

# Medida de aberraciones corneales y oculares

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### **Líneas de Investigación:**

- 1.- Óptica Visual
- 2.- Calidad Óptica y Visual tras Cirugía Refractiva
- 3.- Presbicia y Acomodación.

### **Producción Científica:**

Artículos Internacionales: 91

Patentes: 2

4 Proyectos de Investigación en marcha (IP)

## **Outline**

- 1.- Fundamentals of elevation topography**
- 2.- Irregular astigmatism: Fourier analysis**
- 3.- Wavefront**
- 4.- Wavefront sensing**
- 5.- Zernike polynomials**
- 6.- How does wavefront sensing relate to refractive surgery?**
- 7.- Fourier versus Zernike**



## **1.- FUNDAMENTALS OF ELEVATION MAP TOPOGRAPHY**

***Two Types***

- **General: Placido Disc**
- **Orbscan/Pentacam**

## ***ORBSCAN: MAIN FEATURES***

- **Accurate elevation and curvature information**
- **Anterior and posterior cornea surface's**
- **Full cornea thickness**

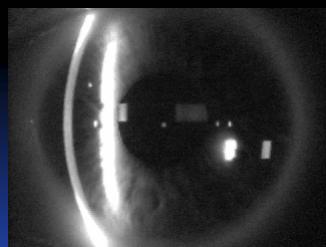
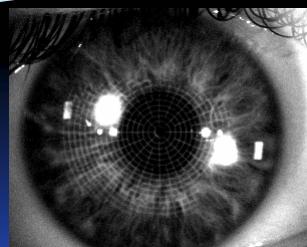
## ***OPTICAL ADQUISITION HEAD***

- **Scans the eye using light slits that are projected at a 45-degree angle.**
- **40 slits in total.**
- **Processing and construction of elevation maps of the anterior & posterior cornea.**
- **Pachimetry: Differences in elevation between the anterior and posterior surface**



## HOW THE INFORMATION DIFFERS TO PLACIDO BASED SYSTEMS?

### Reflective and Slit-scan

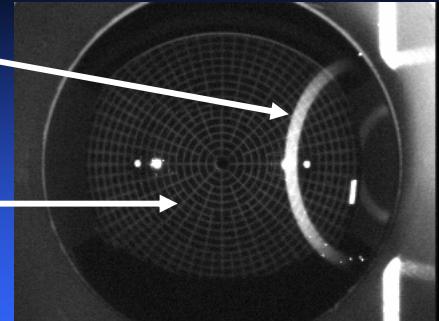


- One image, one surface.
- Angle-dependent specular reflection.
- Measures slope (as a function of distance).
- Multiple images, multiple surfaces.
- Omni-directional diffuse backscatter.
- Triangulates elevation.

Placido reflective systems can only measure the anterior cornea. ORBSCAN measures the anterior cornea, posterior cornea, and the anterior lens and iris.

## Hybrid Technology of ORBSCAN

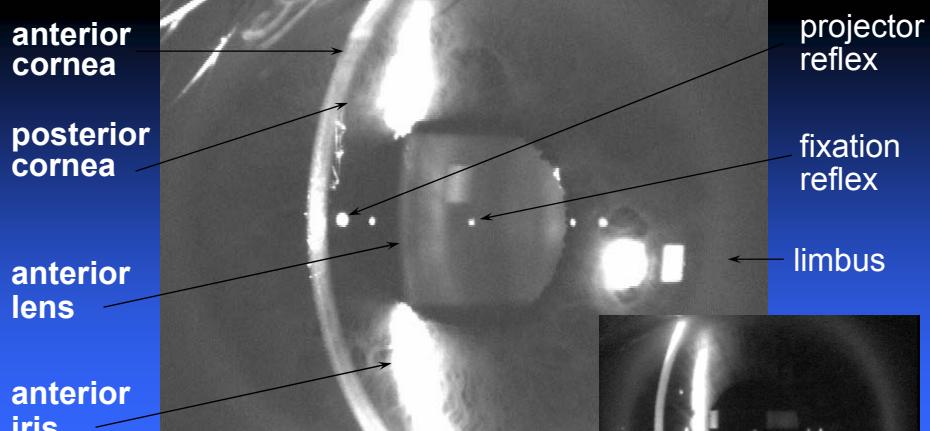
1. Measure **surface elevation directly** by triangulation of backscattered slit-beam.



2. Measure **surface slope directly** using specular reflection, supplemented with triangulated elevation.

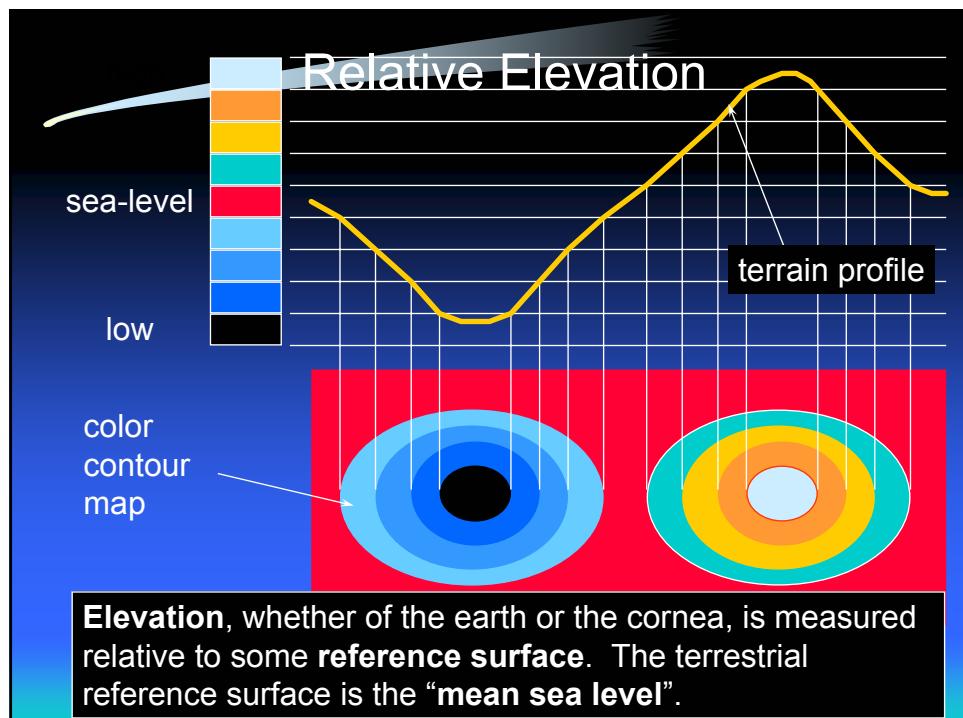
3. **Unify triangulated and reflective data** to obtain accurate surfaces in elevation, slope, and curvature.

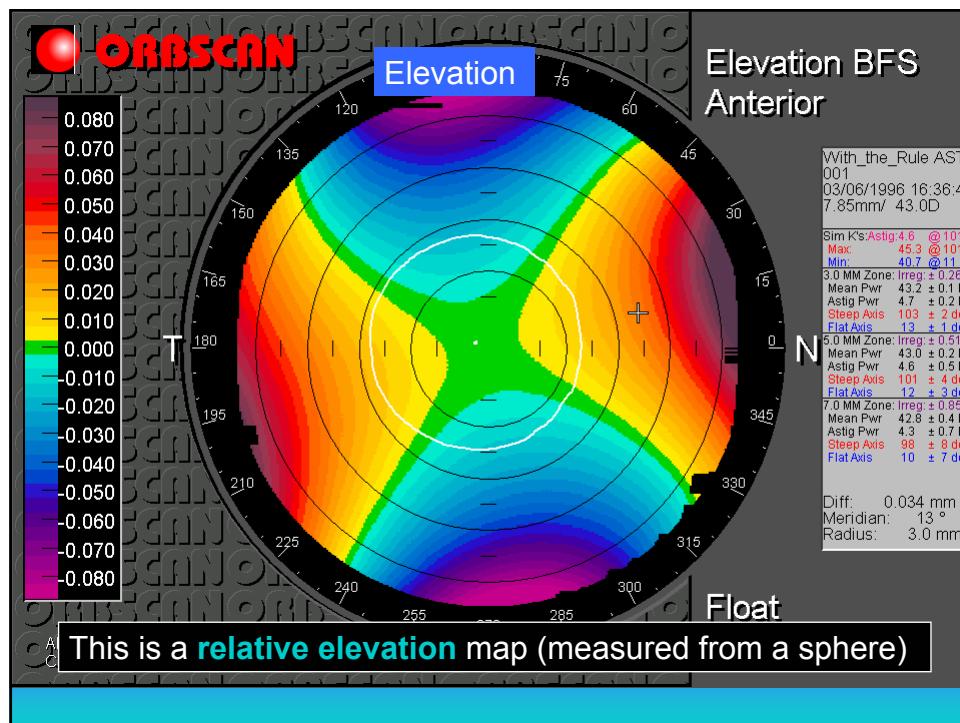
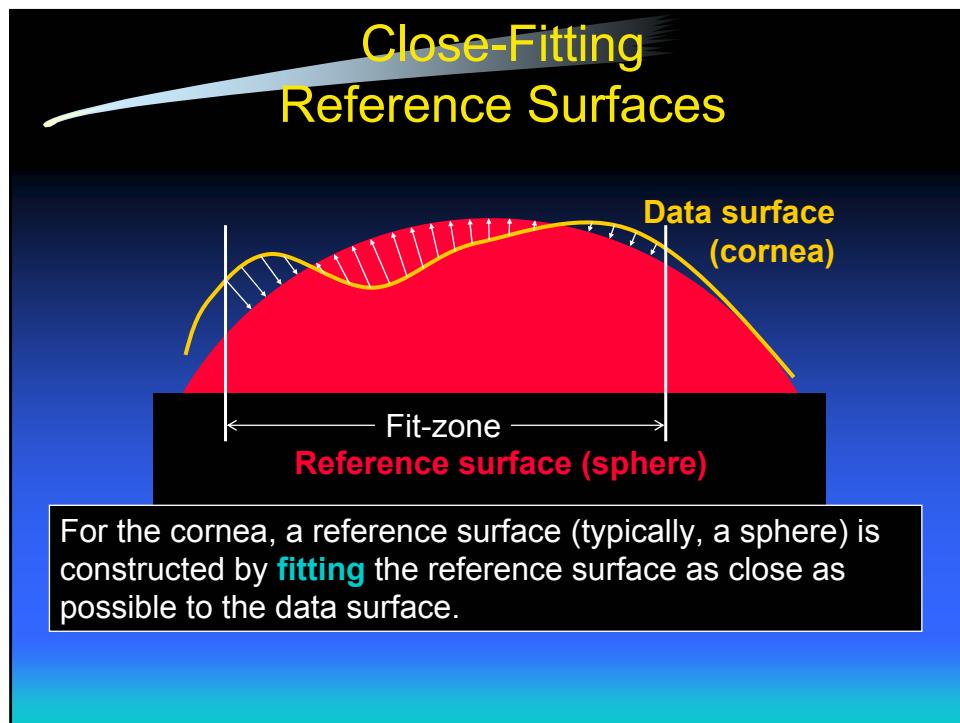
## Scanning slits measure several surfaces

