Book of Abstracts of V CIDAMA

V International Course of Mathematical Analysis in Andalusia

Almería, 2011

V Curso Internacional de Análisis Matemático en Andalucía

Del 12 al 16 de septiembre de 2011 • Almería

MINI-CURSOS

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Almería, 12th to 16th of September of 2011

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Preface

The International Course of Mathematical Analysis in Andalusia has been organized on a biennial basis at various universities of Andalusia. Previous editions were held in Cádiz (2002 and 2009), Granada (2004) and Huelva (2007).
Ten internationally renowned experts in various fields of mathematical analysis will participate giving a lecture or a course which reflects the current state and lines of research in their respective fields. At the same time, we have organized communications and poster sessions for those participants interested in showing their recent work.
Almería is located in the southeast of the Iberian Peninsula, near the Andalusian cities of Granada and Málaga. The Cabo de Gata-Níjar Natural Reserve, the semi-desert landscape of Tabernas, the white villages or the beautiful coastline (along 219 km) are an indication of the geographical diversity of the province. The historical and cultural heritage of the city is linked to its tradition like a bridge between the peoples inhabiting both shores of the Mediterranean sea.

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El Curso Internacional de Análisis Matemático en Andalucía viene desarrollándose con carácter bienal en las distintas universidades andaluzas. Las ediciones anteriores se celebraron en Cádiz (2002 y 2009), Granada (2004) y Huelva (2007).
En esta ocasión, contaremos con diez expertos de reconocido prestigio internacional en distintos ámbitos del Análisis Matemático. Participarán con una conferencia o un curso que refleje el estado actual de algunas de las líneas de investigación de sus respectivas especialidades. Al mismo tiempo, se han contemplado sesiones de comunicaciones y pósteres para que los asistentes interesados puedan hacer públicos sus trabajos recientes.
Almería está situada en el sureste de la península ibérica, muy cerca de las ciudades andaluzas de Granada y Málaga. El parque natural de Cabo de Gata-Níjar, los paisajes semidesérticos de Tabernas, los pueblos interiores de alta montaña o su extensa costa (219 kilómetros) son una muestra de la diversidad geográfica de la provincia. El patrimonio histórico y cultural de la ciudad está ligado a su tradición de puente entre los pueblos que habitan las dos orillas del Mediterráneo.

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Mini-Courses

Measurability and semicontinuity of multi-functions

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Multi-functions (or set-valued functions) naturally appear in analysis and topology (for instance via inequalities, performing unions or intersections with sets indexed in another set, considering the set of points minimizing an expression, etc.) These three lectures will follow the same pattern: in each of them we will present a classical notion, then a well-known result with an application to analysis and finally we will devote part of the time to introduce the audience in some research questions connected with the topic of the lecture. The topics we will deal with are: (a) Lower semi-continuity; Michael's selection theorem; distances to spaces of continuous functions; quantitative perspective of compactness (b) Upper semicontinuity; generation of K-analytic structures: upper semi-continuity for free; WCG Banach spaces are weakly Lindelöf; a metrization result on topology obtained via functional analysis; B_1 selectors; Asplund spaces. (c) Measurability for multi-functions; Kuratowski-Ryll-Narzesdky selection theorem; integration of multifunction (Debreu and Aumann); extension to non separable Banach spaces.

Introduction to Interpolation Theory

Fernando Cobos Díaz
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The theory of interpolation of operators is a branch of functional analysis with important applications to harmonic analysis, partial differential equations, approximation theory, operator theory and other more distant areas of mathematics. In this course, we will describe the main interpolation methods for Banach spaces and some of their properties and applications. It will cover the following topics:

1. Classical theorems on interpolation of operators between Lp-spaces.
2. Main interpolation methods. Examples.
3. Compactness. Factorization of weakly compact operators.

Optimality of function spaces in Sobolev embeddings

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In this minicourse we shall concentrate on the study of sharpness of function spaces appearing in various types of Sobolev embeddings.
We shall first present a general method of reducing a multi-dimensional Sobolev and Poincaré type inequalities involving gradients of scalar-valued functions of several variables to (far simpler) one-dimensional inequalities involving suitable Hardy-type integral operators acting on functions defined on an interval. Such results we call *reduction theorems*.
We shall explain the details of our reduction methods. The first step is the reduction of the *first order Sobolev embedding*. This part is based on a combination of techniques involving isoperimetric inequalities and various variants of the Pólya–Szegö principle. The second major step is the extension of the reduction to *higher order Sobolev embeddings*, which seems to constitute a considerably more difficult task. To this end, we develop a powerful method based on an iteration of sharp first-order results, which are at this stage already at our disposal.
Once the reduction theorems (of all orders) are established, we will apply them to the construction of the *optimal range* space when a domain space is given, and vice versa. We shall work out important examples illustrating the results in several concrete situations involving certain well-established scales of function spaces (Lebesgue spaces, Lorentz spaces, Lorentz–Zygmund spaces, Orlicz spaces, classical Lorentz spaces of type Lambda and Gamma and Lorentz and Marcinkiewicz endpoint spaces). A nontrivial effort has to be spent here in order to work out manageable versions of rather implicit conditions that one has to grapple with.
We will mention how the methods and results could be of use when *compactness* of the Sobolev embeddings is studied.
We shall present three major types of applications of the results obtained. These will involve Euclidean Sobolev embeddings on possibly irregular domains, trace Sobolev embeddings, and Sobolev embeddings on product probability measure spaces (of which the Gaussian measure is a distinctive example).
The techniques we use will include, for example, various inequalities connecting gradients and rearrangements, weighted inequalities for integral and supremal operators, interpolation theory, and more.

