

Approximations of complete modules by complete  
Big Cohen-Macaulay modules over a  
Cohen-Macaulay local ring

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**Abstract**

The Auslander-Buchweitz theory for finitely generated modules over a Cohen-Macaulay local ring with dualizing module is extended to complete modules, finitely generated or not, over an arbitrary Cohen-Macaulay local ring  $A$ .

We prove that every  $A$ -module, complete in its maximal-adic topology, has a complete big Cohen-Macaulay approximation and a complete hull of finite injective dimension.

Among corollaries, we have that the complete big Cohen-Macaulay  $A$ -modules of finite injective dimension are exactly the modules of the form  $W \otimes_{\hat{A}} F$ , where  $W$  is the dualizing module of the maximal-adic completion  $\hat{A}$  of  $A$  and where  $F$  is a complete flat  $A$ -module.

Moreover, among the approximations and hulls of a complete  $A$ -module  $M$ , we are able to find minimal ones.