

# Formality and Deformations of Universal Enveloping Algebras

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## Abstract

The concept of formality of associative algebras, introduced by Maxim Kontsevich on deformation quantization on any Poisson manifold, turned out to be extremely useful for the deformation theory of that algebra. An associative algebra is called formal if its Hochschild complex equipped with the Gerstenhaber graded Lie structure is quasi-isomorphic in the  $L_\infty$  sense to its Hochschild cohomology. If this is the case first order deformations having induced Gerstenhaber bracket equal to zero always integrate to formal deformations. Kontsevich's basic example is the symmetric algebra of a vector space which is formal. We describe, in [2], enveloping algebras of finite-dimensional Lie algebras which are formal. For abelian Lie algebras this is true thanks to the Kontsevich formality theorem. We are using his formality map twisted by the group-like element generated by the linear Poisson structure to simplify the problem, and then study examples. For instance, the universal enveloping algebras of the Lie algebras  $\mathfrak{gl}(n, \mathbb{K}) \oplus \mathbb{K}^n$  are formal. We also recover our rigidity results [1] for enveloping algebras from this new angle.

## References

- 1 M. Bordemann, A. Makhlouf and T. Petit, *Déformation par quantification et rigidité des algèbres enveloppantes*, J. of Algebra, **285** (2005), 623-648.
- 2 M. Bordemann and A. Makhlouf, *Formality and Deformations of Universal Enveloping Algebras*, To appear in International Journal of Theoretical Physics (2007).

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