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QcF Coalgebras and Applications to Hopf Algebras

Abstract. We generalize the results from [M.C.Iovanov, *Co-Frobenius Coalgebras*, J. Algebra 303 (2006), no.1, 146–153] to QcF coalgebras. Two objects M, N in a suitable category will be called σ -weakly isomorphic if $M^{(I)} \simeq N^{(J)}$ (coproducts) for some sets I, J , and π -weakly isomorphic if $M^I \simeq N^J$ (products) for some I, J . We show that a coalgebra C is quasi co-Frobenius if and only if the right comodule C is "weakly" isomorphic to its dual right comodule $\text{Rat}(C^*C^*)$ in either of the above meanings; we give some applications of this result to Hopf Algebras: we obtain as a consequence the a series of well known equivalent characterization of Hopf algebras with integral: they are the same as the co-Frobenius Hopf algebras, QcF coalgebras, semiperfect coalgebras or Hopf Algebras with nonzero rational part. At the same time, the uniqueness of integrals for Hopf algebras follows. Also, as another application of our methods, we give a new proof for the bijectivity of the antipode of co-Frobenius Hopf algebras. The main focus of our applications can be regarded as a plead for the potential of representation theoretic approaches to the theory of Hopf Algebras and quantum groups.