Invariant subspaces for pairs of rotations

Erik Darpö

Matematiska Institutionen, Uppsala Universitet,

Box 480, S-75106 Uppsala, Sweden.

erik.darpo@math.uu.se

A *rotation* in a finite-dimensional Euclidean space V is an orthogonal endomorphism $\delta \in O(V)$ whose matrix with respect to some orthonormal basis equals

$$\begin{pmatrix} R_\alpha & & & \\ & R_\alpha & & \\ & & \ddots & \\ & & & R_\alpha \end{pmatrix}$$

where $R_\alpha = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ and $\alpha \in [0, \pi]$.

We show that whenever (δ, ϵ) is a pair of rotations in a Euclidean space V, there exists a decomposition $V = \bigoplus_i V_i$ of V into (δ, ϵ)-invariant, pairwise orthogonal subspaces, such that $\dim V_i \leqslant 4$ for all i. This leads to a classification of all real representations of the two loop quiver given by pairs of endomorphisms of this type, up to orthogonal isomorphism.