Continua of solutions for quasilinear elliptic problems with natural growth and Gelfand type equations

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We realize a brief summary of recent results for the following parameter boundary value problem

$$\begin{cases} -\Delta u + g(u)|\nabla u|^2 = \lambda f(u) + f_0(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is an open and bounded set in $\mathbb{R}^N$ ($N \geq 3$). Specifically, in [2] it is studied the case in which $0 \leq f_0 \in L^{\frac{2N}{N+2}}(\Omega)$ and $f(u) = u^p$ with $0 \leq p < \frac{N+2}{N-2}$. In addition to the natural growth in the gradient of the quasilinear elliptic differential operator in (1), in some cases, we may also have a singularity at zero, since it is only assumed the continuity of the function $g : (0, +\infty) \to [0, +\infty)$. We study the range of values for the parameter λ, such that (1) admits a positive solution. Combining the results in [1, 4, 5] for $\lambda = 0$ with topological methods we give sufficient conditions to have that this range is bounded or unbounded. It is shown that some differences with the semilinear case ($g \equiv 0$) are due not only to the natural growth but also to the behavior of g. On the other hand, in the case in which $f_0 \equiv 0$, under appropriate conditions of g and f it is proved in [3] that the maximal set of λ for which the problem has at least one solution is a closed interval $[0, \lambda^*]$, $\lambda^* > 0$, and there exists a minimal regular solution for every $\lambda \in [0, \lambda^*)$. The case of radial solutions in which Ω is the unit ball is also studied.

Keywords. Bifurcation; Continua of solutions; Nonlinear elliptic solutions;
Singular natural growth gradient terms.

References

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