Models for growth of heterogeneous sandpiles via Mosco convergence

M. Bocea, 1 M. Mihăilescu,2 M. Pérez-Llanos,3 and J. D. Rossi,4

In this talk we study the asymptotic behavior of several classes of power-law functionals involving variable exponents \( p_n(\cdot) \to \infty \), via Mosco convergence. In the particular case \( p_n(\cdot) = np(\cdot) \), we show that the sequence \( \{H_n\} \) of functionals \( H_n : L^2(\mathbb{R}^N) \to [0, +\infty] \) given by

\[
H_n(u) = \begin{cases} 
\int_{\mathbb{R}^N} \frac{\lambda(x)^n}{np(x)} |\nabla u(x)|^{np(x)} \, dx & \text{if } u \in L^2(\mathbb{R}^N) \cap W^{1,np(\cdot)}(\mathbb{R}^N) \\
+\infty & \text{otherwise,}
\end{cases}
\]

converges in the sense of Mosco to a functional which vanishes on the set

\[
\left\{ u \in L^2(\mathbb{R}^N) : \lambda(x)|\nabla u|^{p(x)} \leq 1 \text{ a.e. } x \in \mathbb{R}^N \right\}
\]

and is infinite in its complement. We also provide an example of a sequence of functionals whose Mosco limit cannot be described in terms of the characteristic function of a subset of \( L^2(\mathbb{R}^N) \).

As an application of our results we obtain a model for the growth of a sandpile in which the allowed slope of the sand depends explicitly on the position in the sample.

Keywords. Mosco convergence, power-law functionals, variable exponent spaces, sandpile models.

References


1 Department of Mathematics, North Dakota State University.
2 Department of Mathematics, University of Craiova.
3 Department of Mathematics, Universidad Autónoma de Madrid.
4 Department of Mathematics, Universidad de Alicante.

mayte.perez@uam.es