A scheme for interpolation by Hankel translates of a basis function

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Golomb and Weinberger [1] described a variational approach to interpolation which reduced the problem to minimizing a norm in a reproducing kernel Hilbert space generated by means of a small number of data points. Later, Duchon [2] defined radial basis function interpolants as functions which minimize a suitable seminorm given by a weight in spaces of distributions closely related to Sobolev spaces. These minimal interpolants could be written as a linear combination of translates of a single function ϕ, the so-called basis function, plus a polynomial. Light and Wayne [3] extended Duchon's class of weight functions, which in turn allowed for non-radial basis functions in their scheme. Following the approach of Light and Wayne, we discuss interpolation of complex-valued functions defined on the positive real axis I by certain spaces of Sobolev type involving the Hankel transformation and powers of the Bessel operator. The set of interpolation points will be a subset $\{a_1, \ldots, a_n\}$ of I and the interpolants will take the form

$$u(x) = \sum_{i=1}^{n} \alpha_i \left(\tau_{a_i}\phi\right)(x) + \sum_{j=0}^{m-1} \beta_j p_{\mu,j}(x) \quad (x \in I),$$

where $\mu > -1/2$, ϕ is the basis function, $p_{\mu,j}(x) = x^{2j+\mu+1/2}$ $(j \in \mathbb{Z}_+,\ 0 \leq j \leq m-1)$ is a Müntz monomial, τ_z $(z \in I)$ denotes the Hankel translation operator of order μ, and α_i, β_j $(i, j \in \mathbb{Z}_+,\ 1 \leq i \leq n,\ 0 \leq j \leq m-1)$ are complex coefficients. An estimate for the pointwise error of the interpolants is also given.

Keywords. Basis function, Bessel operator, Hankel convolution, Hankel translation, Hankel transformation, minimal norm interpolant, Sobolev embedding theorem.

References

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