Keywords. Sobolev spaces, rearrangement-invariant spaces, optimal range, optimal domain, reduction theorems, interpolation, duality, compact embeddings, fundamental functions, boundary traces, Gaussian embeddings

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Derivations and projections on Banach Jordan triples

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We shall use the product rule for differentiation to motivate the study of topological Lie and Jordan algebras and triples. We also discuss the role of contractive projections in the structure theory of Jordan triples and in quantized functional analysis.

Some topics which we plan to cover are:

1. Automatic continuity of derivations
2. Structure of continuous derivations (are they inner?)
3. Projective stability (stability under contractive projections)
4. Projective rigidity (existence of contractive projections)
5. Projections and operator space theory (injectivity)

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Plenary Lectures

Weighted inequalities and extrapolation

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The weighted inequalities for the Hardy-Littlewood maximal operator were characterized by B. Muckenhoupt in 1972. In the following years similar weighted inequalities were shown to hold for many classical operators and the study of the properties of the Muckenhoupt A_p classes became essential. One of their key properties is the extrapolation theorem of J. L. Rubio de Francia (1982). It basically says that the L^p-weighted inequalities for just one value of p give weighted inequalities for (all the) other values of p. Several variants of the extrapolation theorem were subsequently obtained.
In the last decade the study of weighted inequalities has focused attention on the behaviour of the norm of the operator in terms of the A_p constant of the weight. This has led to sharp forms of the extrapolation theorem including the constant of the weights in the assumptions and the conclusions.
After reviewing some of the basic results concerning the A_p classes, we present a simple proof of the sharp form of the extrapolation theorem and give some extensions.

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Sharp one and two-weight norm inequalities

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We present sharp one and two-weight norm inequalities for some of the classical operators of harmonic analysis: the Hilbert and Riesz transforms, the Beurling-Ahlfors operator, the dyadic square function and the vector-valued maximal function. In the one weight case we study the dependence on the A_p constant for the strong inequalities with Muckenhoupt weights. Petermichl and Volberg obtained the sharp results for the Hilbert and Riesz transforms, and for the Beurling-Ahlfors operator by using Haar shift dyadic operators, Bellman functions and two-weight Tb theorems. In the two-weight case we look for sufficient Muckenhoupt type conditions on a pair of weights to obtain strong two-weight estimates.

Tug-of-War games and PDEs

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In this talk we will review some recent results concerning Tug-of-War games and their relation to some well known PDEs. In particular, we will show that solutions to certain PDEs can be obtained as limits of values of Tug-of-War games when the parameter that controls the length of the possible movements goes to zero. Since the equations under study are nonlinear and not in divergence form we will make extensive use of the concept of viscosity solutions.

Keywords. Viscosity solutions, Tug-of-War games

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The Radon-Nikodým theorem for vector measures and factorization of operators on Banach function spaces

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In the theory of Banach function spaces, the classical Radon-Nikodým theorem provides a fundamental tool for representing functionals of the dual space as integrals of concrete measurable functions. In the vector valued case, vector valued versions of this result has also been a classical technique for providing integral representations of operators, and leads to the rich (geometric and topological) theory of Banach spaces with the Radon-Nikodým property. In this talk we consider a slightly different approach to the problem of integral representations. Consider a couple of Banach space valued vector measures $m, n : \Sigma \to X$. In which cases it can be said that n can be written as an integral of m? In other words, when there is a (scalar) m-integrable function f such that for every measurable set $A \in \Sigma$,

$$n(A) := \int_A f \, dm.$$

The answer is given by several Radon-Nikodým type theorems for couples of vector measures, that will be explained in this talk. These results give interesting integral representations of operators, that provide a powerful point of view for obtaining factorization of operators through Banach function spaces with applications in general Functional Analysis and Harmonic Analysis.

The Orlicz-Pettis theorem for multiplier convergent series

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An Orlicz-Pettis theorem is a result which asserts that a series in a topological vector space which converges in a weak topology converges in a stronger topology. The original Orlicz-Pettis theorem asserts that a series in a normed space which is subseries convergent in the weak topology is subseries convergent in the norm topology. We consider versions of the Orlicz-Pettis theorem for multiplier convergent series.. If λ is a scalar sequence spaces and Z is a topological vector space a series $\sum_j z_j$ in Z is λ multiplier convergent if the series $\sum_{j=1}^{\infty} t_j z_j$ converges in Z for every $t = \{t_j\} \in \lambda$. For example, if $\lambda = m_0$, the space of sequences with finite range, a series is m_0 multiplier convergent iff the series is subseries convergent. We consider conditions on the multiplier space λ which guarantee that a series which is λ multiplier convergent in the weak topology of a locally convex space is λ multiplier convergent in some stronger topology such as the Mackey topology.

