules

Cases to be evaluated in this talk:

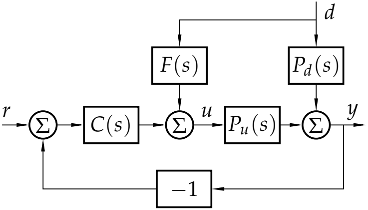
- Non-realizable delay inversion.
- Right-half plan zeros.
- Integrating poles.

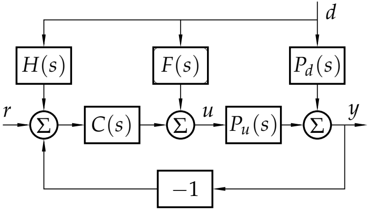
Feedforward tuning rules: non-realizable delay



Feedforward tuning rules: non-realizable delay

Two approaches:





Feedforward tuning rules: non-realizable delay

First approach

To deal with the non-realizable delay case, the first approach was to work with the following:

- Use the classical feedforward control scheme.
- Remove the overshoot observed in the response.
- Proposed a tuning rule to minimize Integral Absolute Error (IAE).
- The rules should be simple and based on the feedback and model parameters.

Feedforward tuning rules: non-realizable delay

To remove the overshoot, the feedback control action is taken into account to calculate the feedforward gain, K_{ff}.

$$\Delta u = \frac{K}{T_i} \int e dt = \frac{K}{T_i} IE \cdot d$$

So, in the new rule, the goal is to take the control signal to the correct stationary level $-\Delta u$ in order to take the feedback control signal into account and reduce the overshoot. The gain is therefore reduced to

$$K_{ff} = \frac{K_3}{K_1} - \frac{K}{T_i} IE$$

Closed-loop design

Feedforward tuning rules: non-realizable delay

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Closed-loop design

IE estimation:

$$Y = (P_d - P_u F)D = P_d D - P_u F D$$

$$y(t) - y_{sp} = \begin{cases} k_d \left(1 - e^{-\frac{t}{\tau_d}}\right) d & 0 \leq t \leq L_b \\ k_d \left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-L_b}{T_b}}\right)\right) d & L_b < t \end{cases}$$

$$L_b = \max(0, L_u - L_d), \quad T_b = \tau_u + T_p - T_z$$

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Feedforward tuning rules: non-realizable delay

IE estimation:

$$
\begin{aligned}
IE \cdot d &= \int_0^\infty (y(t) - y_{sp})dt \\
&= k_d \int_0^{L_b} \left(1 - e^{-\frac{t}{\tau_d}}\right) d\, dt + k_d \int_{L_b}^\infty \left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-L_b}{T_b}}\right) d\, dt \\
&= k_d \left[t + \tau_d e^{-\frac{t}{\tau_d}}\right]_0^{L_b} d + k_d \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-L_b}{T_b}}\right]_{L_b}^\infty d \\
&= k_d \left(L_b + \tau_d e^{-\frac{L_b}{T_a}} - \tau_d - \tau_d e^{-\frac{L_b}{\tau_d}} + T_b\right) d \\
&= k_d \left(L_b - \tau_d + T_b\right) d
\end{aligned}
$$

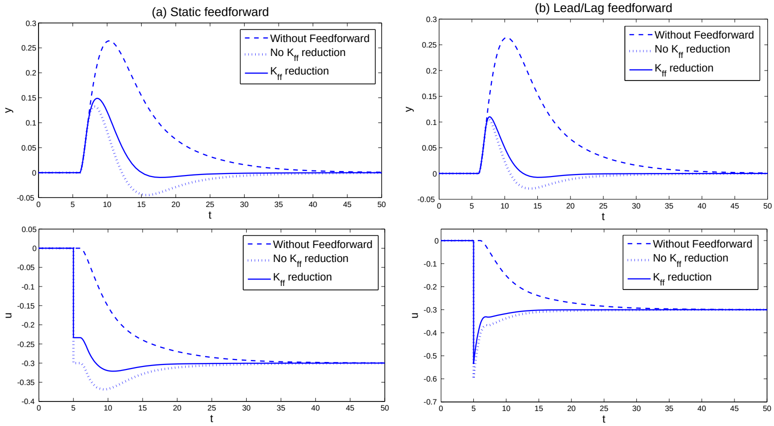
Feedforward tuning rules: non-realizable delay

IE estimation:

$$IE = \begin{cases} k_d(\tau_u - \tau_d + T_p - T_z) & L_d \geq L_u \\ k_d(L_u - L_d + \tau_u - \tau_d + T_p - T_z) & L_d < L_u \end{cases}$$

$$K_{ff} = \frac{K_3}{K_1} - \frac{K}{T_i} IE$$

Gain reduction rule:



Feedforward tuning rules: non-realizable delay

Once the overshoot is reduced, the second goal is to design T_z and T_p to minimize the IAE value. In this way, we keep $T_z = \tau_u$ to cancel the pole of P_u and fix the pole of the compensator:

$$IAE = \int_0^\infty |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^\infty y(t)dt$$

where t_0 is the time when y crosses the setpoint, with $y_{sp} = 0$ and $d = 1$.

$$y(t) - y_{sp} = \begin{cases} k_d\left(1 - e^{-\frac{t}{\tau_d}}\right)d & 0 \leq t \leq L_b \\ k_d\left(\left(1 - e^{-\frac{t}{\tau_d}}\right) - \left(1 - e^{-\frac{t-L_b}{T_b}}\right)\right)d & L_b < t \end{cases}$$

$$IAE = \int_0^\infty |y(t)|dt = \int_0^{t_0} y(t)dt - \int_{t_0}^\infty y(t)dt$$

$$\frac{t_0}{\tau_d} = \frac{t_0 - L_b}{T_b} = \frac{\tau_d L_b}{\tau_d - T_b} = \frac{\tau_d}{\tau_u - T_p}L_b$$

Feedforward tuning rules: non-realizable delay

$$
\begin{aligned}
IAE &= \int_0^{L_b}\left(1-e^{-\frac{t}{\tau_d}}\right)dt + \int_{L_b}^{t_0}\left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-L_b}{T_b}}\right)dt - \int_{t_0}^{\infty}\left(-e^{-\frac{t}{\tau_d}} + e^{-\frac{t-L_b}{T_b}}\right)dt \\
&= \left[t+\tau_d e^{-\frac{t}{\tau_d}}\right]_0^{L_b} + \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-L_b}{T_b}}\right]_{L_b}^{t_0} - \left[\tau_d e^{-\frac{t}{\tau_d}} - T_b e^{-\frac{t-L_b}{T_b}}\right]_{t_0}^{\infty} \\
&= L_b - \tau_d + T_b + 2\tau_d e^{-\frac{t_0}{\tau_d}} - 2T_b e^{-\frac{t_0-L_b}{T_b}} \\
&= L_b - \tau_d + T_b + 2\tau_d e^{-\frac{L_b}{\tau_d - T_b}} - 2T_b e^{-\frac{L_b}{\tau_d - T_b}} \\
&= L_b - \tau\left(1 - 2e^{-\frac{L_b}{\tau}}\right)
\end{aligned}
$$

with $\tau = \tau_d - T_p$.

