Models for growth of heterogeneous sandpiles via Mosco convergence

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In this talk we study the asymptotic behavior of several classes of power-law functionals involving variable exponents $p_n(\cdot) \to \infty$, via Mosco convergence. In the particular case $p_n(\cdot) = np(\cdot)$, we show that the sequence $\{H_n\}$ of functionals $H_n : L^2(\mathbb{R}^N) \to [0, +\infty]$ given by

$$H_n(u) = \begin{cases} \displaystyle\int_{\mathbb{R}^N} \frac{\lambda(x)^n}{np(x)} |\nabla u(x)|^{np(x)}\, dx & \text{if } u \in L^2(\mathbb{R}^N) \cap W^{1,np(\cdot)}(\mathbb{R}^N) \\ +\infty & \text{otherwise,} \end{cases}$$

converges in the sense of Mosco to a functional which vanishes on the set

$$\left\{ u \in L^2(\mathbb{R}^N) \ : \ \lambda(x)|\nabla u|^{p(x)} \leq 1 \text{ a.e. } x \in \mathbb{R}^N \right\}$$

and is infinite in its complement. We also provide an example of a sequence of functionals whose Mosco limit cannot be described in terms of the characteristic function of a subset of $L^2(\mathbb{R}^N)$.
As an application of our results we obtain a model for the growth of a sandpile in which the allowed slope of the sand depends explicitly on the position in the sample.
Keywords. Mosco convergence, power-law functionals, variable exponent spaces, sandpile models.

References

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