Essential norm of composition operators on Banach spaces of Hölder functions

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Let (X, d) be a pointed compact metric space, let $0 < \alpha < 1$, and let $\varphi : X \to X$ be a base point-preserving Lipschitz map. We show that the essential norm of the composition operator C_φ induced by the symbol φ on the Lipschitz spaces $\mathrm{lip}_0(X, d^\alpha)$ and $\mathrm{Lip}_0(X, d^\alpha)$ is given by the formula

$$\|C_\varphi\|_e = \lim_{t \to 0} \sup_{0 < d(x,y) < t} \frac{d(\varphi(x), \varphi(y))^\alpha}{d(x, y)^\alpha}$$

whenever the dual space $\mathrm{lip}_0(X, d^\alpha)^*$ has the approximation property. This happens in particular when X is an infinite compact subset of a finite-dimensional normed linear space.

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