On duality of aggregation operators

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In [2] a duality relation is studied for pairs of classes of binary operations in the unit inteval $\mathbb{I} = [0, 1]$, involving members of a class of aggregation functions which satisfy certain boundary conditions. Specifically, from the solution of a functional equation system related to the De Rham system, it is established that for any F in the above class two unique aggregation functions G and $N_{k,k'}$ exist, so that the pair (F, G) is $N_{k,k'}$-dual.

In this talk we concern ourselves with an explicit expression of function $N_{k,k'}$, and we study interesting properties for this function. The key tool to obtain the proof of the rest of properties, is stated as follows:

Theorem 1 *For $k, k' \in \left]0,1\right[$, $N_{k,k'} : \mathbb{I} \to \mathbb{I}$ is given as follows: if*

$$\begin{aligned} x \quad = \quad & k^{t_0} + \cdots + k^{t_0} (1-k)^{s_0} + k^{t_1} (1-k)^{s_0+1} + \cdots + k^{t_1} (1-k)^{s_1} + \cdots \\ & + k^{t_d} (1-k)^{s_{d-1}+1} + \cdots + k^{t_d} (1-k)^{s_d} + \cdots \end{aligned}$$

then

$$\begin{aligned} N_{k,k'}(x) \quad : \quad & = k' + k' (1-k') + \cdots + k' (1-k')^{t_0-2} + \\ & + k'^{s_0+2} (1-k')^{t_0-1} + \cdots + k'^{s_0+2} (1-k')^{t_1-2} + \\ & + k'^{s_1+2} (1-k')^{t_1-1} + \cdots + k'^{s_1+2} (1-k')^{t_2-2} + \cdots \\ & + k'^{s_{d-1}+2} (1-k')^{t_{d-1}-1} + \cdots + k'^{s_{d-1}+2} (1-k')^{t_d-2} + \cdots \end{aligned}$$

Keywords. De Rham system; singular function; aggregation operator; generalized dyadic representation system; k-negation; fractal dimension

References

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