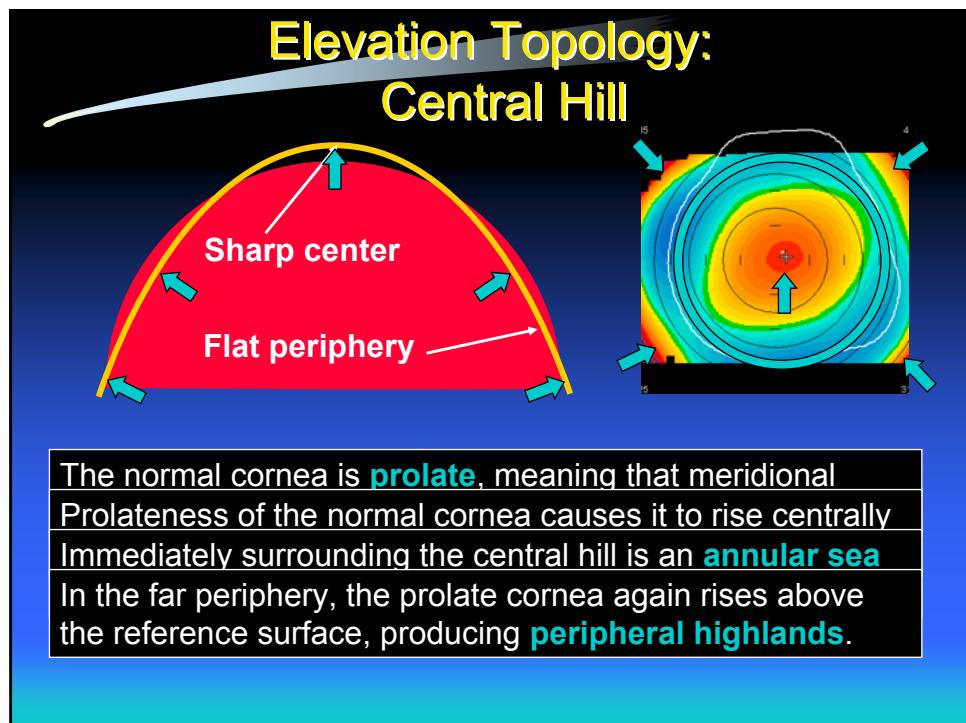
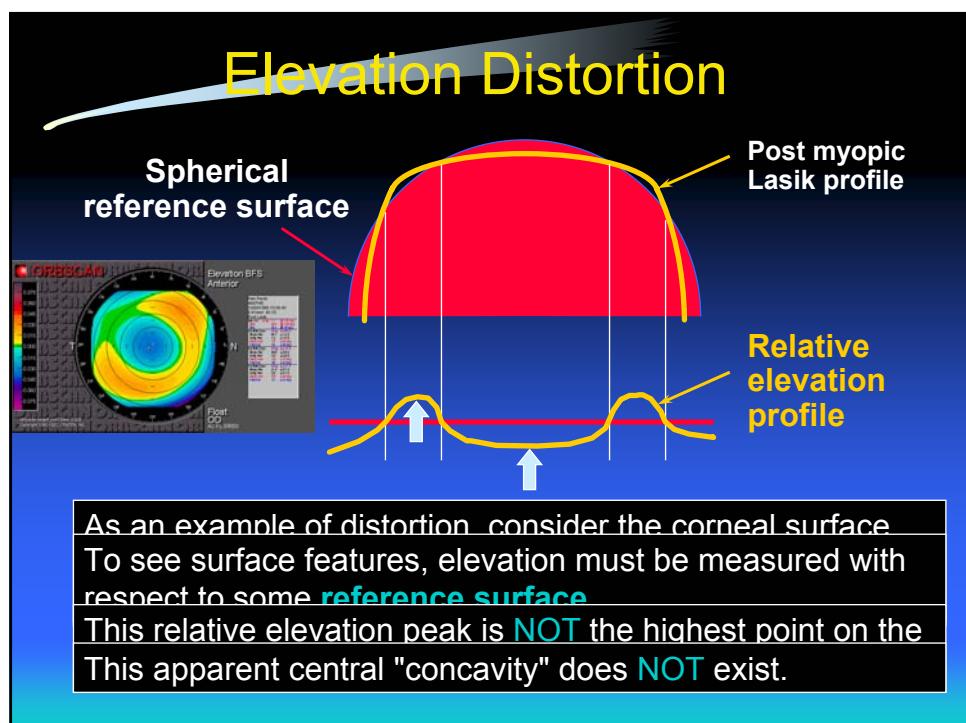


## HOW TO READ CORNEAL ELEVATION MAPS

- Corneal Elevation Topography is viewed relative to a reference surface
- Standardization of the reference surface

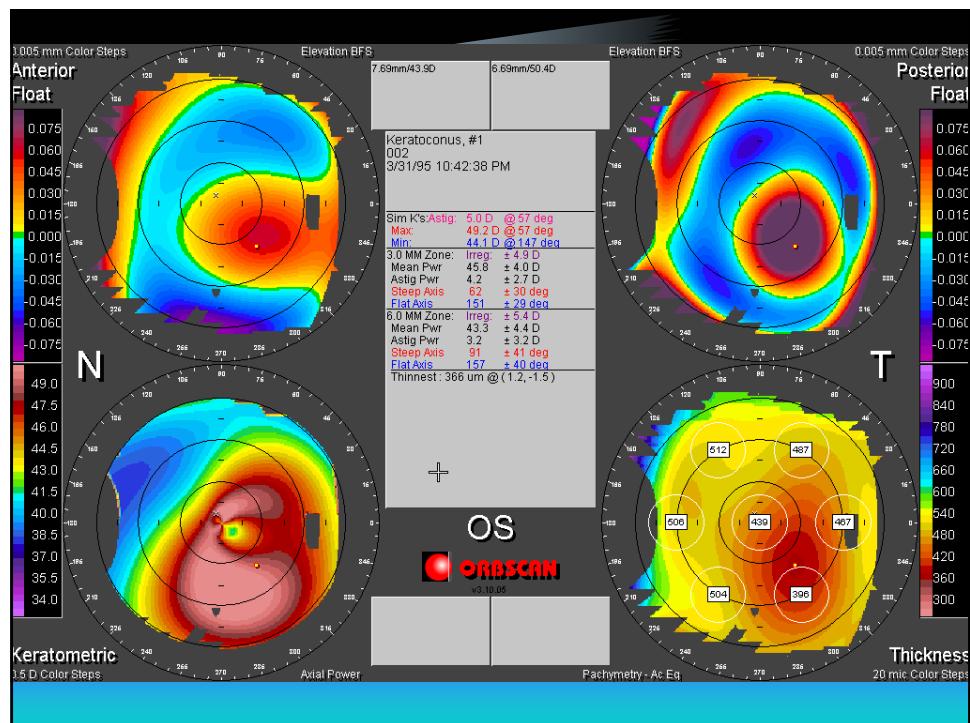






## Importance of The Post Surface of The Cornea

- Keratoconus will show as localized posterior elevation with associated thinning. Patients with thin corneas without posterior elevation are unlikely to be keratoconic.





## 2.- IRREGULAR ASTIGMATISM: FOURIER ANALYSIS

### ASTIGMATISMO IRREGULAR

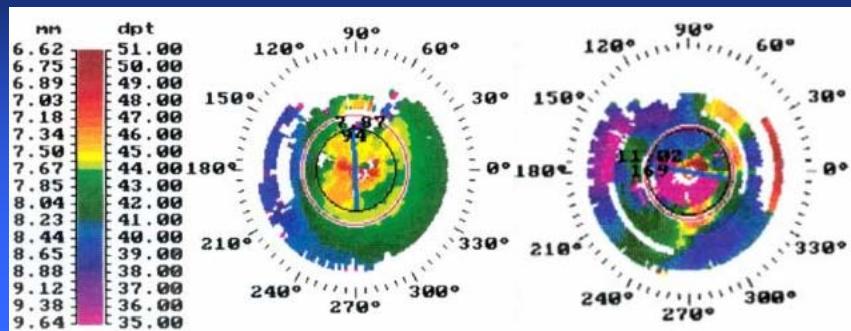
**Astigmatismo regular** ⇒ meridianos principales perpendiculares entre sí, y corrección con lentes esferocíndricas

Cornea con forma irregular que no puede describirse con una sección esférica, tórica o cónica ⇒ **Astigmatismo irregular**

**Causas comunes:** ojo seco, degeneraciones corneales, traumas, cirugía de la catarata y refractiva.

# Problem

Impossibility to evaluate topographies without pattern



# Análisis de Fourier

Es un procedimiento matemático que permite la descomposición de cualquier objeto periódico en una suma de términos sinusoidales de frecuencias crecientes y amplitudes determinadas, lo que se conoce como **espectro de Fourier** de dicha función.

# Solution

To apply Fourier Analysis to  
videoqueratographic data

## Serie of Fourier

Function  $f(x)$  periodical



Sum of discrete function  $f(x)$   
Sinusoidal terms:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cdot \cos(2\pi nx / p) + \sum_{n=1}^{\infty} b_n \cdot \sin(2\pi nx / p)$$

$$a_0 = \frac{1}{p} \int_0^p f(x) dx$$

$$a_n = \frac{1}{p} \int_0^p f(x) \cdot \cos(2\pi nx / p) dx$$

$$b_n = \frac{1}{p} \int_0^p f(x) \cdot \sin(2\pi nx / p) dx$$

## Fourier Transform

Possibility to apply to non-periodical functions using the Fourier Transform (FT):

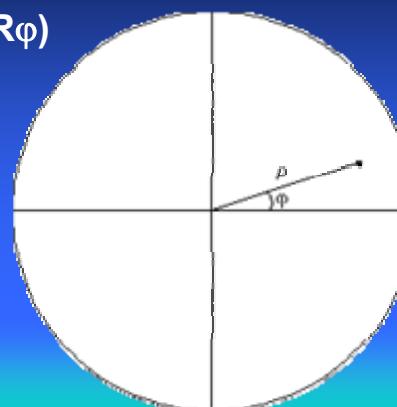
$$T.F.\{f(x)\} = F(w) = \int f(x) \cdot \exp(-i2\pi wx) dx$$