Feedforward tuning rules: non-realizable delay

$$\frac{d}{d\tau}IAE = -1 + 2e^{-\frac{L_b}{\tau}} + 2\frac{L_b}{\tau}e^{-\frac{L_b}{\tau}} = -1 + 2(1+x)e^{-x} = 0$$

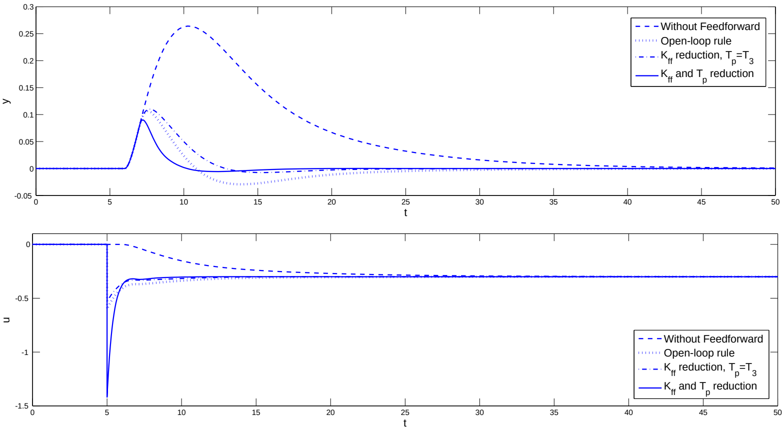
where $x = L_b/\tau$. A numerical solution of this equation gives $x \approx 1.7$, which gives

$$T_p = T_b - \tau_d + \tau_u = \tau_d - \tau \approx \tau_d - \frac{L_b}{1.7}$$

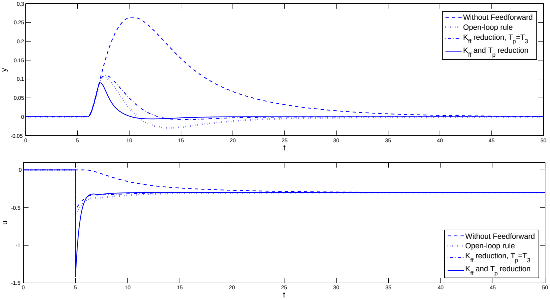
$$T_p = \begin{cases} \tau_u & L_u - L_d \leq 0 \\ \tau_d - \dfrac{L_u - L_d}{1.7} & 0 < L_u - L_d < 1.7\tau_d \\ 0 & L_u - L_d > 1.7\tau_d \end{cases}$$

Feedforward tuning rules: non-realizable delay

Gain and T_p reduction rule:



Gain and T_p reduction rule:



	No FF	Open-loop rule	K_{ff} reduction	K_{ff} &T_p reduction
IAE	9.03	1.76	1.37	0.59

Feedforward tuning rules: non-realizable delay

First approach: Guideline summary

1. Set $T_z = T_1$ and calculate T_p as:

$$T_p = \begin{cases} T_3 & L_1 - L_3 \leq 0 \\ T_3 - \dfrac{L_1 - L_3}{1.7} & 0 < L_1 - L_3 < 1.7T_3 \\ 0 & L_1 - L_3 > 1.7T_3 \end{cases}$$

2. Calculate the compensator gain, K_{ff}, as

$$\begin{aligned} K_{ff} &= \frac{K_3}{K_1} - \frac{K}{T_i} IE \\ IE &= \begin{cases} K_2K_3(T_1 - T_3 + T_p - T_z) & L_3 \geq L_1 \\ K_2K_3(L_1 - L_3 + T_1 - T_3 + T_p - T_z) & L_3 < L_1 \end{cases} \end{aligned}$$

Feedforward tuning rules: non-realizable delay

Second approach

To deal with the non-realizable delay case, the second approach was to work with the following:

- Use the non-interacting feedforward control scheme (feedback effect removed).
- Obtain a generalized tuning rule for T_p for moderate, aggressive and conservative responses.
- The rules should be simple and based on the feedback and model parameters.

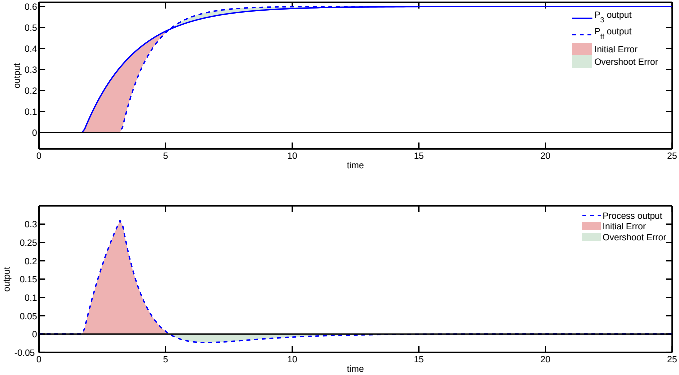
Second approach

The main idea of this second approach relies on analyzing the residual term appearing when perfect cancelation is not possible:

$$\frac{y}{d} = P_d - P_u F = P_d - P_{ff}, \quad P_{ff} = P_u F$$

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-L_d s} - \frac{k_d}{T_p s + 1} e^{-L_u s}$$

Feedforward tuning rules: non-realizable delay



$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-L_d s} - \frac{k_d}{T_p s + 1} e^{-L_u s}$$

Feedforward tuning rules: non-realizable delay

From the previous analysis, it can be concluded that in order to totally remove the overshoot for the disturbance rejection problem by using a lead-lag filter, the settling times of both transfer functions must be the same:

$$\frac{y}{d} = \frac{k_d}{\tau_d s + 1} e^{-L_d s} - \frac{k_d}{T_p s + 1} e^{-L_u s}$$

$$T_p = \frac{4\tau_d + L_d - L_u}{4} = \tau_d - \frac{L_b}{4}$$

$$T_p = \tau_d - \frac{L_u - L_d}{1.7} = \tau_d - \frac{L_b}{1.7}$$

Feedforward tuning rules: non-realizable delay

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Feedforward tuning rules: non-realizable delay

