Lattice structure of $L^1_w(\nu)$ and Representation Theorems of Banach Lattices

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The space of integrable functions with respect to a vector measure which is already interesting by itself, finds applications in important problems as the integral representation and the study of the optimal domain of linear operators or the representation of abstract Banach lattices as spaces of functions. Classical vector measures ν are considered to be defined on a σ-algebra and with values in a Banach space, and the corresponding spaces $L^1(\nu)$ and $L^1_w(\nu)$ of integrable and weakly integrable functions respectively have been studied in depth by many authors being their behavior well understood. However, this framework is not enough, for instance, for applications to operators on spaces which do not contain the characteristic functions of sets or Banach lattices without weak unit. These cases require ν to be defined on a weaker structure than σ-algebra, namely, a δ-ring. In this talk we are mainly interested in providing the properties which guarantee the representation of a Banach lattice by means of a space of integrable functions using vector measures defined on a δ-ring. To solve this problem it was necessary to study first the lattice structure of the corresponding space $L^1_w(\nu)$. It will be also the aim of this talk to present the results concerning the effect of certain properties of ν on the lattice properties of this space. Concretely, we analyze order continuity, order density and Fatou type properties for $L^1_w(\nu)$.

Keywords. Banach function spaces, Banach lattices, Vector Measures, Integration.

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