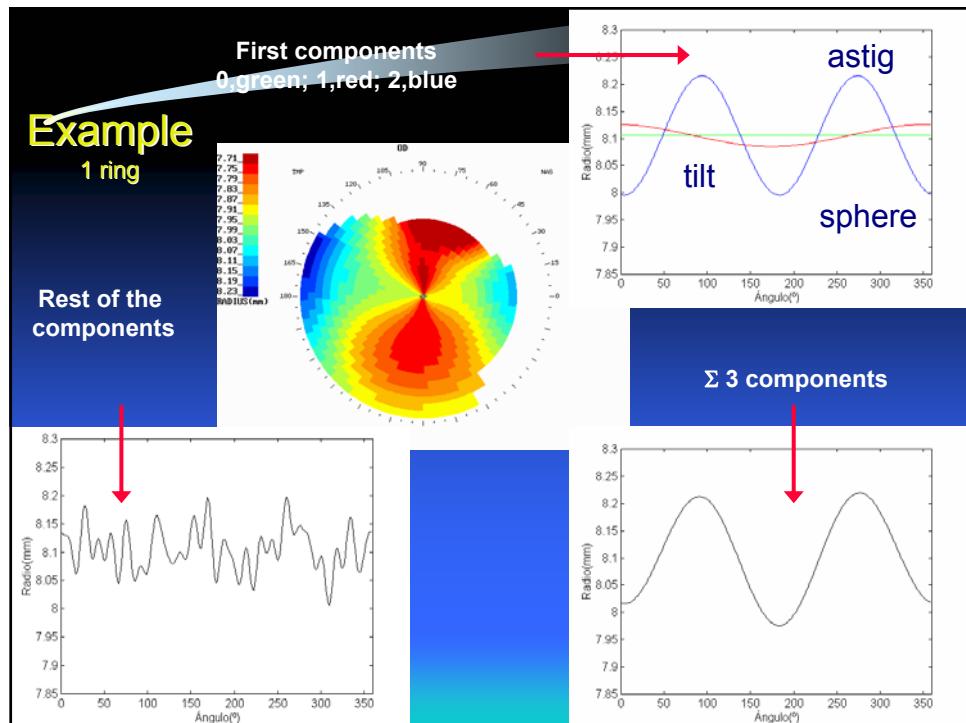
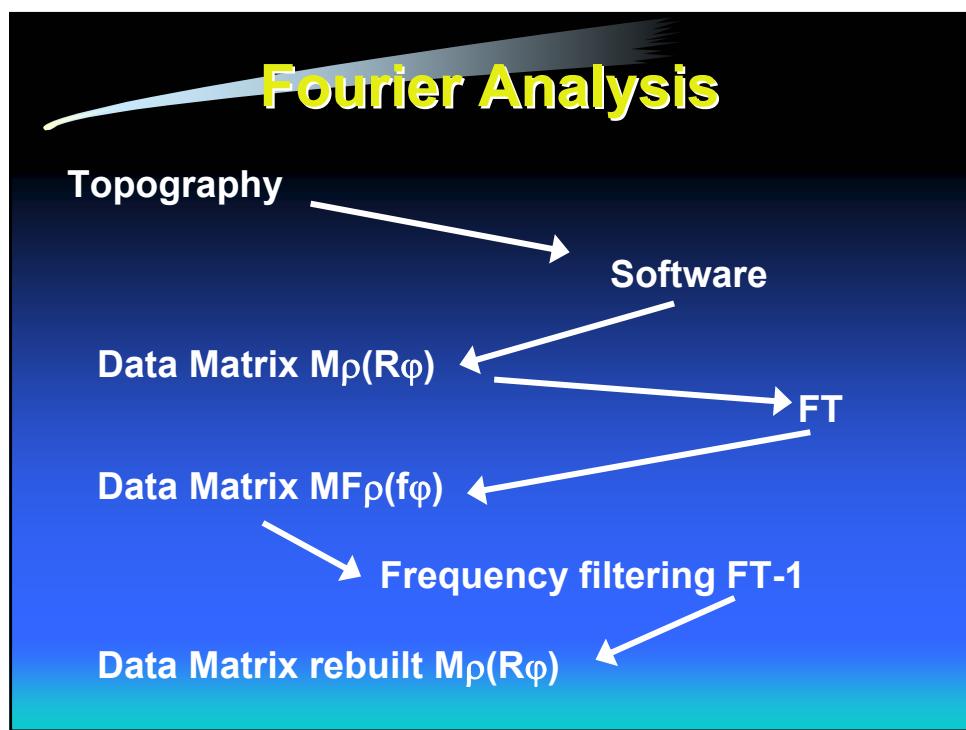
To rebuilt the original function  $f(x)$  we apply the inverse transform to the function  $F(W)$ :

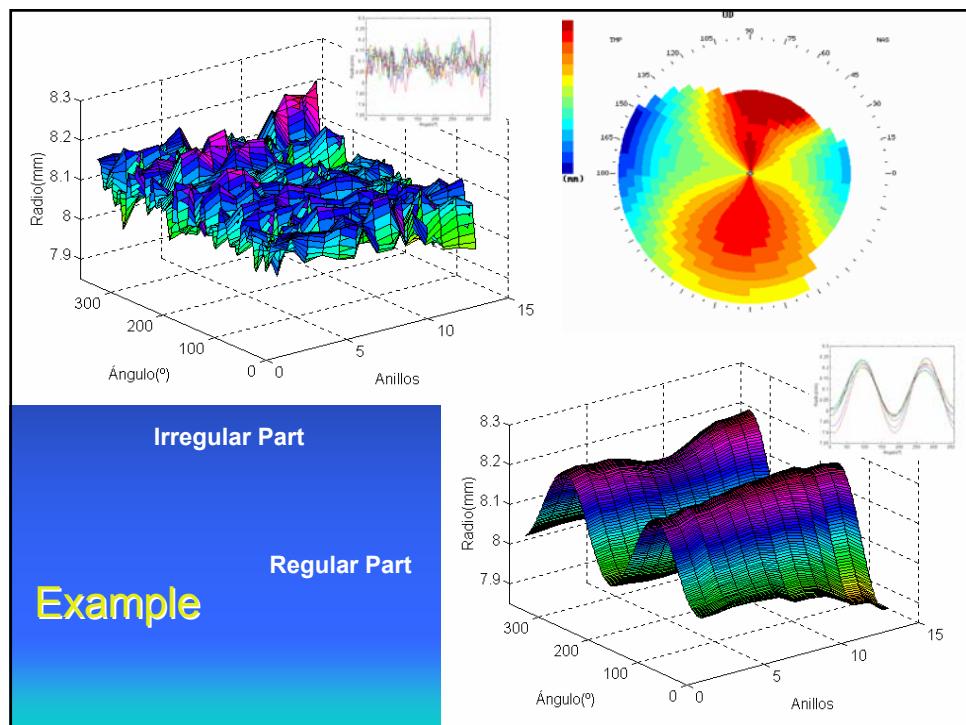
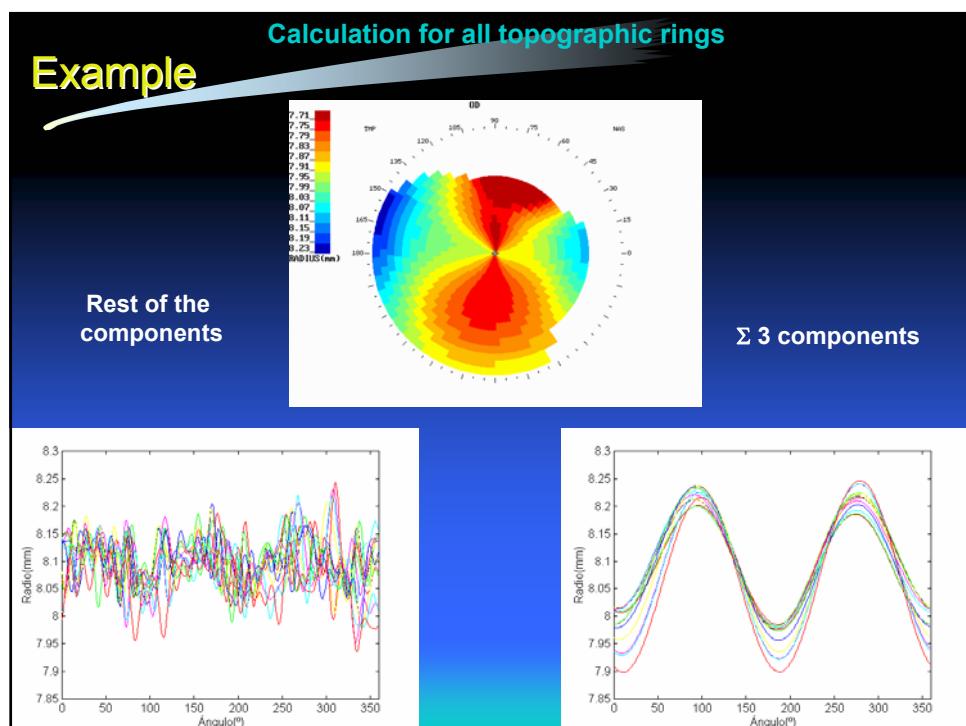
$$f(x) = T.F.^{-1}\{F(w)\} = \int F(w) \cdot \exp(i2\pi xw) dw$$

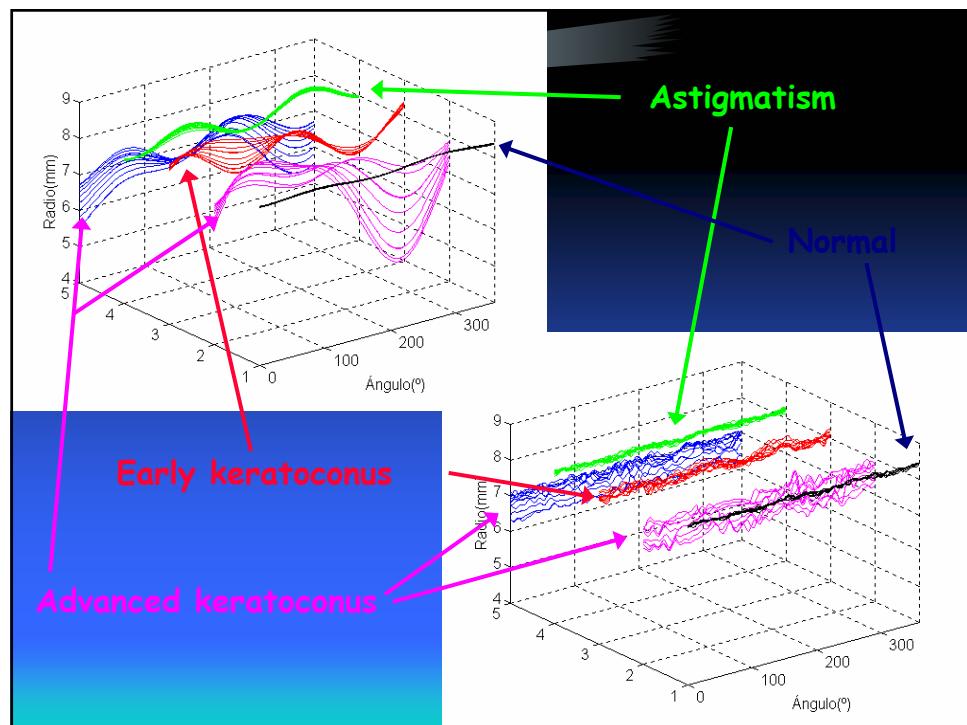
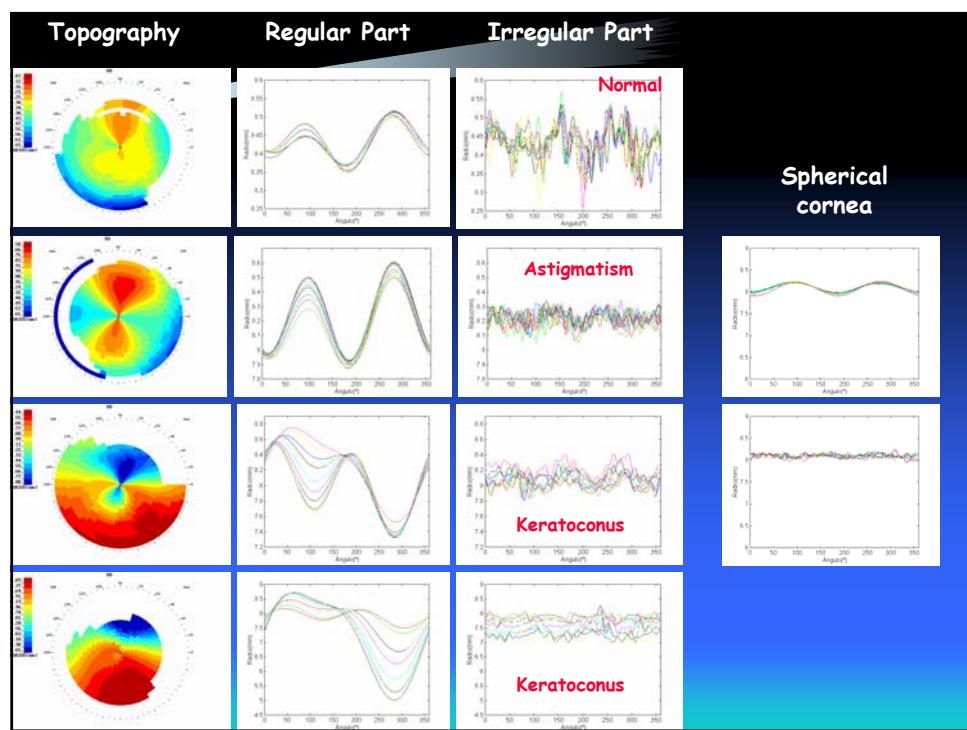
## Fourier Analysis

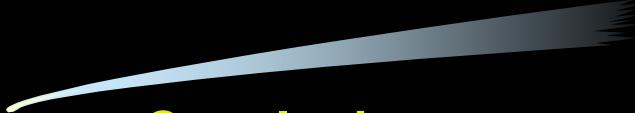
Topographic image is a matrix of data  $M_\rho(R\phi)$  containing radii as a function of the angle ( $R\phi$ ) for each ring of radius  $\rho$ .