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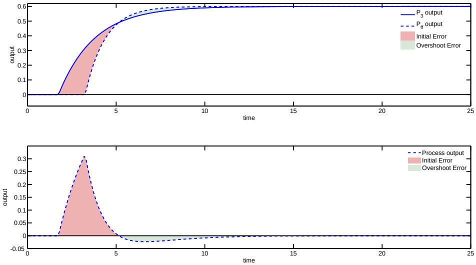
$$T_p = \frac{4\tau_d + L_d - L_u}{4} = \tau_d - \frac{L_b}{4}$$

$$T_p = \tau_d - \frac{L_u - L_d}{1.7} = \tau_d - \frac{L_b}{1.7}$$

Feedforward tuning rules: non-realizable delay

Notice that the new rule for T_p implies a natural limit on performance. If parameter T_p is chosen larger, performance will only get worse because of a late compensation. The only reasons why T_p should be even larger is to decrease the control signal peak:

$$T_p = \tau_d - \frac{L_b}{4}$$



Feedforward tuning rules: non-realizable delay

So, two tuning rules are available:

$$T_p = \frac{4\tau_d + L_d - L_u}{4} = \tau_d - \frac{L_b}{4}$$

$$T_p = \tau_d - \frac{L_u - L_d}{1.7} = \tau_d - \frac{L_b}{1.7}$$

And a third one (a more agreessive rule) can be calculated to minimize Integral Squared Error (ISE) instead of IAE such as proposed in the first approach.

Feedforward tuning rules: non-realizable delay

ISE minimization:

$$\text{ISE} = \int_{L_b}^{\infty} \left(e^{-\frac{(t-L_b)}{T_p}} - e^{-\frac{t}{T_3}} \right)^2 dt$$

$$= \int_{L_b}^{\infty} \left(e^{-\frac{2(t-L_b)}{T_p}} - 2e^{-\frac{\tau_d(t-L_b)+T_p t}{\tau_d T_p}} + e^{-\frac{2t}{\tau_d}} \right) dt$$

$$= -\frac{T_p}{2} \left[e^{-\frac{2(t-L_b)}{T_p}} \right]_{L_b}^{\infty} + 2\frac{\tau_d T_p}{\tau_d + T_p} \left[e^{-\frac{\tau_d(t-L_b)+T_p t}{\tau_d T_p}} \right]_{L_b}^{\infty} - \frac{\tau_d}{2} \left[e^{-\frac{2t}{\tau_d}} \right]_{L_b}^{\infty}$$

$$= \frac{T_p}{2} - 2\tau_d \frac{T_p}{\tau_d + T_p} e^{-\frac{L_b}{\tau_d}} + \frac{\tau_d}{2} e^{-\frac{2L_b}{\tau_d}}$$

Feedforward tuning rules: non-realizable delay

ISE minimization:

$$\frac{d\ \text{ISE}}{d\ T_p} = \frac{1}{2} - 2\tau_d e^{-\frac{L_b}{\tau_d}} \left(\frac{1}{\tau_d + T_p} + \frac{-T_p}{(\tau_d + T_p)^2} \right) = \frac{1}{2} - \frac{2\tau_d^2}{(\tau_d + T_p)^2} e^{-\frac{L_b}{\tau_d}} = 0$$

$$T_p^2 + 2\tau_d T_p + \tau_d^2 (1 - 4e^{-\frac{L_b}{\tau_d}}) = 0$$

$$T_p = \frac{-2\tau_d + \sqrt{4\tau_d^2 - 4\tau_d^2 (1 - 4e^{-\frac{L_b}{\tau_d}})}}{2} = \tau_d \left(2\sqrt{e^{-\frac{L_b}{\tau_d}}} - 1 \right)$$

Feedforward tuning rules: non-realizable delay

Thus, three tuning rules are available:

$$T_p = \tau_d - \frac{L_b}{4}$$
$$T_p = \tau_d - \frac{L_b}{1.7}$$
$$T_p = \tau_d \left(2\sqrt{e^{-\frac{L_b}{\tau_d}}} - 1 \right)$$

which can be generalized as:

$$T_p = \tau_d - \frac{L_b}{\alpha}$$

Feedforward tuning rules: non-realizable delay

Second approach: Guideline summary

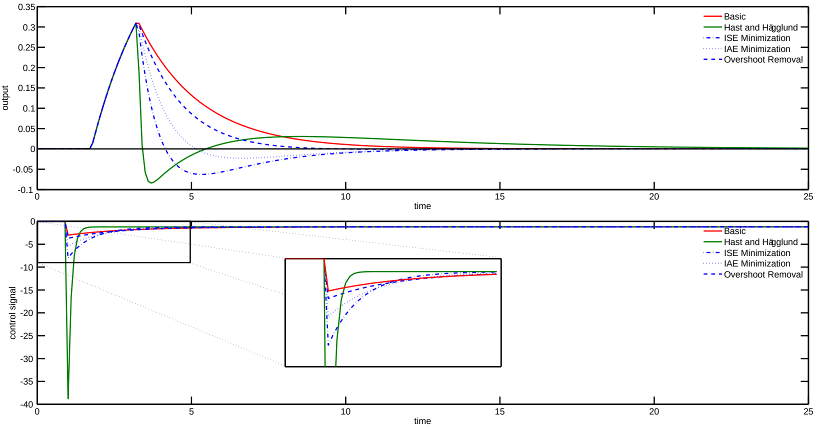
1. Set $T_z = T_1$, $K_{ff} = k_d/k_u$ and calculate T_p as:

$$T_p = \begin{cases} T_3 & L_b \leq 0 \\ T_3 - \frac{L_b}{\alpha} & 0 < L_b < 4T_3 \\ 0 & L_b \geq 4T_3 \end{cases}$$

2. Determine T_p with $L_b/T_3 < \alpha < \infty$ using:

$$\alpha = \begin{cases} \frac{L_b}{2T_3\left(1-\sqrt{e^{-L_b/T_3}}\right)} & \text{aggressive (ISE minimization)} \\ 1.7 & \text{moderate (IAE minimization)} \\ 4 & \text{conservative (Overshoot removal)} \end{cases}$$

Feedforward tuning rules: non-realizable delay



Feedforward tuning rules: non-realizable delay

	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	**0.0739**	0.6423	38.7800	2.5710	0.8979
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	**0.5641**	5.3680	**0.9113**	**0.8315**
Overshoot Removal	0.1277	0.6833	**3.6920**	0.9323	0.8870

$$J_1(F,B) = \frac{1}{2}\left(\frac{\text{ISE}(F)}{\text{ISE}(B)} + \frac{\text{ISC}(F)}{\text{ISC}(B)}\right), \quad \text{ISC} = \int_0^\infty u(t)^2\,\mathrm{d}t$$

