The central elements of the universal enveloping algebra of higher orders and the construction of Knizhik-Zamolodchikov type equations for root systems of types A,D,B

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The talk is devoted to some generalization of the classical Riemann-Hilbert problem of construction of a Pfaffian system of fuchsian type, whose singular set is a collection of reflection hyperplanes defined by a system of roots B_n. Also some new results are obtained for the root systems A and D.

In construction of Knizhnik-Zamolodchikov equations, whose singular set is associated with the root system A_n, the Casimir element of the second order is used. In the case of root system B_n a similar construction was done by A. Leibman in one parameter case, while the corresponding equations must depend on two parameters, as the number of orbits of the corresponding root system equals to two. For other root systems such constructions are not known.

The Knizhnik-Zamolodchikov equations associated with the root system B have the following form:

$$dy = \lambda\Big(\sum_{i<j} \frac{\tau_{ij}}{x_i - x_j} d(x_i - x_j) + \frac{\mu_{ij}}{x_i + x_j} d(x_i + x_j) + \sum_i \frac{\nu_i}{x_i} dx_i\Big) y, \;\; i,j = 1,...,n$$

The Frobenius condition of integrability is equivalent to a system of commutation relations on the coefficients $\tau_{ij}, \mu_{ij}, \nu_i$. Some nontrivial solutions of this system of commutation relations is found in the Hopf algebra $U(U(o_N)) \otimes ... \otimes U(U(o_N))$, where the tensor product is taken n times and N is arbitrary. The key role in this construction is played by central elements of $U(o_N)$, which can have order higher then 2.