Communications

Hausdorff dimension of the level sets of Takagi's function

Enrique de Amo Artero[1], **Manuel Díaz Carrillo**[2] **and Juan Fernández-Sánchez**[3]

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In this talk we show that the Hausdorff dimension of each level set of the Takagi function is at most 1/2. This conjecture was recently posed by Maddock [3]. We prove this conjecture using the seft-affinity of the function of Takagi and the existing relationship between the Hausdorff and box-counting dimension.
Precisely, the main result states that:
$<$ *The Hausdorff and box-counting dimensions of the level sets* L_y *of the Takagi function* T *are at most* 1/2. $>$

Keywords. Takagi's function, level set, box-counting, Hausdorff dimension

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Interpolating sequences in weighted Bergman spaces

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Let $W : \mathbb{D} \to [0, \infty)$ be a measurable function and $0 < p < \infty$. $A^p(W)$ is the space of all analytic functions f in $\mathbb{D}$ such that

$$\|f\|_{p,W}^p = \int_{\mathbb{D}} |f(z)|^p W(z) dA(z) < \infty.$$

A sequence $(z_n) \subset \mathbb{D}$ is said to be an interpolating sequence for $A^p(W)$ if for any $(a_n) \in \ell^p$ there exists $f \in A^p(W)$ such that

$$f(z_n)(1 - |z_n|^2)^{-2/p}(\inf_{w \in D(z_n, r)} W(w))^{-1/p} = a_n, \quad n \in \mathbb{N}(0 < p \leq 1)$$

and

$$f(z_n)(1 - |z_n|^2)^{2/p}(\frac{1}{|D(z_n, r)|} \int_{D(z_n,r)} W^{-p'/p})^{-1/p'} = a_n, \quad n \in \mathbb{N}(1 < p < \infty),$$

where $D(z, r)$ denotes the hyperbolic disk.
Aleman and Vukotic proved ([1]) that any uniformly separated sequence is interpolating for $A^p(W)$ in the case of radial weights which are normal. We extend this result to non radial weights under certain A_1-condition for the averaging operator, extending the previously mentioned result.
Keywords. Interpolating sequences, weighted Bergman spaces, A_1-condition

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p-variation and p-semivariation on L^p of a vector measure

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Let $1 < p \leq 1$, X and Y (real) Banach spaces and (Ω, Σ, μ) a finite and positive measure space. It is well-known that the space of vector valued measures $m : \Sigma \to X$ having p-semivariation (resp. p-variation) finite can be identified with the space of linear and continuous maps (resp. with the space of absolutely summing operators) from $L^{p'}(\mu)$ into X (see [1]).
In this talk we present the vector valued version of this results when we deal with the spaces $L^{p'}(\mu)$ consisting of scalar functions that are integrable with respect to the (countable additive) vector measure $\nu : \Sigma \to Y$. We will discuss about the properties of the integration map (see [3]) and finally different examples will be given.

Keywords. Vector measures, integration, absolutely summing operators

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Stability and Lyapunov-Sobolev inequalities for periodic conservative systems

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In this talk we present some new results on stability properties for systems of equations

$$u''(t) + P(t)u(t) = 0,\ t \in \mathbb{R}, \tag{1}$$

where the matrix function $P(\cdot) \in \Lambda$, and Λ is defined as

The set of real $n \times n$ symmetric matrix valued function $P(\cdot)$, with continuous and $T-$periodic element functions $p_{ij}(t),\ 1 \leq i, j \leq n$, such that (1) has not nontrivial constant solutions and

[Λ]
$$\int_0^T \langle P(t)k, k\rangle \; dt \geq 0,\ \forall\ k \in \mathbb{R}^n.$$

Equation (1) models many phenomena in applied sciences (for example in engineering and physics, including problems in mechanics, astronomy, the theory of electric circuits, of the electric conductivity of metals, of the cyclotron, etc., see [5]). In particular, if $n = 1$, (1) is the very well known Hill equation.

The study of stability properties of (1) is of special interest. To this respect, the results proved by Krein in [4] show that the problem is closely related to Lyapunov-Sobolev inequalities. In fact, the stability properties of (1) strongly depend on the fact that the smallest positive eigenvalue of the antiperiodic eigenvalue problem

$$u''(t) + \lambda P(t)u(t) = 0,\ t \in \mathbb{R},\ u(0) + u(T) = u'(0) + u'(T) = 0 \tag{2}$$

be greater than one.

We establish some new conditions to get this last property (the detailed proofs can be seen in [1], [2], [3]).

Keywords. Periodic conservative systems, stability, Lyapunov-Sobolev inequalities.