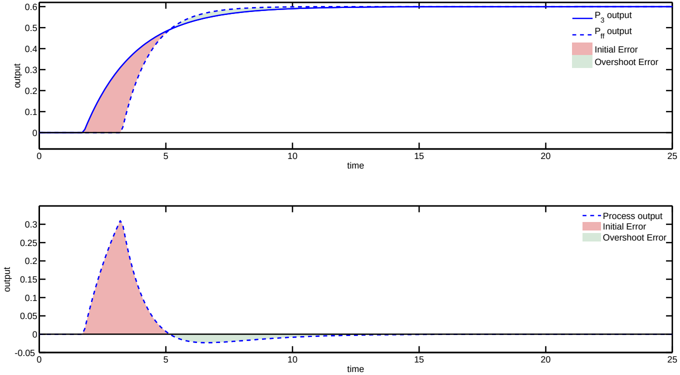
$$J_2(F,B) = \frac{1}{2}\left(\frac{\text{IAE}(F)}{\text{IAE}(B)} + \frac{\text{IAC}(F)}{\text{IAC}(B)}\right), \quad \text{IAC} = \int_0^\infty |u(t)|\,\mathrm{d}t$$

Feedforward tuning rules: non-realizable delay

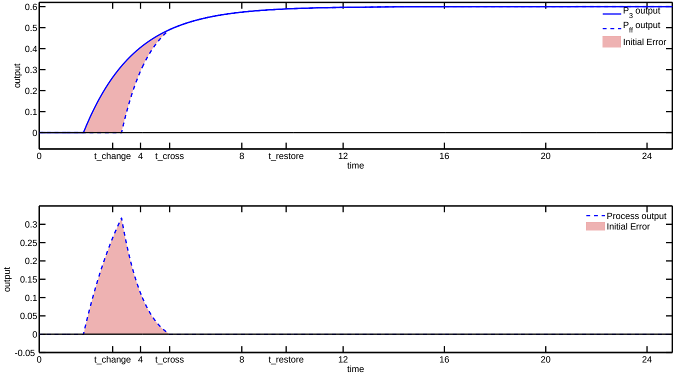
Second approach: A switching solution

It is clear that if the compensation is made too fast, the output will suffer a bigger overshoot error, while if it is too slow, the compensator will take too much time to reject the disturbance and it will have a bigger residual error. Therefore, a switching rule can be proposed in such a way that the feedforward compensator reacts fast before the outputs cross in order to decrease the residual error, and slower after this time to avoid the overshoot because of the residual error.

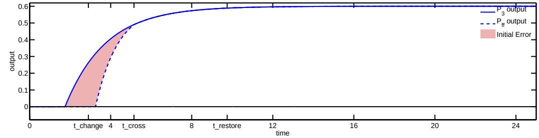
Second approach: A switching solution

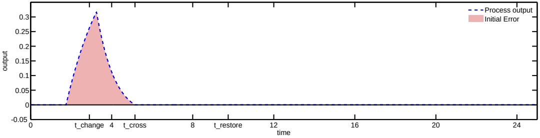


Second approach: A switching solution



Feedforward tuning rules: non-realizable delay





$$t_{cross} = \frac{\tau_d L_u - T_p L_d}{\tau_d - T_p} + t_d \qquad t_{change} = t_{cross} - L_u$$

$$t_{restore} = 4\tau_d + L_d + t_d$$

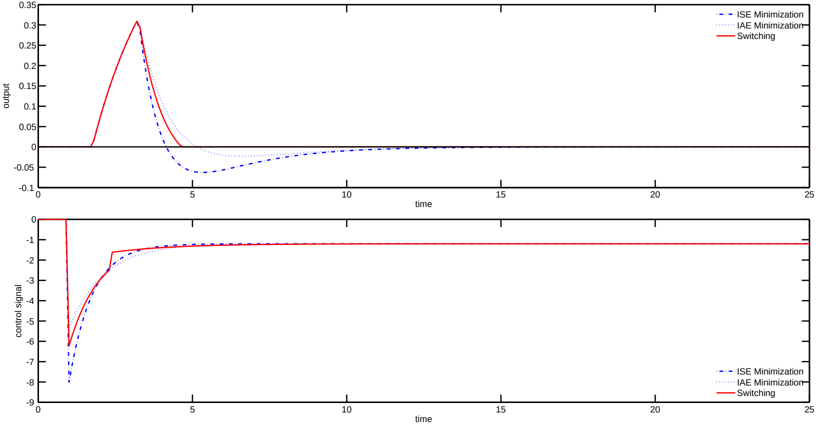
Feedforward tuning rules: non-realizable delay

Second approach: the switching solution guideline

1. Set T_p to a value as close to 0 as possible (tradeoff with the control signal peak).
2. Wait until a step load disturbance is detected at time instant t_d. Define t_{cross} and $t_{restore}$. Set t_{change} = $t_{cross} - L_1$.
3. Using a non-interacting scheme, set C_{ff} and H as follows:

$$C_{ff}(s) = \begin{cases} \frac{K_3}{K_1}\frac{1+T_1 s}{1+T_3 s} & t_{change} \leq t \leq t_r \\ \frac{K_3}{K_1}\frac{1+T_1 s}{1+T_p s} & \text{otherwise} \end{cases}$$

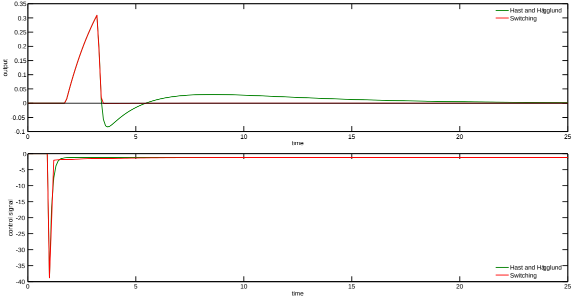
4. Go to step 2.


