Best proximity points for cyclic mappings

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Abstract

Given A and B two subsets of a metric space, a mapping $T : A \cup B \to A \cup B$ is said to be cyclic if $T(A) \subseteq B$ and $T(B) \subseteq A$. It is known that, if A and B are nonempty and complete and the cyclic map verifies for some $k \in (0,1)$ that $d(Tx,Ty) \leq kd(x,y)$ $\forall$ $x \in A$ and $y \in B$, then $A \cap B \neq \emptyset$ and the mapping T has a unique fixed point. A generalization of this situation was studied under the assumption of $A \cap B = \emptyset$. In this case, it was also assumed that there exists $k \in (0,1)$ such that

$$d(Tx,Ty) \leq kd(x,y) + (1-k)\operatorname{dist}(A,B)$$

for all $x \in A$ and $y \in B$ to obtain existence, uniqueness and convergence of iterates to the so-called best proximity points; that is, a point x either in A or B such that $d(x,Tx) = \operatorname{dist}(A,B)$. This was first studied for uniformly convex Banach spaces by A. Anthony Eldred and P. Veeramani. Later, T. Suzuki, M. Kikkawa and C. Vetro tackled the same problem within the wider framework of metric spaces, but in this case from a different point of view. Now the hypothesis were imposed to the sets A and B instead of to the whole space X. More precisely, the property UC was introduced for a pair (A,B) of subsets of a metric space so a result on existence, uniqueness and convergence of iterates stands in general metric spaces. In fact, this result generalized the one obtained for uniformly convex Banach spaces. In this poster, we introduce two new properties for pairs of sets (A,B), the so-called property WUC and HW, which are proved to happen under less restrictive conditions than property UC and for which we obtain similar existence,uniqueness and convergence results. Moreover, we also give a partial affirmative answer to a question raised by Eldred and Veeramani about the existence of best proximity points in reflexive spaces.