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An application of Interpolation Theory to renorming of Lorentz-Karamata type spaces

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Operators acting on cones of measurable functions have been considered by many different authors in the past years. In this context, interpolation results for operators have proved to be very useful. See for example the paper by Sagher [4] in which the Fourier coefficients for some classes of functions were studied. Thus, interpolation of operators acting on cones of functions became a subject of interest and a number of papers in this topic were published. In this comunication we apply interpolation techniques to obtain renormings of Lorentz-Karamata type spaces. The paper, [2], is inspired by the ideas of Edmunds and Opic in [1] where they present new characterizations of Lorentz-Karamata spaces by means of quasi-norms equivalent to the classical ones. In that paper Lorentz-Karamata spaces are presented as big family of spaces containing classical Lebesgue spaces, Lorentz spaces, Lorentz-Zygmund spaces and generalized Lorentz-Zygmund spaces. Nevertheless we will see Lorentz-Karamata type spaces as particular cases of ultrasymmetric spaces. This large class of rearrangement invariant spaces were introduced by Pustylnik in [3].

Following this approach we define Lorentz-Karamata type spaces as follows. Let (Ω, μ) be a σ-finite measure spaces with a non-atomic measure. Given b a slowly varying function, a real parameter $1 \leq p \leq \infty$ and a rearrangement invariant space E, the Lorentz-Karamata type space $L_{p,b,E}$ consists of all measurable functions on Ω for which

$$\|f\|_{p,b,E} = \|t^{1/p} b(t) f^*(t)\|_E < \infty.$$

We will prove that for any $q \neq p \in \mathbb{R}$, any slowly varying function a and any rearrangement invariant space F,

$$\|t^{1/p} b(t) f^*(t)\|_E \sim \left\| t^{1/q} \frac{b(t)}{a(t)} \|u^{1/p-1/q} a(u) f^*(u)\|_{\widetilde{F}(t,\infty)} \right\|_{\widetilde{E}},$$

and in case $q < 0 < p$,

$$\|t^{1/p} b(t) f^*(t)\|_E \sim \left\| t^{1/q} \frac{b(t)}{a(t)} \|u^{1/p-1/q} a(u) f^*(u)\|_{\widetilde{F}(0,t)} \right\|_{\widetilde{E}}.$$

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On duality of aggregation operators

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In [2] a duality relation is studied for pairs of classes of binary operations in the unit inteval $\mathbb{I} = [0,1]$, involving members of a class of aggregation functions which satisfy certain boundary conditions. Specifically, from the solution of a functional equation system related to the De Rham system, it is established that for any F in the above class two unique aggregation functions G and $N_{k,k'}$ exist, so that the pair (F,G) is $N_{k,k'} - dual$.
In this talk we concern ourselves with an explicit expression of function $N_{k,k'}$, and we study interesting properties for this function. The key tool to obtain the proof of the rest of properties, is stated as follows:

Theorem 1. *For $k, k' \in]0,1[$, $N_{k,k'} : \mathbb{I} \to \mathbb{I}$ is given as follows, if*

$$\begin{aligned} x =& k^{t_0} + \ldots + k^{t_0}(1-k)^{s_0} + k^{t_1}(1-k)^{s_0+1} + \ldots + k^{t_1}(1-k)^{s_1} + \ldots \\ &+ k^{t_d}(1-k)^{s_{d-1}+1} + \ldots + k^{t_d}(1-k)^{s_d} + \ldots \end{aligned}$$

then

$$\begin{aligned} N_{k,k'}(x) :=& k' + k'(1-k') + \ldots + k'(1-k')^{t_0-2} + k'^{s_0+2}(1-k')^{t_0-1} + \ldots \\ &+ k'^{s_0+2}(1-k)^{t_1-2} + k'^{s_1+2}(1-k')^{t_1-1} + \ldots + k'^{s_1+2}(1-k')^{t_1-2} + \ldots \\ &+ k'^{s_{d-1}+2}(1-k')^{t_{d-1}-1} + \ldots + k'^{s_{d-1}+2}(1-k')^{t_d-2} + \ldots \end{aligned}$$

Keywords. De Rham system, singular function, aggregation operator, generalized dyadic representation system, k-negation, fractal dimension

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Renormings, fixed point property and stability

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Let $(X, \|\cdot\|)$ be a Banach space and C a subset of X. We say that a mapping T is non-expansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A set C has the fixed point property if for every non-expansive mapping $T : C \to C$ there exists $x \in C$ such that $Tx = x$. It is said that a Banach space X satisfies the fixed point property (FPP) if every closed convex bounded set $C \subset X$ has the fixed point property.
It is not difficult to show that the Banach spaces ℓ_1 and c_0 endowed with their usual norms do not have the FPP. For a long time it was conjectured that every Banach space with the Fixed Point Property (FPP) was reflexive. In 2008, P. K. Lin proved that there exists an equivalent norm $|||\cdot|||$ on ℓ_1 such that $(\ell_1, |||\cdot|||)$ has the FPP [4], which disproves the conjecture. Lin's example turned out to be the first known non-reflexive Banach space with the FPP. In this talk we extend P.K. Lin's techniques to more general spaces obtaining new non-reflexive Banach spaces with the FPP [1, 2]. Also, we apply our result to some subspaces of the Banach spaces $L_1[0, 1]$ and we analyze the stabiility of the FPP in ℓ_1 [3].