ISE Minimization
IAE Minimization
Switching
output
time
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
0
5
10
15
20
25
control signal
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
ISE Minimization
IAE Minimization
Switching
time

Feedforward tuning rules: non-realizable delay

	ISE	IAE	u_{init}	J_1	J_2
ISE Minimization	0.0896	0.6021	8.0090	0.9993	0.8615
IAE Minimization	0.0975	0.5641	**5.3680**	0.9113	0.8315
Switching	**0.0889**	**0.4252**	6.2160	**0.9062**	**0.7527**

Feedforward tuning rules: non-realizable delay



	ISE	IAE	u_{init}	J_1	J_2
Hast and Hägglund	0.0739	0.6423	38.78	**2.5710**	0.8979
Switching	**0.0630**	**0.2878**	38.78	2.6650	**0.7149**

Feedforward tuning rules: RH plane zeros

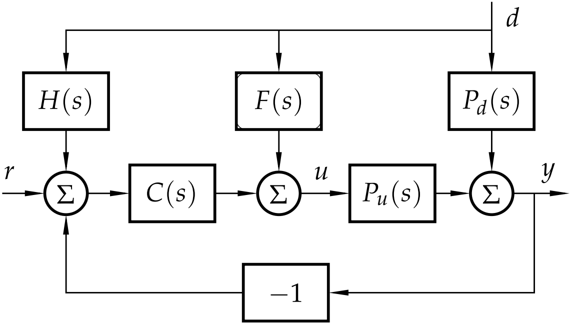
Right-half plane zeros

$$P_u(s) = \frac{k_u\left(-\beta_u s + 1\right)}{D_u^-(s)} e^{-L_u s} \quad \beta_u > 0$$

$$P_d(s) = \frac{k_d}{D_d^-(s)} e^{-L_d s}$$

such that $D_u^-(s) = 1 + \sum_{i=1}^{n_u} a_u[i]s^i$ and $D_d^-(s) = 1 + \sum_{i=1}^{n_d} a_d[i]s^i$ are polynomials with n_u and n_d degree, respectively, such that all their roots are located in the LHP (left-half plane). Moreover, $L_u \leq L_d$.

Feedforward tuning rules: RH plane zeros



$$H(s) = P_d(s) - P_u(s)F(s)$$

Feedforward tuning rules: RH plane zeros

$$\frac{y(s)}{d(s)} = e^{-L_d s}\left(\frac{k_d}{D_d^-(s)} - F(s)\frac{k_u\left(-\beta_u s+1\right)}{D_u^-(s)}e^{-(L_u-L_d)s}\right)$$

$$F(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(T_p s+1\right)^{n_{ff}}}e^{-(L_d-L_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-L_d s}}{D_d^-(s)}\left(1-\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s+1\right)}{\left(T_p s+1\right)^{n_{ff}}}\right)$$

Feedforward tuning rules: RH plane zeros

$$\frac{y(s)}{d(s)} = e^{-L_d s}\left(\frac{k_d}{D_d^-(s)} - F(s)\frac{k_u\left(-\beta_u s + 1\right)}{D_u^-(s)}e^{-(L_u - L_d)s}\right)$$

$$F(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(T_p s + 1\right)^{n_{ff}}}e^{-(L_d - L_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-L_d s}}{D_d^-(s)}\left(1 - \frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s + 1\right)}{\left(T_p s + 1\right)^{n_{ff}}}\right)$$

$$\frac{y(s)}{d(s)} = e^{-L_d s}\left(\frac{k_d}{D_d^-(s)} - F(s)\frac{k_u\left(-\beta_u s + 1\right)}{D_u^-(s)}e^{-(L_u - L_d)s}\right)$$

$$F(s) = \frac{k_d}{\kappa_u}\cdot\frac{D_u^-(s)}{D_d^-(s)}\cdot\frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)}{\left(T_p s + 1\right)^{n_{ff}}}e^{-(L_d - L_u)s}$$

$$\frac{y(s)}{d(s)} = \frac{k_d e^{-L_d s}}{D_d^-(s)}\left(1 - \frac{\left(1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i\right)\left(-\beta_u s + 1\right)}{\left(T_p s + 1\right)^{n_{ff}}}\right)$$

Feedforward tuning rules: RH plane zeros

By using the binomial theorem, the previous expression results in:

$$\frac{y(s)}{d(s)} = \frac{k_d P_0 s}{(T_p s + 1)^{n_u}} \cdot \frac{P(s)}{D_d^-(s)} e^{-L_d s}$$

with

$$P(s) = P_0^{-1}\left(\beta_u \sum_{i=1}^{n_d} \beta_{ff}[i]s^i - \sum_{i=1}^{n_d-1} \beta_{ff}[i+1]s^i + \sum_{i=1}^{n_u-1} \frac{n_u!}{(i+1)!\,(n_u-i-1)!} T_p^{i+1} s^i\right) + 1 \quad (1)$$

$$P_0 = n_u T_p + \beta_u - \beta_{ff}[1]$$

Feedforward tuning rules: RH plane zeros

After solving $\beta_{ff}[i]$ coefficients and cancelling $D_d^-(s)$, it is obtained that

$$G_d(s) = \frac{y(s)}{d(s)} = \frac{\kappa_{y/d}s}{\left(T_p s + 1\right)^{n_u}} e^{-L_d s}$$

with

$$\kappa_{y/d} = k_d \frac{\beta_u^{n_d - n_u + 1}\left(\beta_u + T_p\right)^{n_u}}{\beta_u^{n_d} + \sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d - l}}$$

And where the unitary step response is given by

$$y(t) = \frac{\kappa_{y/d}\left(t - L_d\right)^{n_u - 1}}{T_p^{n_u}\left(n_u - 1\right)!} e^{-\frac{(t - L_d)}{T_p}}$$

Feedforward tuning rules: RH plane zeros

After solving $\beta_{ff}[i]$ coefficients and cancelling $D_d^-(s)$, it is obtained that

$$G_d(s) = \frac{y(s)}{d(s)} = \frac{\kappa_{y/d}s}{\left(T_p s + 1\right)^{n_u}} e^{-L_d s}$$

with

$$\kappa_{y/d} = k_d \frac{\beta_u^{n_d - n_u + 1}\left(\beta_u + T_p\right)^{n_u}}{\beta_u^{n_d} + \sum_{l=1}^{n_d} a_d[l]\beta_u^{n_d - l}}$$

And where the unitary step response is given by

$$y(t) = \frac{\kappa_{y/d}\left(t - L_d\right)^{n_u - 1}}{T_p^{n_u}\left(n_u - 1\right)!} e^{-\frac{\left(t - L_d\right)}{T_p}}$$

Feedforward tuning rules: RH plane zeros

Three different tuning rules are proposed for T_p looking for

- Obtaining a desired settling time.
- Minimize the H_∞ norm.
- Minimize the H_2 norm.

