Regularity for quasilinear elliptic systems with critical growth. Critical groups computations and applications

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We study the existence of positive solutions of the following nonlinear system

$$\begin{cases} -\mathrm{div}\left((\alpha + |\nabla u|^{p-2})\nabla u\right) = \mathrm{D}_u F(x,u,v), & x \in \Omega \\ -\mathrm{div}\left((\alpha + |\nabla v|^{r-2})\nabla v\right) = \mathrm{D}_v F(x,u,v), & x \in \Omega \\ u = v = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where Ω is a smooth bounded domain of $\mathbb{R}^N$, p,r are real numbers larger than 2, $\alpha \geq 0$ and $N \geq \max\{p^2, r^2\}$ and F having a critical growth, for example $F(u,v) = \frac{1}{p}|u|^p + \frac{1}{r}|v|^r + \frac{2}{\gamma+\beta}|u|^\gamma |v|^\beta$ where $\gamma, \beta > 1$ satisfy $\frac{\gamma}{p^*} + \frac{\beta}{r^*} = 1$.
In [1] we prove C^1 regularity up to the boundary for solutions (see [3] for the scalar case). This allows a finite dimensional reduction for the critical group computation of the associated functional [2]. To overcame the lack of compactness we prove a local Palais-Smale condition around critical points.

References

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