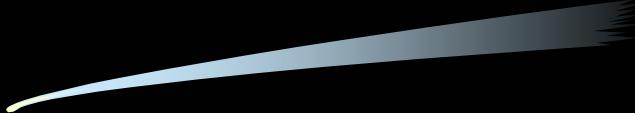




## Conclusions

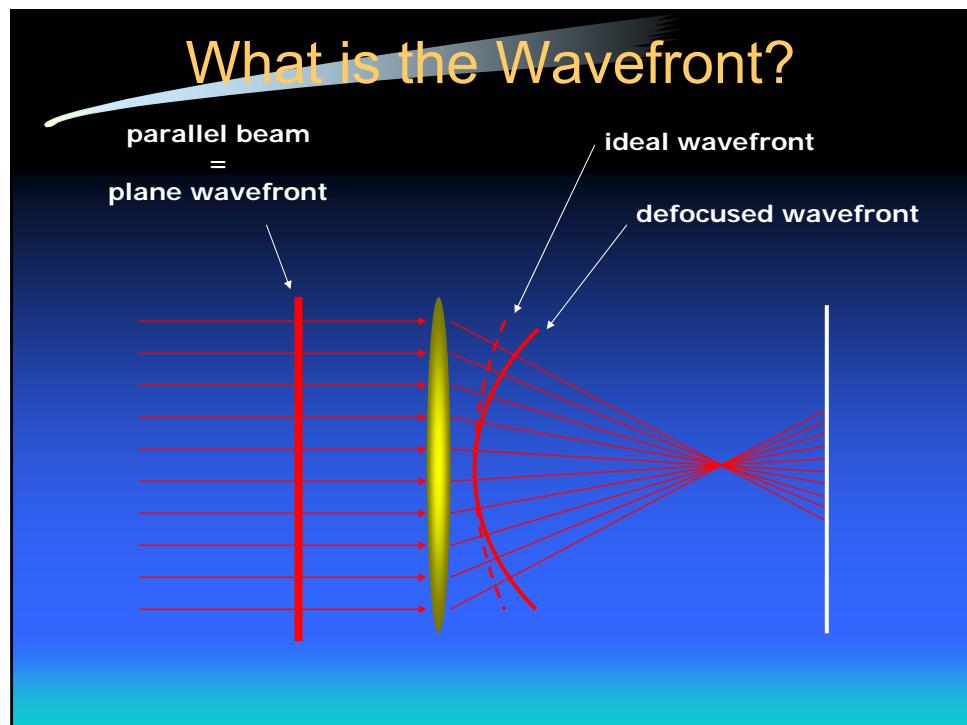
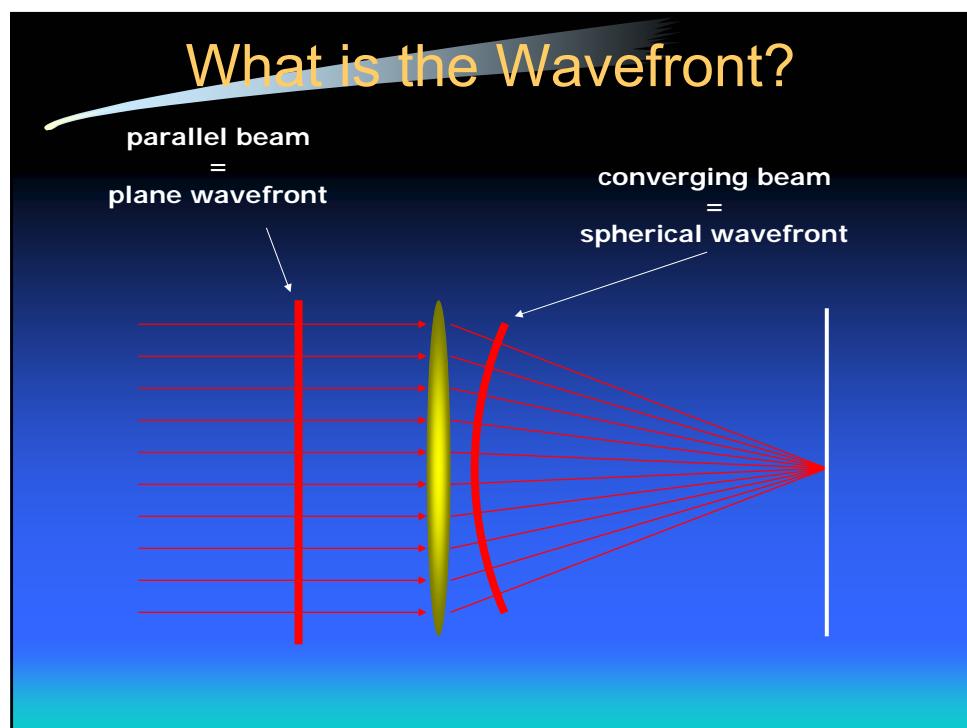
We can divide topographic information between regular and irregular parts

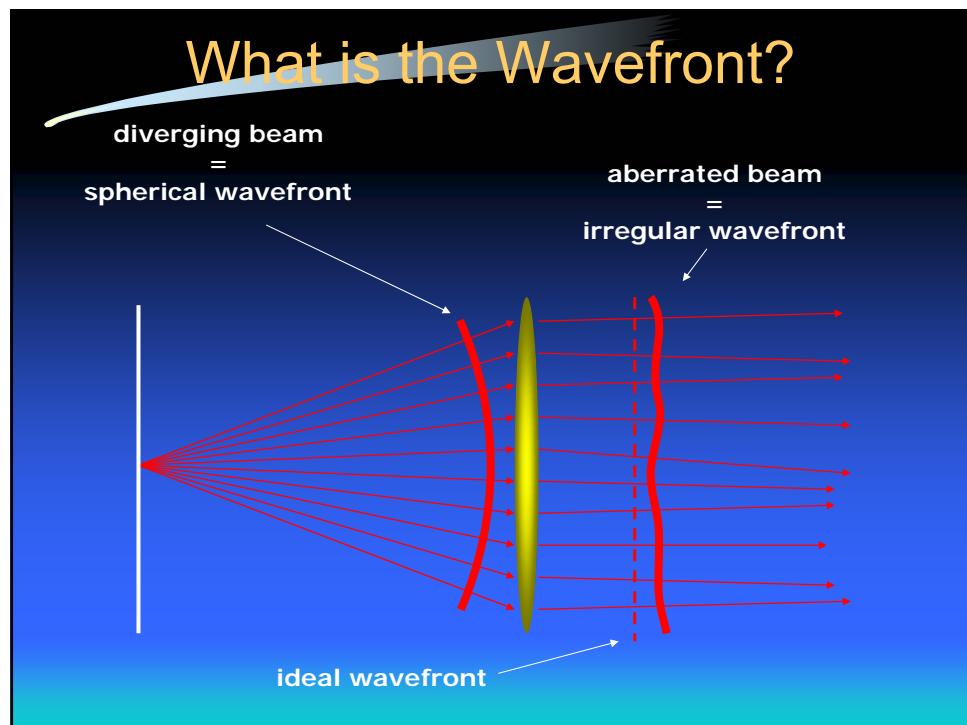
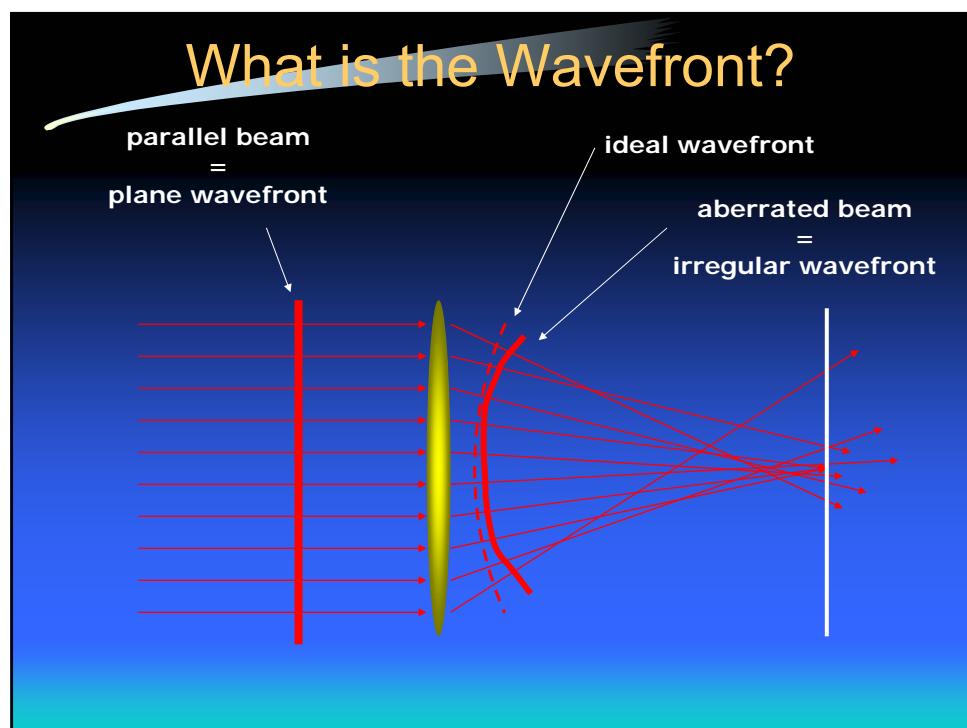
We can quantify the corneal irregularity by means two parameters, defined from the regular and irregular parts.



## 3.- Wavefront

We will describe the wavefront. This is the one of the most fundamental and useful description of the optical properties of the eye, from which most of the image quality metrics can be derived.

