DEFORMATIONS OF SIMPLE FINITE GROUP SCHEMES

FILIPPO VIVIANI

Simple finite group schemes over an algebraically closed field of positive characteristic p are of two types: either the constant group schemes associated to simple finite groups or the group schemes of height one associated to the universal p-envelopes of simple modular Lie algebras. Two great achievements of the mathematics of the last century was a complete classification of these two kind of simple objects (the simple Lie algebras have been classified only for $p \neq 2, 3$).

Simple finite groups were classified during the years 1955-1985 thanks to the contribution of many mathematicians (see [SOL95], [SOL01], [ASC04] for a nice historical account).

Simple Lie algebras over an algebraically closed field F of characteristic $p \neq 2, 3$ have recently been classified by Block-Wilson-Premet-Strade (see [BW84], [SW91], [STR04]). The classification says that for $p \geq 7$ the simple Lie algebras can be of two types: of classical type and of generalized Cartan type. The algebras of *classical type* are obtained by considering the simple Lie algebras in characteristic zero, by taking a model over the integers and then reducing modulo the prime p. The algebras of *generalized Cartan type* are the finite-dimensional analogues of the four classes of infinite-dimensional complex simple Lie algebras, which occurred in Cartan's classification of Lie pseudogroups. In characteristic $p = 5$, apart from the above two types of algebras, there is one more family of simple Lie algebras called Melikian algebras. In characteristic $p = 2, 3$, there are many exceptional simple restricted Lie algebras and the classification seems still far away.

In this talk, we considered the problem of determining the *infinitesimal deformations* of such simple finite group schemes. The group schemes associated to simple finite group schemes and to classical simple Lie algebras are known to be rigid apart from some bad characteristic of the base field. We show that this is never the case for the group schemes associated to simple Lie algebras of Cartan type. In particular we determine the infinitesimal deformations of the group schemes of height one associated to the restricted simple Lie algebras of Cartan type. We do this by computing the second ordinary and restricted cohomology group (see [HOC54]) of these algebras with values in the adjoint representation.

The results presented in this talk are contained in the preprints [VIV1] [VIV2], [VIV3], [VIV4], [VIV5].

References

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