Morita-Takeuchi Contexts and Elementary Corngs

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Abstract

Given a *Morita semi-context* $(T, S, P, Q, <, >_T)$ for rings T, S and bimodules $_T P_S$ and $_S Q_T$, it is well known that the bimodule morphism $<, >_T : P \otimes_S Q \to T$ induces on $Q \otimes_T P$ and $P \otimes_S Q$ structures of rngs (i.e. associative rings not necessarily with units). These rngs, called *elementary rngs*, yield Morita contexts that are -in case we begin with a Morita context- homomorphic to the initial context; and have units, for example, if the initial context is strict. Dually, given a *Morita-Takeuchi semi-context* $(\mathcal{D}, \mathcal{C}, M, N, f_{\mathcal{D}})$ for corings $\mathcal{D}, \mathcal{C}$ and *compatible* bimodules $M \in {}^{\mathcal{D}}\mathbb{M}^{\mathcal{C}}$, $N \in {}^{\mathcal{C}}\mathbb{M}^{\mathcal{D}}$, the bicomodule morphism $f_{\mathcal{D}} : \mathcal{D} \to M \square_{\mathcal{C}} N$ induces on $M \square_{\mathcal{C}} N$ and $N \square_{\mathcal{D}} M$ structures of corngs (i.e. coassociative corings not necessarily with counits). We show that these corngs, which we call *elementary corngs,* yield Morita-Takeuchi contexts that are -in case we begin with a Morita-Takeuchi context- homomorphic to the initial context. We study the structure of such corngs, investigate their comodules categories, and clarify when they have counits.