

# Drazin inverse and generalized inverse strongly preservers

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Let  $A$  and  $B$  be Banach algebras. We say that an element  $a \in A$  has Drazin inverse  $b = a^D$  if

$$ab = ba, \quad bab = b \quad \text{and} \quad a^kba = a^k \quad \text{for some } k \in \mathbb{N}$$

If  $k = 1$  satisfies the equation above we will say that  $b = a^\sharp$  is the group inverse of  $a$ . We characterize additive maps that preserve strongly Drazin (resp. group) invertibility, that is,  $T : A \rightarrow B$  such that  $T(a^D) = T(a)^D$  (resp.  $T(a^\sharp) = T(a)^\sharp$ ).

Let now  $A$  and  $B$  be C\*-algebras. We say that an element  $a \in A$  has generalized inverse  $b = a^\wedge$  if

$$Q(a)(b) = a, \quad Q(b)(a) = b \quad \text{and} \quad Q(a)Q(b) = Q(b)Q(a)$$

where  $Q(x)(y) = xy^*x$ . We characterize additive maps that preserve strongly generalized invertibility, that is,  $T : A \rightarrow B$  such that  $T(a^\wedge) = T(a)^\wedge$ .

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