

Local spectral theory for strongly compact operators

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An algebra of bounded linear operators on a Banach space is said to be *strongly compact* if its unit ball is precompact in the strong operator topology, and a bounded linear operator on a Banach space is said to be *strongly compact* if the unital algebra generated by the operator is strongly compact. Our interest in this notion stems from the work of Lomonosov [4] on the existence of invariant subspaces.

A characterization of strongly compact, normal operators in terms of their spectral representation was given by Lomonosov, Rodríguez-Piazza, and the first author [2]. Necessary and sufficient conditions were also obtained for a unilateral weighted shift to be strongly compact in terms of the sliding products of its weights.

It was shown by Fernández-Valles and the first author [1] that an operator with a total set of eigenvectors must be strongly compact, and moreover, if the corresponding eigenvalues have finite multiplicity then its commutant is a strongly compact algebra. This result was applied to test strong compactness for several classes of operators, namely, bilateral weighted shifts, Cesàro operators, and composition operators.

A result of a different nature is needed in absence of eigenvalues. The aim of this talk is to provide a local spectral condition that is sufficient for a bounded linear operator on a Banach space to be strongly compact. The condition requires from the operator that the origin must lie in the interior of its full spectrum and that there must be a total set of vectors at which the local spectral radius is strictly less than the distance from the origin to the boundary of its full spectrum. This condition is then applied to describe a large class of strongly compact, injective bilateral weighted shifts on Hilbert spaces, completing and extending earlier work of Fernández-Valles and the first author [1]. Finally, further applications are easily derived, for instance, a strongly compact, invertible bilateral weighted shift is constructed in such a way that its inverse fails to be a strongly compact operator.

Keywords. Strongly compact operator, local spectral condition, bilateral weighted shift.

References

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