Keywords. Fixed point theory, Renorming theory, Nonexpansive mapping, Stability

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Lattice structure of $L^1_w(\nu)$ and representation theorems of Banach lattices

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The space of integrable functions with respect to a vector measure which is already interesting by itself, finds applications in important problems as the integral representation and the study of the optimal domain of linear operators or the representation of abstract Banach lattices as spaces of functions. Classical vector measures ν are considered to be defined on a σ-algebra and with values in a Banach space, and the corresponding spaces $L^1(\nu)$ and $L^1_w(\nu)$ of integrable and weakly integrable functions respectively have been studied in depth by many authors being their behavior well understood. However, this framework is not enough, for instance, for applications to operators on spaces which do not contain the characteristic functions of sets or Banach lattices without weak unit. These cases require ν to be defined on a weaker structure than σ-algebra, namely, a δ-ring. In this talk we are mainly interested in providing the properties which guarantee the representation of a Banach lattice by means of a space of integrable functions using vector measures defined on a δ-ring. To solve this problem it was necessary to study first the lattice structure of the corresponding space $L^1_w(\nu)$. It will be also the aim of this talk to present the results concerning the effect of certain properties of ν on the lattice properties of this space. Concretely, we analyze order continuity, order density and Fatou type properties for $L^1_w(\nu)$.

Keywords. Banach function spaces, Banach lattices, Vector Measures, Integration.

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Local spectral theory for strongly compact operators

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An algebra of bounded linear operators on a Banach space is said to be *strongly compact* if its unit ball is precompact in the strong operator topology, and a bounded linear operator on a Banach space is said to be *strongly compact* if the unital algebra generated by the operator is strongly compact. Our interest in this notion stems from the work of Lomonosov [4] on the existence of invariant subspaces.

A characterization of strongly compact, normal operators in terms of their spectral representation was given by Lomonosov, Rodríguez-Piazza, and the first author [2]. Necessary and sufficient conditions were also obtained for a unilateral weighted shift to be strongly compact in terms of the sliding products of its weights.

It was shown by Fernández-Valles and the first author [1] that an operator with a total set of eigenvectors must be strongly compact, and moreover, if the corresponding eigenvalues have finite multiplicity then its commutant is a strongly compact algebra. This result was applied to test strong compactness for several classes of operators, namely, bilateral weighted shifts, Cesàro operators, and composition operators.

A result of a different nature is needed in absence of eigenvalues. The aim of this talk is to provide a local spectral condition that is sufficient for a bounded linear operator on a Banach space to be strongly compact. The condition requires from the operator that the origin must lie in the interior of its full spectrum and that there must be a total set of vectors at which the local spectral radius is strictly less than the distance from the origin to the boundary of its full spectrum. This condition is then applied to describe a large class of strongly compact, injective bilateral weighted shifts on Hilbert spaces, completing and extending earlier work of Fernández-Valles and the first author [1]. Finally, further applications are easily derived, for instance, a strongly compact, invertible bilateral weighted shift is constructed in such a way that its inverse fails to be a strongly compact operator.

Keywords. Strongly compact operator, local spectral condition, bilateral weighted shift.

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Lipschitz estimates for solutions of elliptic equations singular at the boundary

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Let Ω be a bounded smooth domain in $\mathbb{R}^N$, $N \geq 2$, and let us denote by $d(x) = \mathrm{dist}(x, \partial\Omega)$. We study a class of singular Hamilton-Jacobi equations, arising from stochastic control problems, whose simplest model is

$$-\alpha\Delta u + u + \frac{\nabla u \cdot B(x)}{d(x)} + c(x)|\nabla u|^2 = f(x) \quad \text{in } \Omega,$$

where f belongs to $W^{1,\infty}_{\mathrm{loc}}(\Omega)$ and is (possibly) singular at $\partial\Omega$, $c \in W^{1,\infty}(\Omega)$ (with no sign condition) and the field $B \in W^{1,\infty}(\Omega)^N$ has the outward direction and satisfies $B \cdot \nu \geq \alpha$ at $\partial\Omega$ (ν is the outward normal). Despite the singularity in the equation, we prove gradient bounds up to the boundary and the existence of a (globally) Lipschitz solution. In some cases, we show that this is the unique bounded solution. The main tool is a refined weighted version of the classical Bernstein's method to get gradient bounds (see [4], [5], [6] for classical results); the key role is played here by the orthogonal transport component of the Hamiltonian. We also discuss the stability of such estimates with respect to α, as α vanishes, obtaining Lipschitz solutions for first order problems with similar features (see [3]). One of the main interest in studying this kind of problem relies on the application to a stochastic control problem with state constraint (see [1] and [2]).