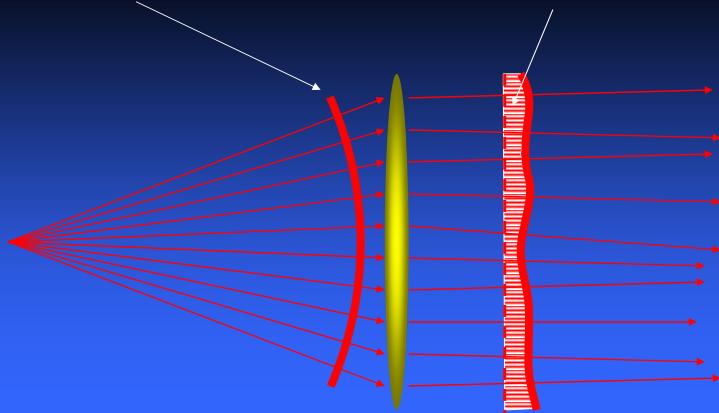




## What is the *Wave Aberration*?

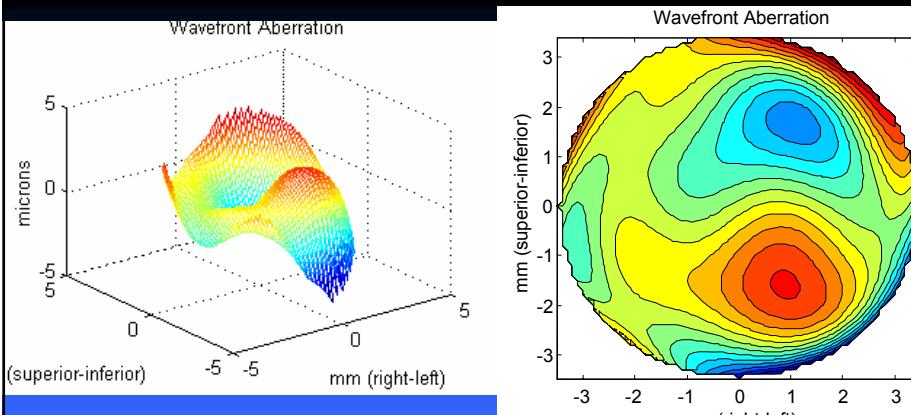
diverging beam  
=  
spherical wavefront

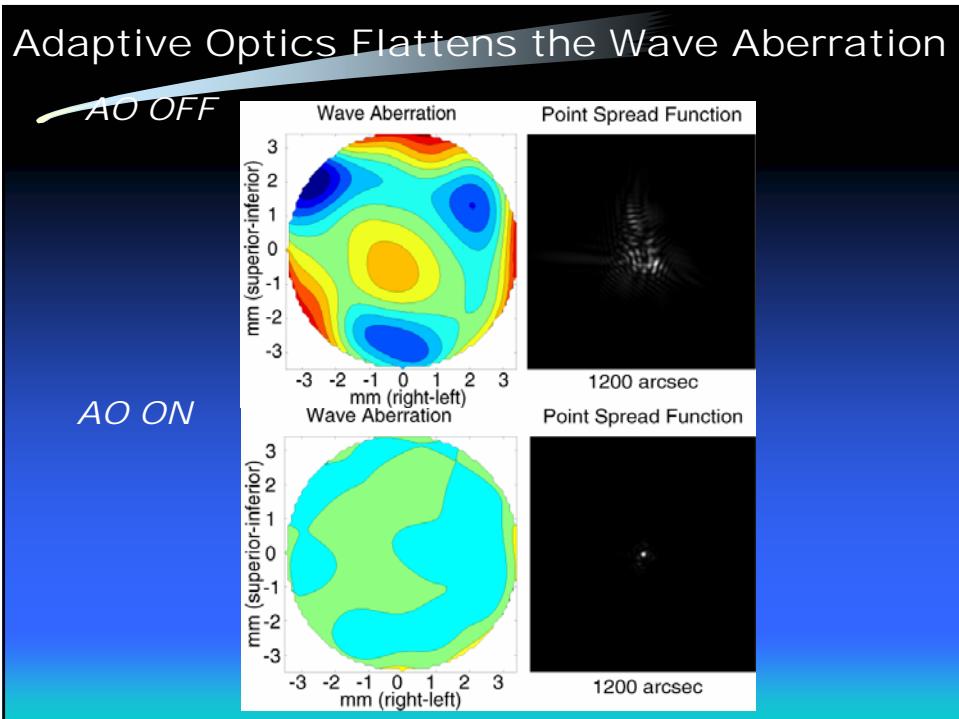
wave aberration



Wave aberration is a measure of the difference between the ideal wavefront and the actual wavefront. You are able to choose whatever ideal wavefront you want, but you commonly choose the ideal wavefront as one that would focus the light to the image plane

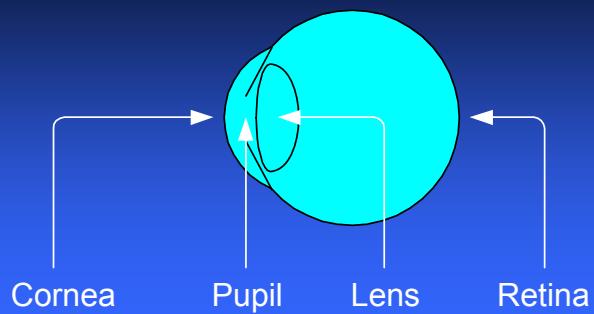
## Wave Aberration of a Surface





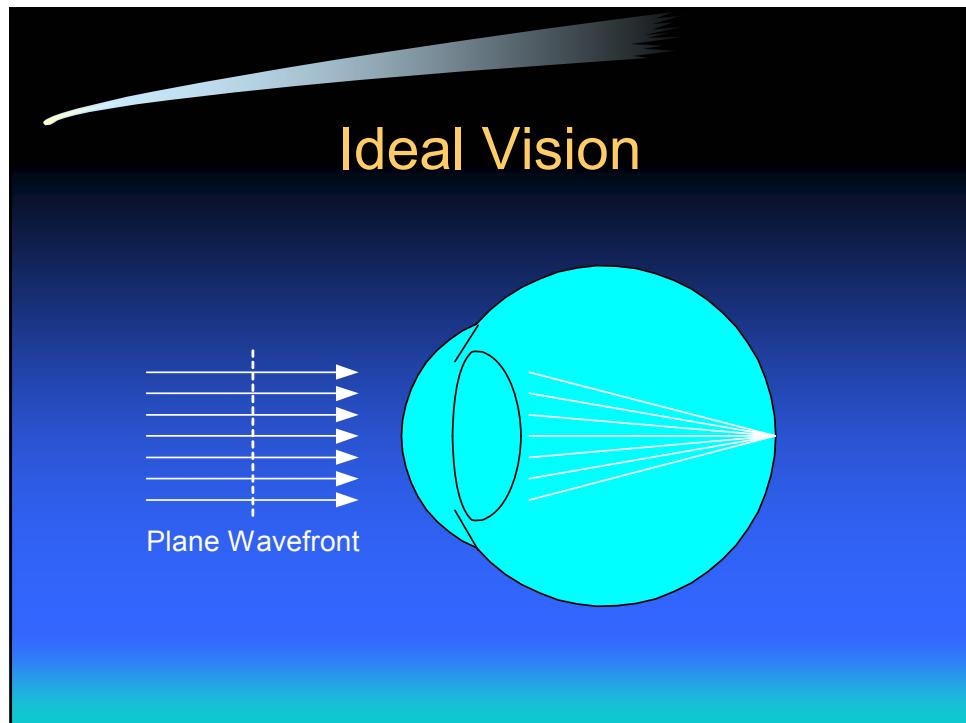
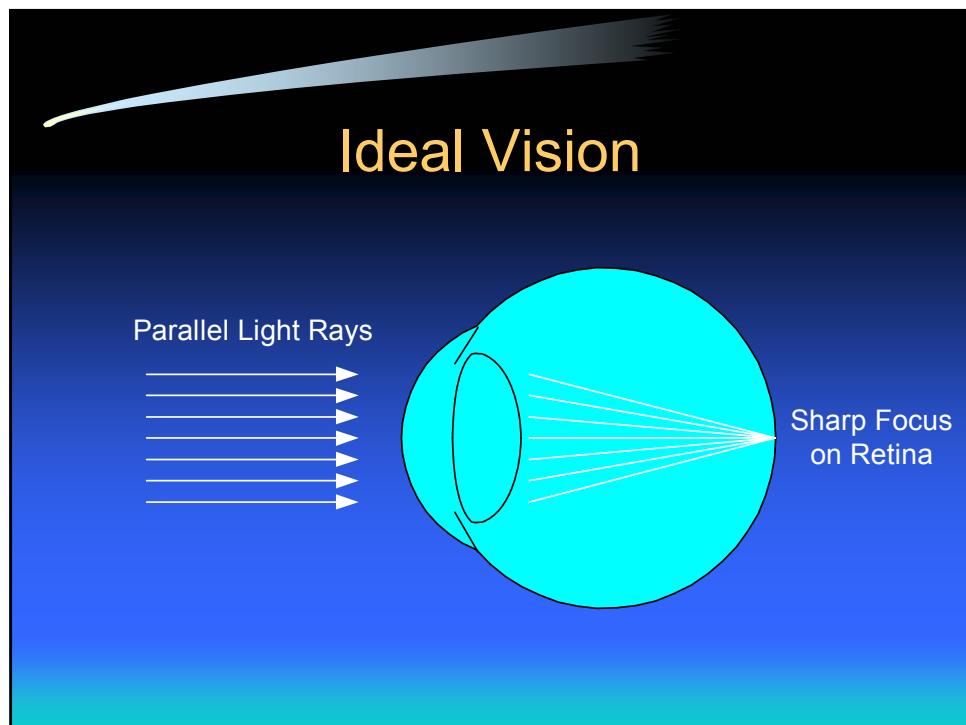
## 4.- Wavefront Sensing

## Optical Anatomy of the Eye



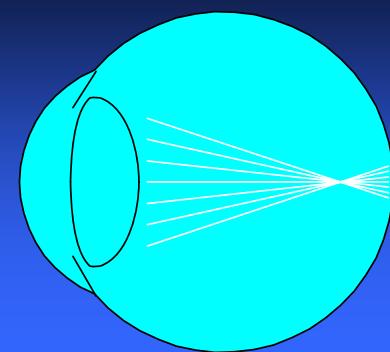
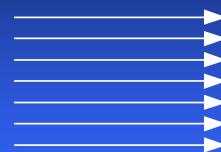
## Wavefront Sensing Clinical Utility

- Measures integrated function of optical system
- Allows accurate calculation of effective clinical prescription
- Also provides details of higher order aberrations
- Quick measurement easily made in clinical setting



## Simple Near-Sightedness (myopia)

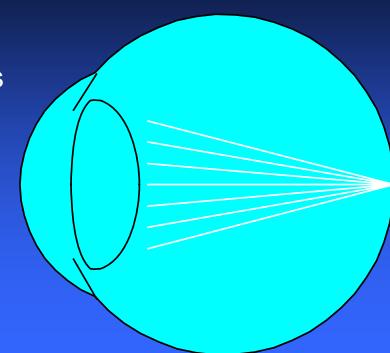
Parallel Light Rays



Focus in  
Front of  
Retina

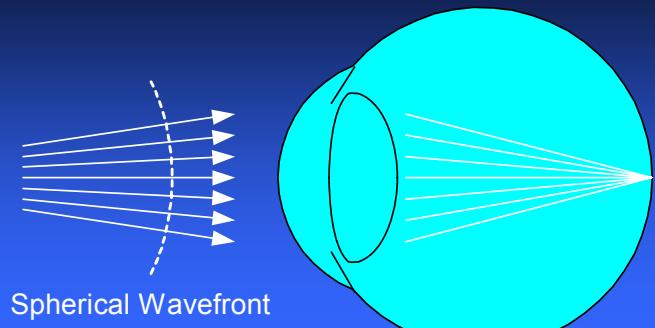
## Simple Near-Sightedness (myopia)