Feedforward tuning rules: RH plane zeros

Settling time rule

$$y(t) = \frac{\kappa_{y/d}\left(t - L_d\right)^{n_u - 1}}{T_p^{n_u}\left(n_u - 1\right)!} e^{-\frac{\left(t - L_d\right)}{T_p}}$$

The settling time is defined as the time that the system takes to reach around 5% of its maximum value

$$y(t_{5\%}) = 0.05 M_{peak}$$

$$\frac{dy(t)}{dt} = 0 \Rightarrow t_{peak} \Rightarrow M_{peak} \Rightarrow t_{5\%}$$

Settling time rule

$$t_{5\%} = L_d + xT_p, \quad 0.05 - \frac{x^{n_u-1}}{(n_u-1)^{n_u-1}} e^{-x+n_u-1} = 0$$

$$T_p = \frac{(t_{5\%} - L_d)}{x}$$

For $n_u = 1$, the following solution is obtained

$$T_p \approx \frac{t_{5\%-L_d}}{3}$$

Feedforward tuning rules: RH plane zeros

Settling time rule: Example

$$P_u(s) = \frac{-0.8s+1}{s^2+s+1}, \quad P_d(s) = \frac{0.45}{0.75s+1}$$

$$C_{ff}(s) = 0.45\frac{s^2+s+1}{0.75s+1} \cdot \frac{\beta_{ff}[1]s+1}{\left(\tau_{ff}s+1\right)^2}$$

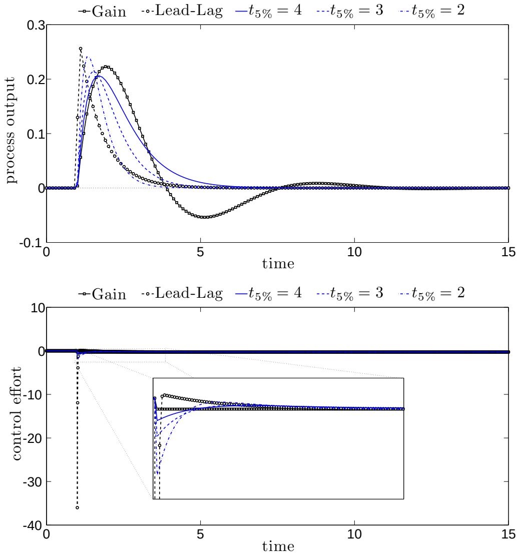
To cancel the stable pole of $P_d(s)$, it is necessary to set

$$\beta_{ff}[1] = -0.6452\tau_{ff}^2 + 0.9677\tau_{ff} + 0.3871$$

Then, T_p is selected according to the desired settling time

$$T_p \approx \frac{t_{5\%}}{5.74}$$

Feedforward tuning rules: RH plane zeros



Feedforward tuning rules: RH plane zeros

Feedforward controller	$\beta_{ff}[1]$	T_p
$t_{5\%} = 4$	0.75	0.70
$t_{5\%} = 3$	0.72	0.52
$t_{5\%} = 2$	0.65	0.35

Feedforward tuning rules: RH plane zeros

H_∞*-norm rule*

$$y(t) = \frac{\kappa_{y/d}\left(t - L_d\right)^{n_u - 1}}{T_p^{n_u}\left(n_u - 1\right)!} e^{-\frac{\left(t - L_d\right)}{T_p}}$$

An H_∞ optimal feedforward compensator to minimize the maximum value of the disturbance response can be found by minimizing the absolute value of the maximum peak:

$$\frac{d\left\|y(t)\right\|_\infty}{dT_p} = 0$$

$$\left(\beta_u + T_p\right)^{n_u - 1}\left(n_u T_p - \left(\beta_u + T_p\right)\right) = 0 \Rightarrow T_p = \frac{\beta_u}{n_u - 1}$$

Feedforward tuning rules: RH plane zeros

H_2-norm rule

$$y(t) = \frac{\kappa_{y/d}\left(t - L_d\right)^{n_u - 1}}{T_p^{n_u}\left(n_u - 1\right)!} e^{-\frac{\left(t - L_d\right)}{T_p}}$$

An H_2 optimal feedforward compensator of the disturbance response can be found by minimizing the absolute value of the output:

$$\frac{d\left\|y(t)\right\|_2}{dT_p} = 0$$

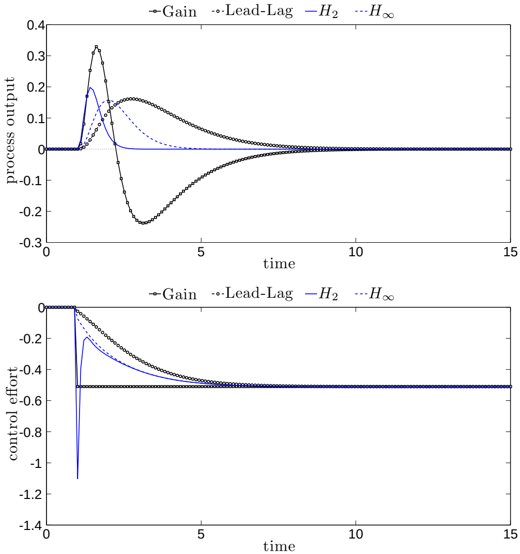
$$T_p^{-1.5}\left(\beta_u + T_p\right)^{n_u - 1}\left(n_u T_p - 0.5\left(\beta_u + T_p\right)\right) = 0 \Rightarrow T_p = \frac{\beta_u}{2n_u - 1}$$

H_∞ and H_2 rules: Example

$$P_u(s) = \frac{-s+1}{(0.25s+1)^4}, \quad P_d(s) = \frac{0.85}{(0.9s+1)^3}$$

$$C_{ff}(s) = 0.85\frac{(0.25s+1)^4}{(0.9s+1)^3} \cdot \frac{1+\sum_{i=1}^{3}\beta_{ff}[i]s^i}{(\tau_{ff}s+1)^4}$$

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	T_p
H_2	1.32	0.77	0.18	0.14
H_∞	1.87	1.30	0.32	0.33


Gain
Lead-Lag
H_2
H_∞
process output
time
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
0
5
10
15
control effort
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4

Feedforward tuning rules: RH plane zeros

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	$\|y(t)\|_\infty$
Gain	80.47	3.85	0.33
Lead-lag	51.51	2.39	0.16
H_2	**12.68**	**1.33**	0.20
H_∞	23.50	1.61	**0.16**

Feedforward tuning rules: RH plane zeros

1. Set T_p according to the desired specification:

$$\begin{aligned} \text{Settling time}: \quad & T_p = \left(t_{5\%} - L_d\right)/x \\ H_\infty: \quad & T_p = \frac{\beta_u}{n_u - 1} \\ H_2: \quad & T_p = \frac{\beta_u}{2n_u - 1}. \end{aligned}$$