Keywords. Hamilton-Jacobi equations, Lipschitz estimates

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Strongly preserver problems in Banach algebras and C*-algebras

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Let A and B be Banach algebras. We say that an element $a \in A$ has Drazin inverse $b = a^D$ if

$$ab = ba, \qquad bab = b \quad \text{and} \quad a^k ba = a^k \text{ for some } k \in \mathbb{N}$$

If $k = 1$ satisfies the equation above we will say that $b = a^\sharp$ is the group inverse of a. We characterize additive maps that preserve strongly Drazin (resp. group) invertibility, that is, $T : A \to B$ such that $T(a^D) = T(a)^D$ (resp. $T(a^\sharp) = T(a)^\sharp$).

Let now A and B be C*-algebras. We say that an element $a \in A$ has generalized inverse $b = a^\wedge$ if

$$Q(a)(b) = a, \qquad Q(b)(a) = b \quad \text{and} \quad Q(a)Q(b) = Q(b)Q(a)$$

where $Q(x)(y) = xy^*x$. We characterize additive maps that preserve strongly generalized invertibility, that is, $T : A \to B$ such that $T(a^\wedge) = T(a)^\wedge$.

Keywords. Drazin inverse, Banach algebra, generalized inverse, C*-algebra, linear preserver

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Continua of solutions for quasilinear elliptic problems with natural growth and Gelfand type equations

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We realize a brief summary of recent results for the following parameter boundary value problem

$$\begin{cases} -\Delta u + g(u)|\nabla u|^2 = \lambda f(u) + f_0(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{3}$$

where Ω is an open and bounded set in $\mathbb{R}^N$ $(N \geq 3)$. Specifically, in [2] it is studied the case in which $0 \leq f_0 \in L^{\frac{2N}{N+2}}(\Omega)$ and $f(u) = u^p$ with $0 \leq p < \frac{N+2}{N-2}$. In addition to the natural growth in the gradient of the quasilinear elliptic differential operator in (3), in some cases, we may also have a singularity at zero, since it is only assumed the continuity of the function $g : (0, +\infty) \to [0, +\infty)$. We study the range of values for the parameter λ, such that (3) admits a positive solution. Combining the results in [1, 4, 5] for $\lambda = 0$ with topological methods we give sufficient conditions to have that this range is bounded or unbounded. It is shown that some differences with the semilinear case ($g \equiv 0$) are due not only to the natural growth but also to the behavior of g. On the other hand, in the case in which $f_0 \equiv 0$, under appropriate conditions of g and f it is proved in [3] that the maximal set of λ for which the problem has at least one solution is a closed interval $[0, \lambda^*]$, $\lambda^* > 0$, and there exists a minimal regular solution for every $\lambda \in [0, \lambda^*)$. The case of radial solutions in which Ω is the unit ball is also studied.

Keywords. Bifurcation; Continua of solutions; Nonlinear elliptic solutions;
Singular natural growth gradient terms.

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The Kronecker-Bohl lemma for counting zeros of Dirichlet polynomials

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The functions $L_n(z) := 1 - \sum_{k=2}^{n} k^z$, $n \geq 2$, are the prototype of non-lattice Dirichlet polynomials [2], that is, functions of the form $f(z) := 1 - \sum_{k=2}^{n} m_k a_k^z$ with $m_j \in \mathbb{C}$ and $a_j > 0$ such that the additive subgroup $G = \mathbb{Z}\log a_2 + \ldots + \mathbb{Z}\log a_n$ is dense in $\mathbb{R}$. In this paper we prove, by means of Kronecker-Bohl lemma and by using some techniques of [3], the existence of an infinite amount of r-rectangles (rectangular sets with two little semicircle of radius r) in the critical strip of $L_n(z)$ for which the exact number of zeros inside them is given by

$$\left[\frac{T\log n}{2\pi}\right] + 1 \tag{4}$$

where T is the height of the r-rectangle and [ ] represents the integer part. In the extensive literature on the related question with the topic of the zeros, the formulae to determine the number of them in certain regions, mainly rectangles, contain a term which expresses the maximum error, namely $n-1$ or $\frac{n}{2}$ in the best of the cases (see for example [4] and [1] respectively), where n is the "degree" of the exponential polynomial; however, in our r-rectangles, the error in the formula (1) is reduced to 0.

Keywords. Zeros of entire functions, exponential polynomials, Dirichlet polynomials

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M-norms on C*-algebras

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Two elements a, b in a C*-algebra A are said to be *(algebraically) orthogonal* ($a \perp b$) whenever $ab^* = b^*a = 0$. It is well known that orthogonal elements in A are *(geometrically) M-orthogonal* in the underlying Banach space, that is, $\|a \pm b\|_0 = \max\{\|a\|_0, \|b\|_0\}$ whenever $a \perp b$.

A norm $\|.\|_1$ on A is said to be an *M-norm* (resp., a *semi-M-norm*) if for every a, b in A with $a \perp b$ we have $\|a+b\|_1 = \max\{\|a\|_1, \|b\|_1\}$ (resp., $\|a+b\|_1 \geq \max\{\|a\|_1, \|b\|_1\}$). The original C*-norm, $\|.\|_0$, is an M-norm on A. However, not every M-norm on A satisfies the Gelfand-Naimark axiom.

