Renormings, Fixed Point Property and Stability

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Let $(X, \|\cdot\|)$ be a Banach space and C a subset of X. We say that a mapping T is non-expansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. A set C has the fixed point property if for every non-expansive mapping $T : C \to C$ there exists $x \in C$ such that $Tx = x$. It is said that a Banach space X satisfies the fixed point property (FPP) if every closed convex bounded set $C \subset X$ has the fixed point property.
It is not difficult to show that the Banach spaces ℓ_1 and c_0 endowed with their usual norms do not have the FPP. For a long time it was conjectured that every Banach space with the Fixed Point Property (FPP) was reflexive. In 2008, P. K. Lin proved that there exists an equivalent norm $|||\cdot|||$ on ℓ_1 such that $(\ell_1, |||\cdot|||)$ has the FPP [4], which disproves the conjecture. Lin's example turned out to be the first known non-reflexive Banach space with the FPP. In this talk we extend P.K. Lin's techniques to more general spaces obtaining new non-reflexive Banach spaces with the FPP [1, 2]. Also, we apply our result to some subspaces of the Banach spaces $L_1[0,1]$ and we analyze the stabiility of the FPP in ℓ_1 [3].
Keywords. Fixed point theory, Renorming theory, Nonexpansive mapping, Stability

References

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