2. Obtain the coefficients $\beta_{ff}[i]$ to cancel $D_d^-(s)$.
3. Define the feedforward compensator $F(s)$ as

$$F(s) = \frac{k_d}{k_u} \cdot \frac{D_u^-(s)}{D_d^-(s)} \cdot \frac{\left(1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i] s^i\right)}{\left(T_p s + 1\right)^{n_{ff}}} e^{-(L_d - L_u)s}$$

4. Set $H(s) = P_{ff}(s) = P_d(s) - F(s)P_u(s)$.

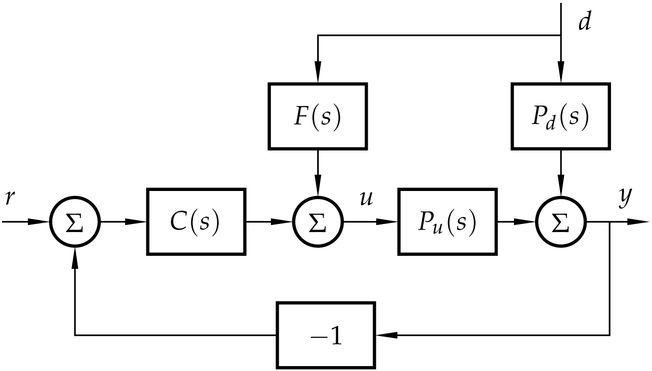
Integrating poles

$$P_u(s) = \frac{k_u}{D_u(s)s^{t_u}}$$

$$P_d(s) = \frac{k_d}{D_d^-(s)}$$

such that $D_u(s) = 1 + \sum_{i=1}^{n_u} a_u[i]s^i$ is a polynomial of degree n_u and $D_d^-(s) = 1 + \sum_{i=1}^{n_d} a_d[i]s^i$ is a polynomial of degree n_d with all its roots in the left half plane (LHP), and t_u is the type of process $P_u(s)$.

Feedforward tuning rules: integrators



Feedforward tuning rules: integrators

In this case, the feedback controller will be defined as follows

$$C_{fb}(s) = \kappa_{fb}\frac{N_{fb}(s)}{D_{fb}(s)s^{t_{fb}}}$$

such that t_{fb} is the type of $C_{fb}(s)$.

And the reference tracking response can be expressed as

$$\frac{y(s)}{r(s)} = \frac{N_{fb}(s)}{D_{cl}(s)}$$

where $D_{cl}(s)$ is a polynomial of degree n_{cl} that represents the closed-loop system dynamics.

Feedforward tuning rules: integrators

$$\frac{y(s)}{d(s)} = \left(\frac{k_d}{D_d^-(s)} - F(s)\frac{k_u}{D_u(s)}s^{-t_u}\right)\frac{D_u(s)s^{t_u}D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$

$$= \left(\frac{k_d d D_u(s)s^{t_u}}{D_d^-(s)} - F(s)k_u\right)\frac{D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$

$$F(s) = \frac{k_d}{k_u}\frac{1}{D_{fb}(s)D_d^-(s)}\frac{1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i}{(T_p s+1)^{n_{ff}}}$$

Feedforward tuning rules: integrators

$$\frac{y(s)}{d(s)} = \left(\frac{k_d}{D_d^-(s)} - F(s)\frac{k_u}{D_u(s)}s^{-t_u}\right)\frac{D_u(s)s^{t_u}D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$

$$= \left(\frac{k_d d D_u(s)s^{t_u}}{D_d^-(s)} - F(s)k_u\right)\frac{D_{fb}(s)s^{t_{fb}}}{D_{cl}(s)}$$

$$F(s) = \frac{k_d}{k_u}\frac{1}{D_{fb}(s)D_d^-(s)}\frac{1+\sum_{i=1}^{m_{ff}}\beta_{ff}[i]s^i}{(T_p s+1)^{n_{ff}}}$$

By substituting the proposed compensator in the disturbance rejection response, it is obtained that

$$\frac{y(s)}{d(s)} = G_{y/d}(s) = \frac{-k_d d s^{t_{fb}}}{\left(T_p s+1\right)^{n_{ff}}} \frac{P(s)}{D_{cl}(s) D_d^-(s)}$$

with

$$P(s) = 1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i] s^i - \left(T_p s+1\right)^{n_{ff}} D_{fb}(s) D_u(s) s^{t_u}$$

The idea is to cancel all stable roots of $D_{cl}(s)$ and $D_d^-(s)$ with $\beta_{ff}[i]$ coefficients.

Feedforward tuning rules: integrators

So, the resulting response will not present any undesired dynamics or undershoot. This fact can be clearly observed by its consequent time response against unitary step

$$y(t) = \frac{-k_d t^{n_{ff}-1}}{\tau_{ff}^{n_{ff}} \left(n_{ff}-1\right)!} e^{-\frac{t}{T_p}}$$

Feedforward tuning rules: integrators

Three different tuning rules are proposed for T_p looking for

- Obtaining a desired settling time.
- Optimal solution for a tradeoff between maximum peak and settling time.

Settling time rule

$$y(t) = \frac{-k_d t^{n_{ff}-1}}{\tau_{ff}^{n_{ff}}\left(n_{ff}-1\right)!} e^{-\frac{t}{T_p}}$$

The settling time is defined as the time that the system takes to reach around 5% of its maximum value

$$y(t_{5\%}) = 0.05 M_{peak}$$

$$\frac{dy(t)}{dt} = 0 \Rightarrow t_{peak} \Rightarrow M_{peak} \Rightarrow t_{5\%}$$

Settling time rule

$$t_{5\%} = \frac{x}{T_p}, \quad 0.05 - \frac{x^{n_{ff}-1}}{\left(n_{ff}-1\right)^{n_{ff}-1}} e^{-x+n_{ff}-1} = 0$$

$$T_p = \frac{t_{5\%}}{x}$$

For $n_u = 1$, the following solution is obtained

$$T_p \approx \frac{t_{5\%}}{3}$$

Settling time rule: Example

$$P_u(s) = \frac{1}{s\left(0.25s+1\right)}$$

$$P_d(s) = \frac{0.5}{0.9s+1}$$

To obtain a reference tracking response with the closed-loop dynamics given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 2\frac{0.56s^2 + 1.5s + 1}{s\left(0.5s+1\right)}$$