We shall present in this talk the latest advance in the study of the following problem:

> Is every complete semi-M-norm on a C*-algebra automatically continuous with respect to the original C*-norm?

In a recent paper, obtained in collaboration with Timur Oikhberg (University of California - Irvine) and Maribel Ramírez (Universidad de Almería), we establish that every complete (semi-)M-norm on a von Neumann algebra or on a compact C*-algebra A is equivalent to the original C*-norm of A.

Models for growth of heterogeneous sandpiles via Mosco convergence

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In this talk we study the asymptotic behavior of several classes of power-law functionals involving variable exponents $p_n(\cdot) \to \infty$, via Mosco convergence. In the particular case $p_n(\cdot) = np(\cdot)$, we show that the sequence $\{H_n\}$ of functionals $H_n : L^2(\mathbb{R}^N) \to [0, +\infty]$ given by

$$H_n(u) = \begin{cases} \displaystyle\int_{\mathbb{R}^N} \frac{\lambda(x)^n}{np(x)} |\nabla u(x)|^{np(x)}\, dx & \text{if } u \in L^2(\mathbb{R}^N) \cap W^{1,np(\cdot)}(\mathbb{R}^N) \\ +\infty & \text{otherwise,} \end{cases}$$

converges in the sense of Mosco to a functional which vanishes on the set

$$\left\{ u \in L^2(\mathbb{R}^N) \ : \ \lambda(x)|\nabla u|^{p(x)} \leq 1 \text{ a.e. } x \in \mathbb{R}^N \right\}$$

and is infinite in its complement. We also provide an example of a sequence of functionals whose Mosco limit cannot be described in terms of the characteristic function of a subset of $L^2(\mathbb{R}^N)$.

As an application of our results we obtain a model for the growth of a sandpile in which the allowed slope of the sand depends explicitly on the position in the sample. Keywords. Mosco convergence, power-law functionals, variable exponent spaces, sandpile models.

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Existence of solution for a singular Liouville equation via variational methods

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We consider a singular Liouville equation on a compact surface, arising from the study of Chern-Simons vortices in a self dual regime. A first novelty in our approach is a definition of barycenter which is very convenient for our purposes. Indeed, we give improved versions of the Moser-Trudinger inequality for functions with barycenter on the singularities. This is the main ingredient or our variational scheme, from which we prove new existence results. This is joint work with Andrea Malchiodi (SISSA, Italy).

Posters

Linear maps strongly preserving Moore-Penrose invertibility

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Let A and B be C*-algebras. We investigate linear maps from A to B strongly preserving Moore-Penrose invertibility where A is unital, and either it is linearly spanned by its projections, or has real rank zero, or has large socle.

Keywords. Moore-Penrose inverse, generalized inverse, C*-algebra, linear preserver

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Regularity for quasilinear elliptic systems with critical growth. Critical groups computations and applications

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We study the existence of positive solutions of the following nonlinear system

$$\begin{cases} -\mathrm{div}\left((\alpha+|\nabla u|^{p-2})\nabla u\right) = \mathrm{D}_u F(x,u,v), & x\in\Omega \\ -\mathrm{div}\left((\alpha+|\nabla v|^{r-2})\nabla v\right) = \mathrm{D}_v F(x,u,v), & x\in\Omega \\ u=v=0, \quad x\in\partial\Omega, \end{cases} \tag{5}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, p,r are real numbers larger than 2, $\alpha \geq 0$ and $N \geq \max\{p^2, r^2\}$ and F having a critical growth, for example $F(u,v) = \frac{1}{p}|u|^p + \frac{1}{r}|v|^r + \frac{2}{\gamma+\beta}|u|^\gamma |v|^\beta$ where $\gamma, \beta > 1$ satisfy $\frac{\gamma}{p^*} + \frac{\beta}{r^*} = 1$.
In [1] we prove C^1 regularity up to the boundary for solutions (see [3] for the scalar case). This allows a finite dimensional reduction for the critical group computation of the associated functional [2]. To overcame the lack of compactness we prove a local Palais-Smale condition around critical points.

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Optimal extensions of operators on Banach function spaces

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Given an operator $T : X \to E$, where X is a Banach function space and E is a Banach space, we provide several optimal extension results for T through spaces of integrable functions with respect to a vector measure ν_T canonically associated to T. Depending on the continuity property of the operator which we want to preserve, the optimal domain is $L^1(\nu_T)$, $L^1_w(\nu_T)$ or the Fatou completion of $L^1(\nu_T)$.