Diverging Light Rays

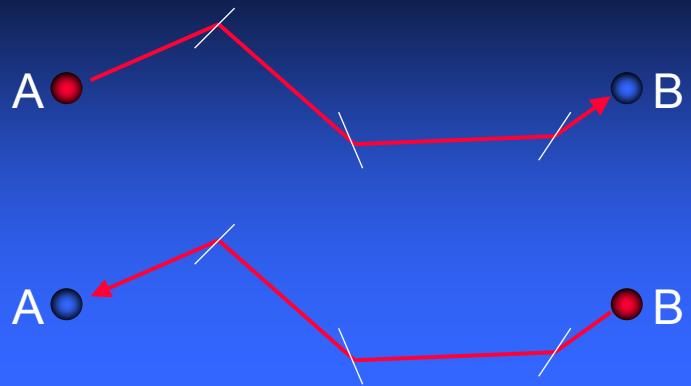


Sharp Focus  
on Retina

## Simple Near-Sightedness (myopia)

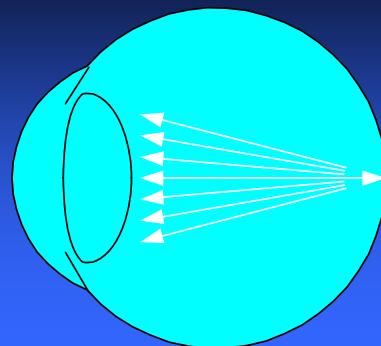


## The Reversible Nature of Light Propagation



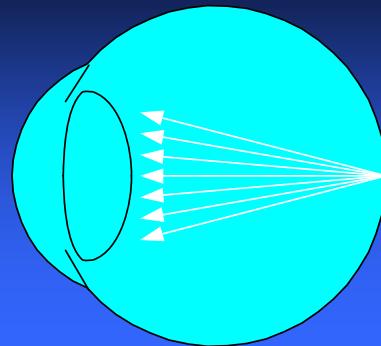
## Wavefront Sensing: Turn the Rays Around!

Probe Light Beam

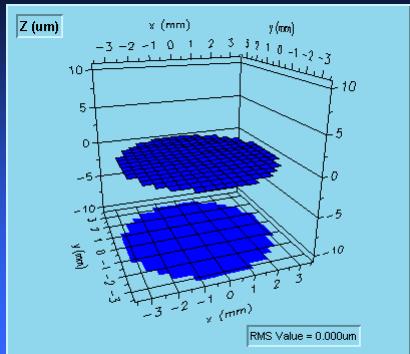


## Re-Emitted Wavefront for an Ideal Eye

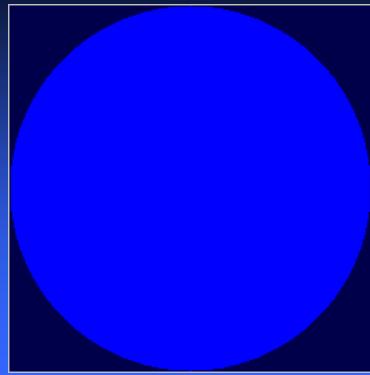
Plane Wavefront



## Wavefront Displays for Ideal Vision

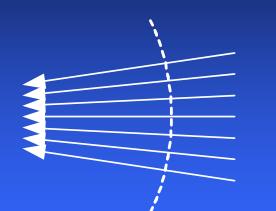


3-D Representation

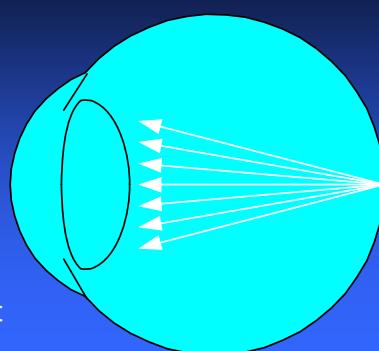


2-D Color Map

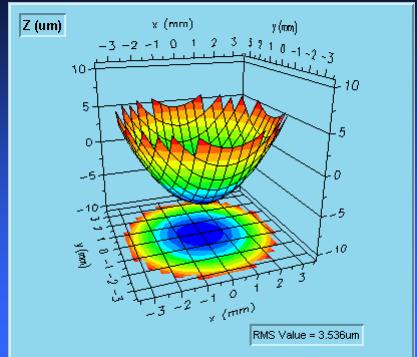
## Re-Emitted Wavefront for an Near-Sighted Eye (myopic)



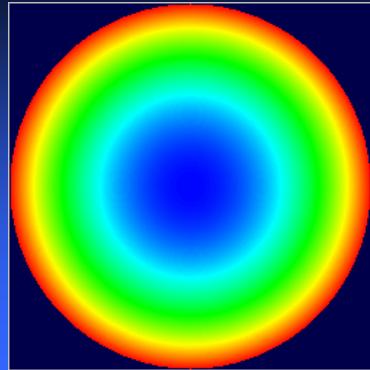
Spherical Wavefront



# Wavefront Displays for Near-Sightedness



3-D Representation



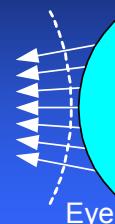
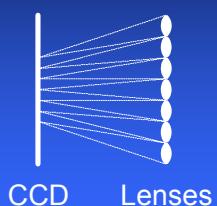
2-D Color Map

How do We Make the  
Wavefront Measurement?  
Wavefront sensors

Usually use ray-tracing methods to reconstruct the wavefront and are classified into the following 3 types:

- Outgoing wavefront aberrometry  
(Hartmann-Shack)
- Ingoing retinal imaging aberrometry  
(cross cylinder, Tscherning aberroscope)
- Ingoing feedback aberrometer  
(spatially resolved refractometer, optical path difference)

