Stability and Lyapunov-Sobolev Inequalities for Periodic Conservative Systems

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In this talk we present some new results on stability properties for systems of equations

$$u''(t) + P(t)u(t) = 0, \ t \in \mathbb{R}, \tag{1}$$

where the matrix function $P(\cdot) \in \Lambda$, and Λ is defined as

[Λ] *The set of real $n \times n$ symmetric matrix valued function $P(\cdot)$, with continuous and $T-$periodic element functions $p_{ij}(t),\ 1 \le i, j \le n$, such that (1) has not nontrivial constant solutions and*

$$\int_0^T \langle P(t)k, k\rangle \ dt \ge 0, \ \forall \ k \in \mathbb{R}^n.$$

Here, $\langle \cdot, \cdot \rangle$ denotes the usual inner product in $\mathbb{R}^n$.
Equation (1) models many phenomena in applied sciences (for example in engineering and physics, including problems in mechanics, astronomy, the theory of electric circuits, of the electric conductivity of metals, of the cyclotron, etc., see [5]). In particular, if $n = 1$, (1) is the very well known Hill equation.
The study of stability properties of (1) is of special interest. To this respect, the results proved by Krein in [4] show that the problem is closely related to Lyapunov-Sobolev inequalities. In fact, the stability properties of (1) strongly depend on the fact that the smallest positive eigenvalue of the antiperiodic eigenvalue problem

$$u''(t) + \lambda P(t)u(t) = 0, \ t \in \mathbb{R}, \ u(0) + u(T) = u'(0) + u'(T) = 0 \tag{2}$$

be greater than one.
We establish some new conditions to get this last property (the detailed proofs can be seen in [1], [2], [3]).

Keywords. Periodic conservative systems, stability, Lyapunov-Sobolev inequalities.

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