Best proximity points for cyclic mappings

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Given A and B two subsets of a metric space, a mapping $T : A \cup B \to A \cup B$ is said to be cyclic if $T(A) \subset B$ and $T(B) \subset A$. It is known that, if A and B are nonempty and complete and the cyclic map verifies for some $k \in (0, 1)$ that $d(Tx, Ty) \leq kd(x, y)$ $\forall x \in A$ and $y \in B$, then $A \cap B \neq \emptyset$ and the mapping T has a unique fixed point. A generalization of this situation was studied under the assumption of $A \cap B = \emptyset$. In this case, it was also assumed that there exists $k \in (0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y) + (1 - k)dist(A, B)$$

for all $x \in A$ and $y \in B$ to obtain existence, uniqueness and convergence of iterates to the so-called best proximity points; that is, a point x either in A or B such that $d(x, Tx) = dist(A, B)$. This was first studied for uniformly convex Banach spaces by A. Anthony Eldred and P. Veeramani. Later, T. Suzuki, M. Kikkawa and C. Vetro tackled the same problem within the wider framework of metric spaces, but in this case from a different point of view. Now the hypotheses were imposed to the sets A and B instead of to the whole space X. More precisely, the property UC was introduced for a pair (A, B) of subsets of a metric space so a result on existence, uniqueness and convergence of iterates stands in general metric spaces. In fact, this result generalized the one obtained for uniformly convex Banach spaces. In this poster, we introduce two new properties for pairs of sets (A, B), the so-called property WUC and HW, which are proved to happen under less restrictive conditions than property UC and for which we obtain similar existence,uniqueness and convergence results. Moreover, we also give a partial affirmative answer to a question raised by Eldred and Veeramani about the existence of best proximity points in reflexive spaces.

Vector measure orthogonal sequences and signal approximation

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In the last decade we have developed new procedures to construct approximations of functions by defining orthogonal series in spaces of square integrable functions with respect to a vector measure. Sequences of real functions that are orthogonal with respect to a vector measure are a natural generalization of the orthogonal systems with respect to a parametric measure. In this work we present the case when the Fourier coefficients are also functions producing a non linear approximation. We study the convergence properties of these series, defining a convenient approximation structure for signal processing involving parametric dependence of the measure. Some examples regarding classical orthogonal polynomials are given.

Keywords. Function approximation, vector measures, integration.

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Density by moduli and statistical convergence

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By using modulus functions we introduce a new concept of density for subsets of $\mathbb{N}$ and $\mathbb{N}^2$. Consequently, we obtain a generalization of the notion of statistical con-vergence which is studied and characterized. As an application, we prove that the ordinary convergence and 'Pringsheim convergence', respectively, are equivalent to 'module statistical convergence for every unbounded modulus function'.

Keywords. Density, modulus function, statistical convergence.

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A scheme for interpolation by Hankel translates of a basis function

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Golomb and Weinberger [1] described a variational approach to interpolation which reduced the problem to minimizing a norm in a reproducing kernel Hilbert space generated by means of a small number of data points. Later, Duchon [2] defined radial basis function interpolants as functions which minimize a suitable seminorm given by a weight in spaces of distributions closely related to Sobolev spaces. These minimal interpolants could be written as a linear combination of translates of a single function, the so-called basis function, plus a polynomial. Light and Wayne [3] extended Duchon's class of weight functions, which in turn allowed for non-radial basis functions in their scheme. Following the approach of Light and Wayne, we discuss interpolation of complex-valued functions defined on the positive real axis I by certain spaces of Sobolev type involving the Hankel transformation and powers of the Bessel operator. The set of interpolation points will be a subset $\{a_1, \ldots, a_n\}$ of I and the interpolants will take the form

$$u(x) = \sum_{i=1}^{n} \alpha_i (\tau_{a_i} \phi)(x) + \sum_{j=0}^{m-1} \beta_j p_{\mu,j}(x) \quad (x \in I),$$

where $\mu > -1/2$, ϕ is the basis function, $p_{\mu,j}(x) = x^{2j+\mu+1/2}$ $(j \in \mathbb{Z}_+, 0 \leq j \leq m-1)$ is a Müntz monomial, $\tau_z (z \in I)$ denotes the Hankel translation operator of order μ, and $\alpha_i, \beta_j (i, j \in \mathbb{Z}_+, 1 \leq i \leq n, 0 \leq j \leq m-1)$ are complex coefficients. An estimate for the pointwise error of the interpolants is also given.

Keywords. Basis function, Bessel operator, Hankel convolution, Hankel translation, Hankel transformation, minimal norm interpolant, Sobolev embedding theorem.

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Essential norm of composition operators on Banach spaces of Hölder functions

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Let (X, d) be a pointed compact metric space, let $0 < \alpha < 1$, and let $\varphi : X \to X$ be a base point-preserving Lipschitz map. We show that the essential norm of the composition operator C_φ induced by the symbol φ on the Lipschitz spaces $\operatorname{lip}_0(X, d^\alpha)$ and $\operatorname{Lip}_0(X, d^\alpha)$ is given by the formula

$$\|C_\varphi\|_e = \lim_{t\to 0} \sup_{0<d(x,y)<t} \frac{d(\varphi(x), \varphi(y))^\alpha}{d(x,y)^\alpha}$$

whenever the dual space $\operatorname{lip}_0(X, d^\alpha)^*$ has the approximation property. This happens in particular when X is an infinite compact subset of a finite-dimensional normed linear space.

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