An application of Interpolation Theory to renorming of Lorentz-Karamata type spaces

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Operators acting on cones of measurable functions have been considered by many different authors in the past years. In this context, interpolation results for operators have proved to be very useful. See for example the paper by Sagher [4] in which the Fourier coefficients for some classes of functions were studied. Thus, interpolation of operators acting on cones of functions became a subject of interest and a number of papers in this topic were published.

In this comunication we apply interpolation techniques to obtain renormings of Lorentz-Karamata type spaces. The paper, [2], is inspired by the ideas of Edmunds and Opic in [1] where they present new characterizations of Lorentz-Karamata spaces by means of quasi-norms equivalent to the classical ones. In that paper Lorentz-Karamata spaces are presented as big family of spaces containing classical Lebesgue spaces, Lorentz spaces, Lorentz-Zygmund spaces and generalized Lorentz-Zygmund spaces. Nevertheless we will see Lorentz-Karamata type spaces as particular cases of ultra-symmetric spaces. This large class of rearrangement invariant spaces were introduced by Pustylnik in [3].

Following this approach we define Lorentz-Karamata type spaces as follows. Let (Ω, μ) be a σ-finite measure spaces with a non-atomic measure. Given b a slowly varying function, a real parameter $1 \leq p \leq \infty$ and a rearrangement invariant space E, the Lorentz-Karamata type space $L_{p,b,E}$ consists of all measurable functions on Ω for which

$$\|f\|_{p,b,E} = \|t^{1/p} b(t) f^*(t)\|_E < \infty.$$

We will prove that for any $q \neq p \in \mathbb{R}$, any slowly varying function a and any rearrangement invariant space F,

$$\|t^{1/p} b(t) f^*(t)\|_E \sim \left\| t^{1/q} \frac{b(t)}{a(t)} \|u^{1/p-1/q} a(u) f^*(u)\|_{\widetilde{F}(t,\infty)} \right\|_{\widetilde{E}},$$

and in case $q < 0 < p$,

$$\|t^{1/p} b(t) f^*(t)\|_E \sim \left\| t^{1/q} \frac{b(t)}{a(t)} \|u^{1/p-1/q} a(u) f^*(u)\|_{\widetilde{F}(0,t)} \right\|_{\widetilde{E}}.$$

Keywords. Interpolation theory, Functions spaces

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