M-norms on C*-algebras

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July, 2011

Abstract

Two elements a, b in a C*-algebra A are said to be *(algebraically) orthogonal* ($a \perp b$) whenever $ab^* = b^*a = 0$. It is well known that orthogonal elements in A are *(geometrically) M-orthogonal* in the underlying Banach space, that is, $\|a \pm b\|_0 = \max\{\|a\|_0, \|b\|_0\}$ whenever $a \perp b$.

A norm $\|.\|_1$ on A is said to be an *M-norm* (resp., a *semi-M-norm*) if for every a, b in A with $a \perp b$ we have $\|a + b\|_1 = \max\{\|a\|_1, \|b\|_1\}$ (resp., $\|a + b\|_1 \geq \max\{\|a\|_1, \|b\|_1\}$). The original C*-norm, $\|.\|_0$, is an M-norm on A. However, not every M-norm on A satisfies the Gelfand-Naimark axiom.

We shall present in this talk the latest advance in the study of the following problem:

> Is every complete semi-M-norm on a C*-algebra automatically continuous with respect to the original C*-norm?

In a recent paper, obtained in collaboration with Timur Oikhberg (University of California - Irvine) and Maribel Ramírez (Universidad de Almería), we establish that every complete (semi-)M-norm on a von Neumann algebra or on a compact C*-algebra A is equivalent to the original C*-norm of A.