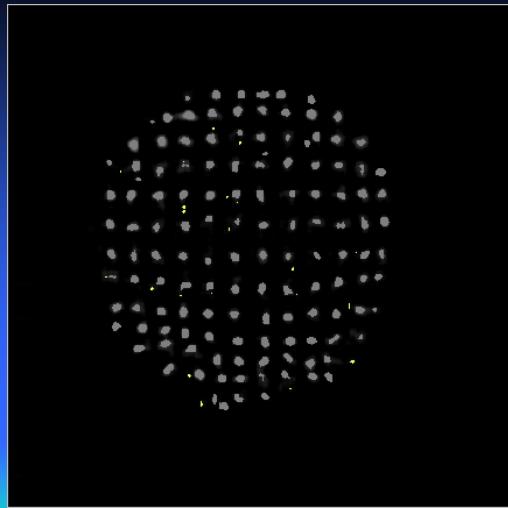
## The Wavefront Sensing Path



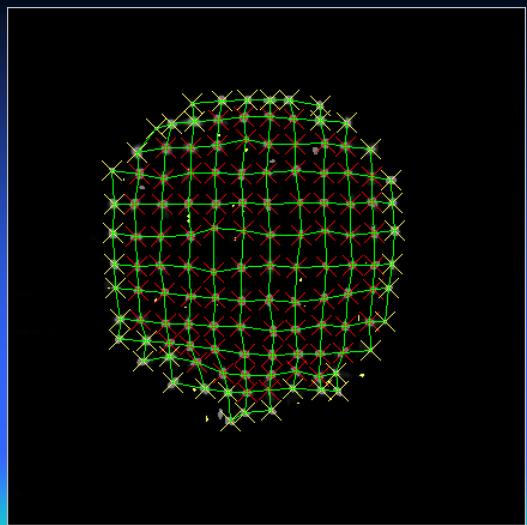
Direct CCD Image



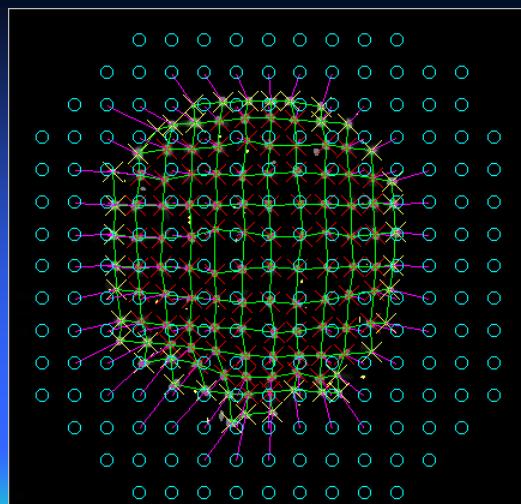
Enhanced CCD Image



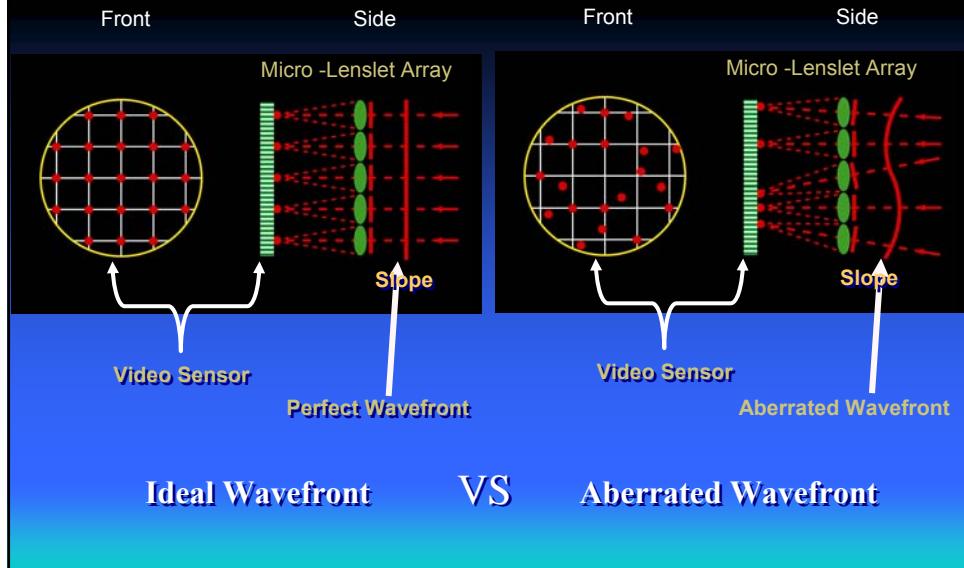
## Focussed Spot Associations



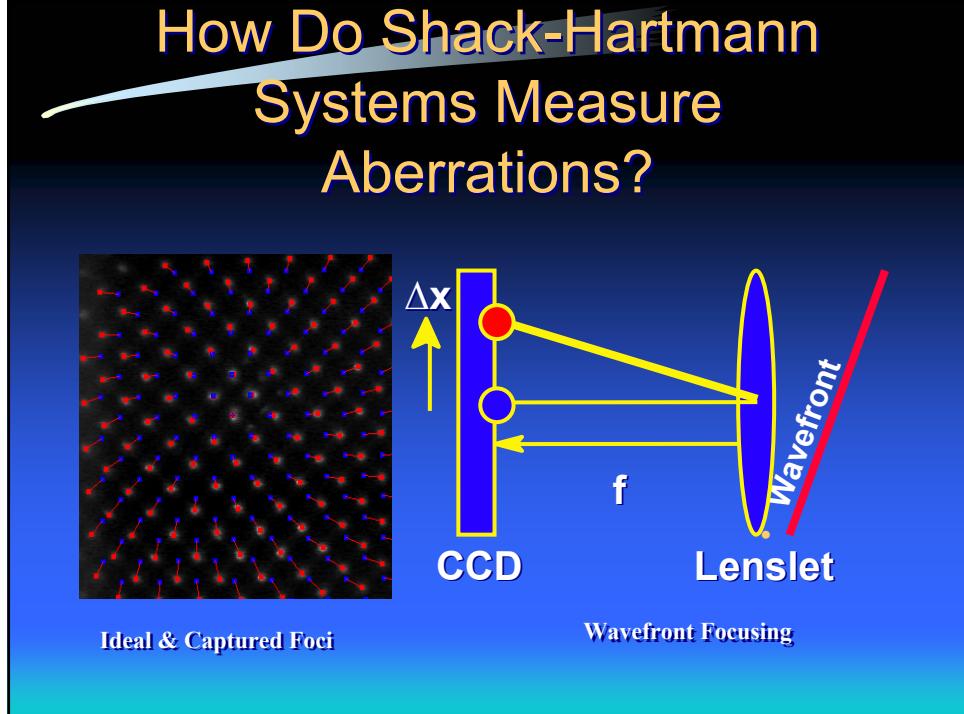
## Comparison to Ideal Pattern

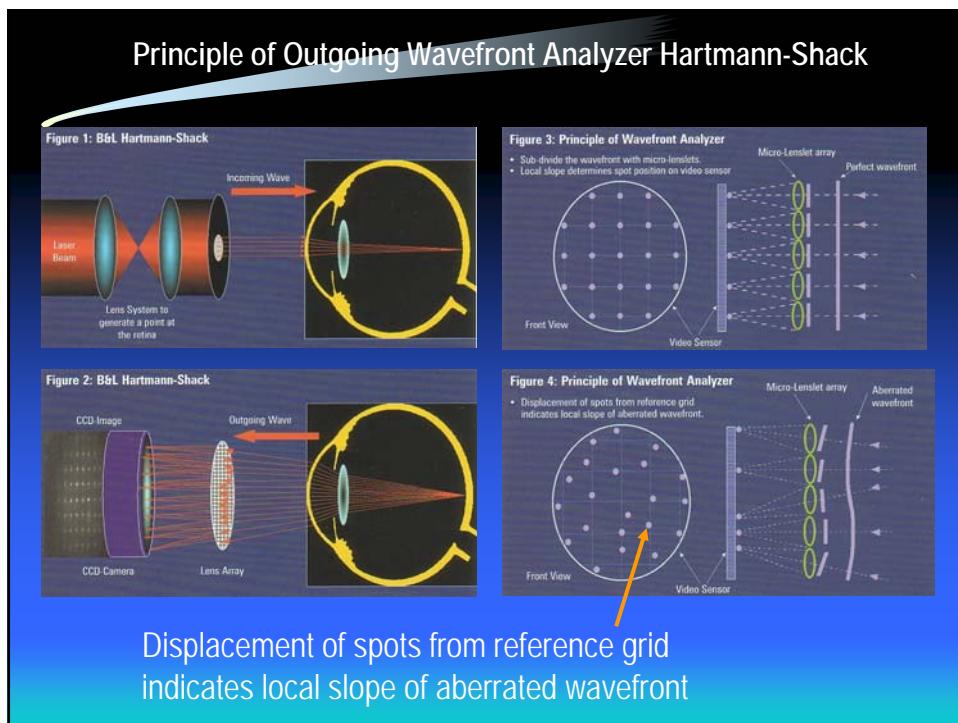
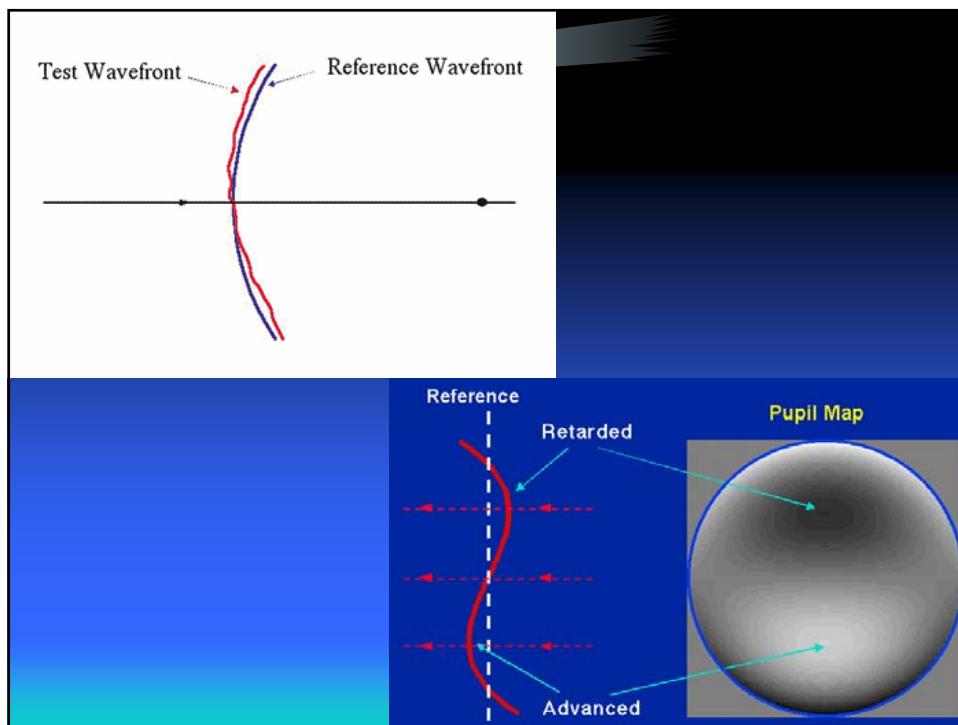


## What Are We Comparing With Our System?

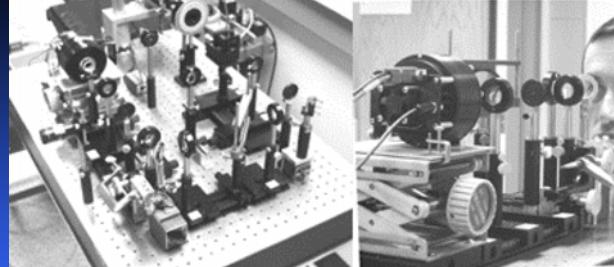


## How Do Shack-Hartmann Systems Measure Aberrations?





## Wavefront Analyzer Hartmann-Shack



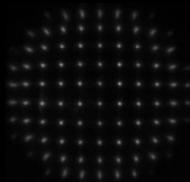
Montaje laboratorio en banco de óptica



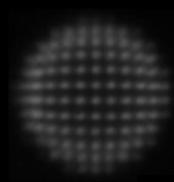
Comercial

## Examples of spots position in a Hartmann-Shack

Emmetropic Model Eye

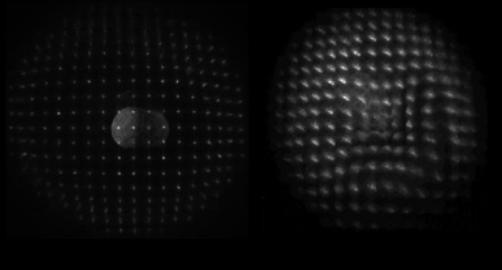


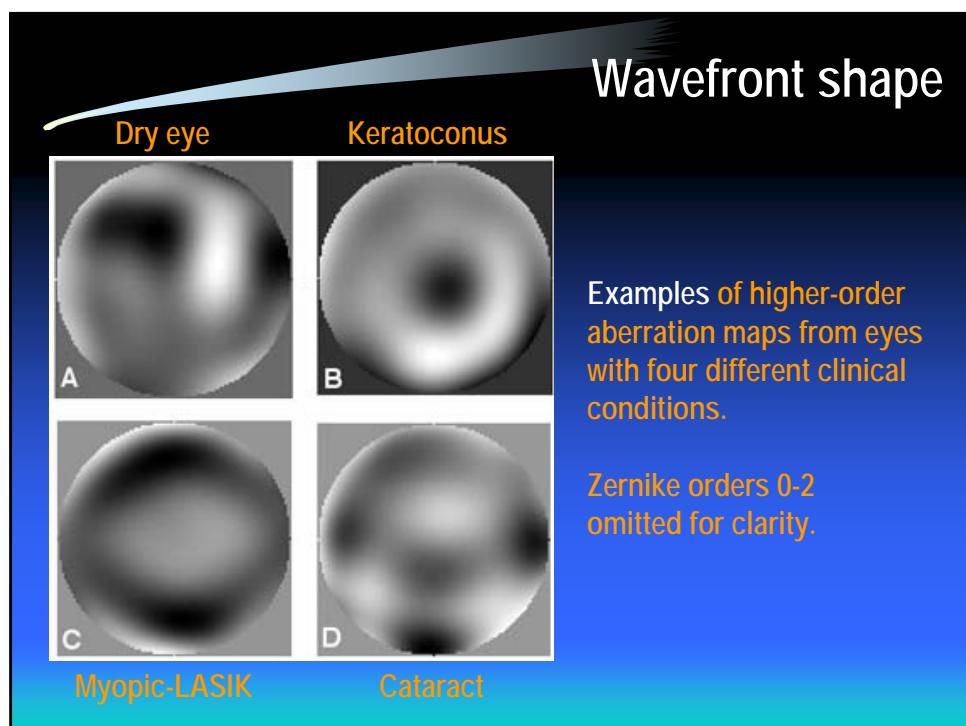
4D Myopic Model Eye



LASIK

Keratoconus





## 5.- Zernike polynomials

## Introducción

- La aproximación más familiar para cuantificar las aberraciones ópticas es la de Seidel, definida para sistemas rotacionalmente simétricos
- Cuando describimos las aberraciones oculares, Seidel no se utiliza ya que la óptica del ojo no es totalmente simétrica
- Los polinomios de Taylor tambien han sido utilizados para describir las aberraciones del ojo
- Recientemente se han utilizado los polinomios de Zernike debido a sus propiedades matemáticas adecuadas para pupilas circulares

## Introducción

- Polinomios de Zernike: consisten en un conjunto ortogonal de polinomios que presentan las aberraciones y además están relacionados con las aberraciones ópticas clásicas
- Parecen el método más deseable para estimaciones precisas del error de frente de onda, debido a sus propiedades de ortogonalidad (independencia de los términos entre sí) y pueden ajustarse por el método de mínimos cuadrados, que es lineal en parámetros

## Introducción: Topografía

- Los topógrafos miden la elevación corneal sólo en un número discreto de puntos y los polinomios de Zernike no son ortogonales sobre un conjunto discreto de puntos
- La técnica de ortogonalización de Gram-Smith permite expandir el conjunto discreto de datos de elevación corneal, en términos de polinomios de Zernike y conseguir las ventajas de una expansión ortogonal. Una vez completada la expansión, las funciones ortogonales se transforman en términos de polinomios de Zernike, resultando un conjunto único de coeficientes de Zernike

