Renormings, Fixed Point Property and Stability

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Let $(X, \| \cdot \|)$ be a Banach space and $C$ a subset of $X$. We say that a mapping $T$ is non-expansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A set $C$ has the fixed point property if for every non-expansive mapping $T : C \to C$ there exists $x \in C$ such that $Tx = x$. It is said that a Banach space $X$ satisfies the fixed point property (FPP) if every closed convex bounded set $C \subset X$ has the fixed point property.

It is not difficult to show that the Banach spaces $\ell_1$ and $c_0$ endowed with their usual norms do not have the FPP. For a long time it was conjectured that every Banach space with the Fixed Point Property (FPP) was reflexive. In 2008, P. K. Lin proved that there exists an equivalent norm $\| \cdot \|$ on $\ell_1$ such that $(\ell_1, \| \cdot \|)$ has the FPP \cite{Lin2008}, which disproves the conjecture. Lin’s example turned out to be the first known non-reflexive Banach space with the FPP. In this talk we extend P.K. Lin’s techniques to more general spaces obtaining new non-reflexive Banach spaces with the FPP \cite{Hernandez-Linares2010, Hernandez-Linares2011}. Also, we apply our result to some subspaces of the Banach spaces $L_1[0, 1]$ and we analyze the stability of the FPP in $\ell_1$ \cite{Hernandez-Linares2013}.

Keywords. Fixed point theory, Renorming theory, Nonexpansive mapping, Stability

References


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