Settling time rule: Example

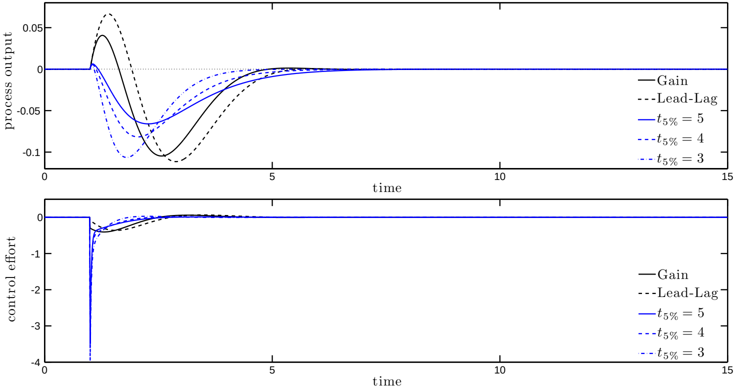
Then, the feedforward compensator is defined as

$$F(s) = \frac{0.5}{(0.025s+1)\,(0.9s+1)\,(0.5s+1)} \frac{1+\sum_{i=1}^{6}\beta_{ff}[i]s^i}{\left(T_p s+1\right)^3}$$

$$T_p = 0.13 t_{5\%}$$

Feedforward controller	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	T_p
$t_{5\%} = 5$	3.42	5.17	4.25	1.90	0.43	0.04	0.65
$t_{5\%} = 4$	3.42	4.78	3.50	1.38	0.27	0.02	0.52
$t_{5\%} = 3$	3.42	4.39	2.85	0.98	0.17	0.01	0.39

Settling time rule: Example



Settling time rule: Example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	18.57	1.16	-0.30
Lead-Lag	22.91	1.32	$\mathbf{-0.08}$
$t_{5\%} = 5$	15.14	**0.83**	-3.47
$t_{5\%} = 4$	15.10	0.92	-3.60
$t_{5\%} = 3$	**15.05**	1.06	-3.96

Optimal tuning rule

A tradeoff arises from the fact that by making T_p small, the settling time is reduced but the maximum peak is increased.

So, a cost function to find a tradeoff between settling time and maximum peak can be proposed as follows

$$J = \alpha t_{5\%} + (1-\alpha)\left|M_{peak}\right| \quad \alpha \in (0,1)$$

where α is a weighting parameter.

Optimal tuning rule

Then, substituting M_{peak} and $t_{5\%}$ equations previously calculated in J, when J is derivative with respect to T_p and is taken equal to zero

$$\frac{dJ}{dT_p} = 0$$

the following solution is obtained

$$T_p = \sqrt{|k_d| \frac{(1-\alpha)}{\alpha} \frac{e^{1-n_{ff}} \left(n_{ff}-1\right)^{n_{ff}-1}}{x \left(n_{ff}-1\right)!}}$$

α can be easily used as a tuning parameter to find a desired tradeoff between settling time and maximum peak values.

Optimal tuning rule: Example

$$P_u(s) = \frac{1}{s\,(s+1)}$$

$$P_d(s) = \frac{0.75}{(0.35s+1)^3}$$

To obtain a reference tracking response with the closed-loop dynamics given by $D_{cl}(s) = \left(0.25s^2 + 0.75s + 1\right)^2$, the feedback controller is selected as a PID controller with a filter in the derivative term such that

$$C_{fb}(s) = 3.2\frac{0.75s^2 + 1.5s + 1}{s\,(0.2s+1)}$$

Feedforward tuning rules: integrators

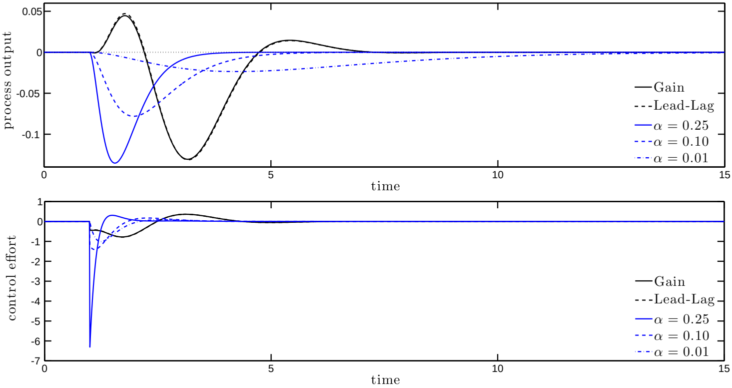
Optimal tuning rule: Example

Then, the feedforward compensator is defined as

$$F(s) = \frac{0.75}{(0.35s+1)^3(0.2s+1)} \frac{1+\sum_{i=1}^{7}\beta_{ff}[i]s^i}{(\tau_{ff}s+1)^3}$$

Feedforward	$\beta_{ff}[1]$	$\beta_{ff}[2]$	$\beta_{ff}[3]$	$\beta_{ff}[4]$	$\beta_{ff}[5]$	$\beta_{ff}[6]$	$\beta_{ff}[7]$	T_p
$\alpha = 0.25$	3.55	5.05	3.54	1.39	0.32	0.04	0.01	0.28
$\alpha = 0.10$	3.55	5.67	4.75	2.17	0.53	0.06	0.01	0.49
$\alpha = 0.01$	3.55	9.06	15.95	15.52	6.89	6.88	0.01	1.62

Optimal tuning rule: Example



Feedforward tuning rules: integrators

Optimal tuning rule: Example

Feedforward controller	$\|y(t)\|_1$	$\|y(t)\|_2$	u_{init}
Gain	23.35	1.40	−0.45
Lead-Lag	23.60	1.41	−0.43
$\alpha = 0.25$	**14.06**	1.15	−6.31
$\alpha = 0.10$	**14.06**	0.87	−1.21
$\alpha = 0.01$	**14.06**	**0.48**	−**0.03**

Feedforward tuning rules: integrators

1. Set T_p according to the desired specification:

$$\text{Settling time}: \quad T_p = t_{5\%}/x$$
$$\text{Optimal}: \quad \text{tuning rule}$$

2. Obtain the coefficients $\beta_{ff}[i]$ to cancel $D_d^-(s)D_{cl}(s)$.
3. Define the feedforward compensator as

$$F(s) = \frac{k_d}{k_u} \frac{1}{D_{fb}(s)D_d^-(s)} \frac{1 + \sum_{i=1}^{m_{ff}} \beta_{ff}[i]s^i}{(T_p s + 1)^{n_{ff}}}$$

Conclusions

- The motivation for feedforward tuning rules was introduced.
- The feedback effect on the feedforward design was analyzed.
- The different non-realizable situations were studied.
- The two available feedforward control schemes were used.
- Simple tuning rules based on the process and feedback controllers parameters were derived.

Collaborators

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   Most of the results presented here are part of his PhD thesis.
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End of the presentation

Thank you for your attention