## Definición y notaciones

**Los polinomios de Zernike son un conjunto infinito de funciones polinómicas, ortogonales en el círculo de radio unidad.**

**Son muy útiles para representar la forma del frente de onda en sistemas ópticos. Su uso está muy extendido y son muy comunes distintas notaciones, normalizaciones y criterios en la asignación de signos.**

Los **polinomios de Zernike** pueden expresarse en coordenadas polares, siendo  $\rho$  la coordenada radial (intervalo de variación  $[0,1]$ ) y  $\theta$  la componente azimutal (intervalo de variación es  $[0,2\pi]$ )

Distinguimos tres componentes

el factor de normalización ( $N$ ),

la dependencia radial

y la dependencia azimutal.

$$Z_n^{\pm m}(\rho, \theta) = \begin{cases} N_n^{\pm m} R_n^{|m|}(\rho) \cos m\theta & \text{for } m \geq 0 \\ -N_n^{\pm m} R_n^{|m|}(\rho) \sin m\theta & \text{for } m < 0 \end{cases}$$

La dependencia radial es polinómica y la azimutal es armónica.

$$N_n^m = \sqrt{\frac{2(n+1)}{1 + \delta_{m0}}}$$
$$R_n^{|m|}(\rho) = \sum_{s=0}^{(n-|m|)/2} \frac{(-1)^s (n-s)!}{s! [0.5(n+|m|)-s]! [0.5(n-|m|)-s]!} \rho^{n-2s}$$

Se identifica al polinomio con dos índices “n” y “m”, donde “n” indica la potencia más alta (orden) en la componente polinómica radial y “m” es la frecuencia azimutal en la componente armónica

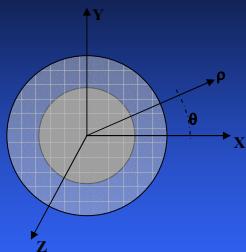
## Representación de las aberraciones.

La función aberración de onda  $W(\rho, \theta)$  puede expresarse como **combinación lineal de los polinomios de Zernike:**

$$W = \sum_{j=1 \dots N} C_j Z_j$$

donde  $C_j$  son los Coeficientes de Zernike que se expresan en micras y miden el valor de las distintas aberraciones presentes en el sistema.

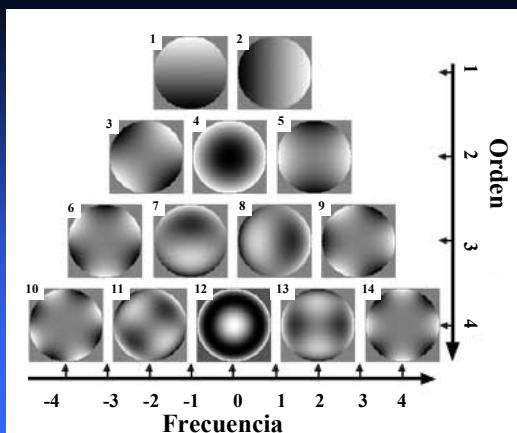
Para describir las aberraciones oculares se toma como sistema de referencia un triedro a derechas con origen en la pupila de entrada del ojo, el semieje positivo Y apuntando hacia arriba, el X apuntando hacia la izquierda del sujeto y Z apuntando en dirección emergente al ojo.



Sistema de referencia para la descripción de la aberración ocular en función de los polinomios de Zernike. Se muestra vista frontal del ojo. Los semiejes positivos se toman de la misma manera en ambos ojos.

Al usar coordenadas polares  $\theta$  se mide respecto del semieje positivo X y  $\rho$  es la distancia respecto del origen medida en unidades normalizadas al radio pupilar

La siguiente figura muestra la forma del frente de onda representado por cada polinomio de Zernike, la aberración total se expresa como combinación lineal de esos patrones característicos



Visualización los 14 primeros polinomios de Zernike en escala de grises (color claro para adelante de fase y oscuro para retraso). Cada patrón se identifica con su índice j, cada fila corresponde a un orden n y cada columna a una frecuencia m.

Las **aberraciones de bajo orden** vienen representadas por los polinomios de ordenes  $n = 0, 1$  y  $2$ .

Para  $n = 0$  tenemos un único polinomio de valor constante unidad y para  $n = 1$  encontramos dos polinomios denominados “**tilts**” Éstos representan traslaciones y rotaciones del sistema de referencia

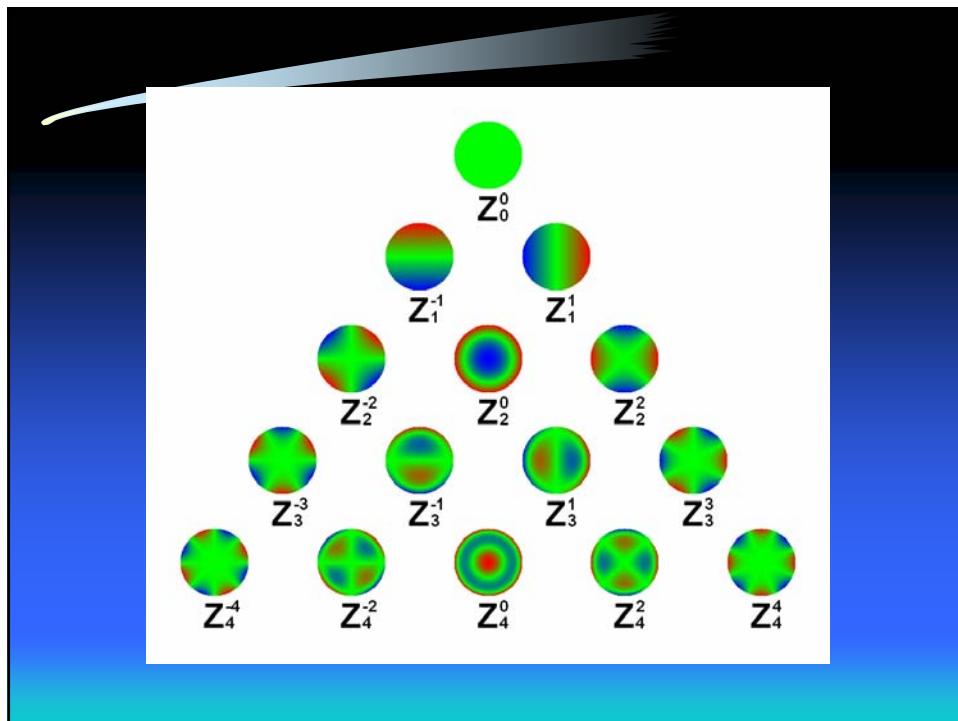
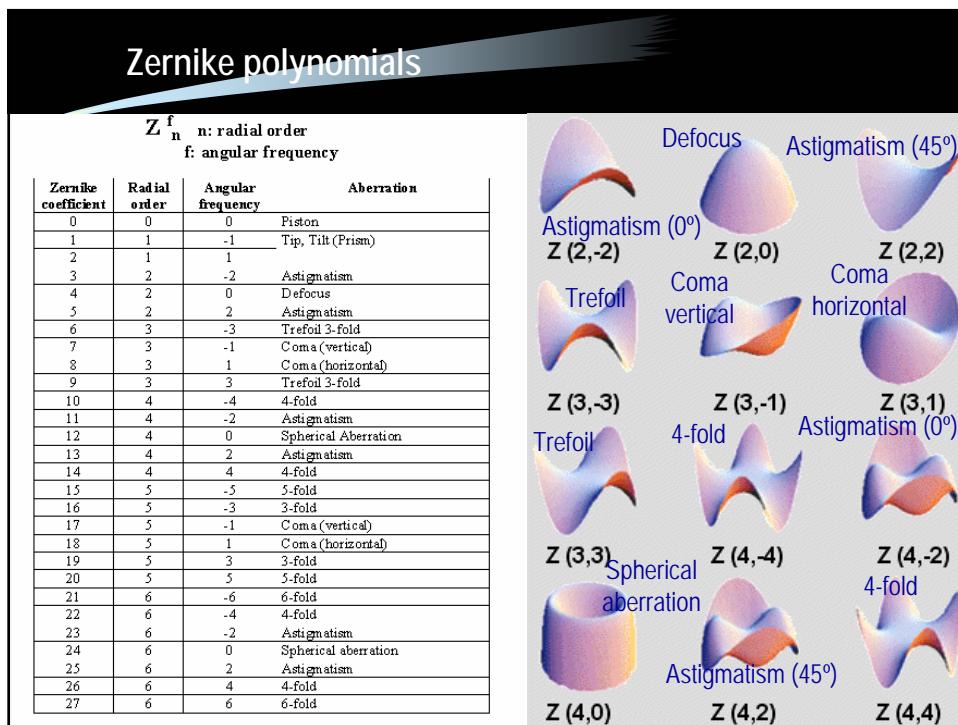
Las **aberraciones de 2º orden** están descritas por los 3 polinomios de Zernike correspondientes a  $n = 2$ . Estos polinomios representan el desenfoque ( $j=4$ ) y astigmatismo ( $j=3$  y  $5$ )

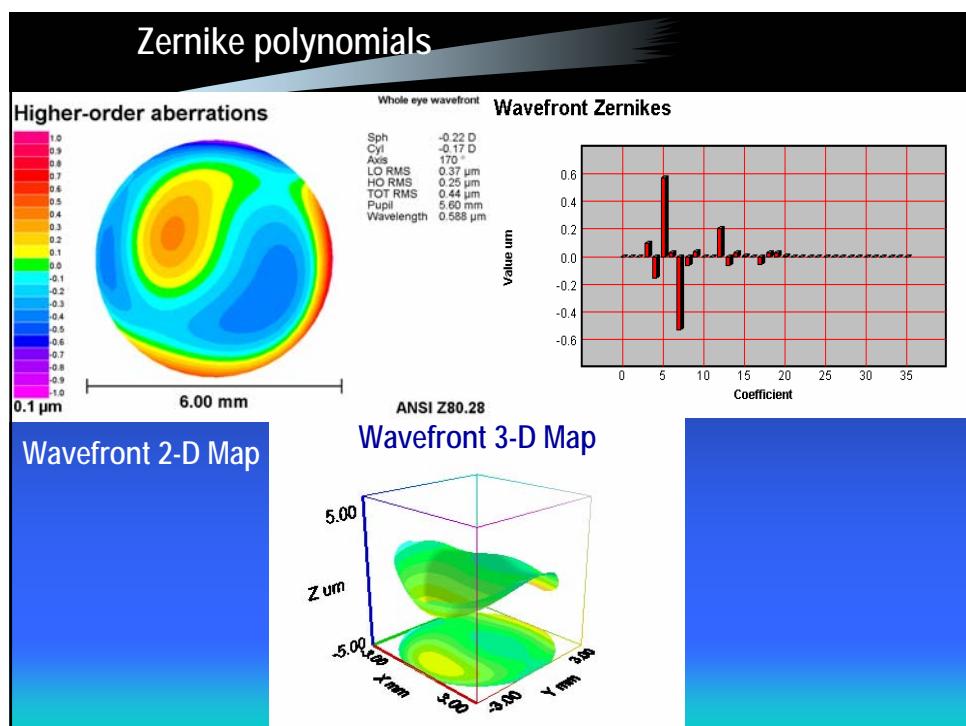
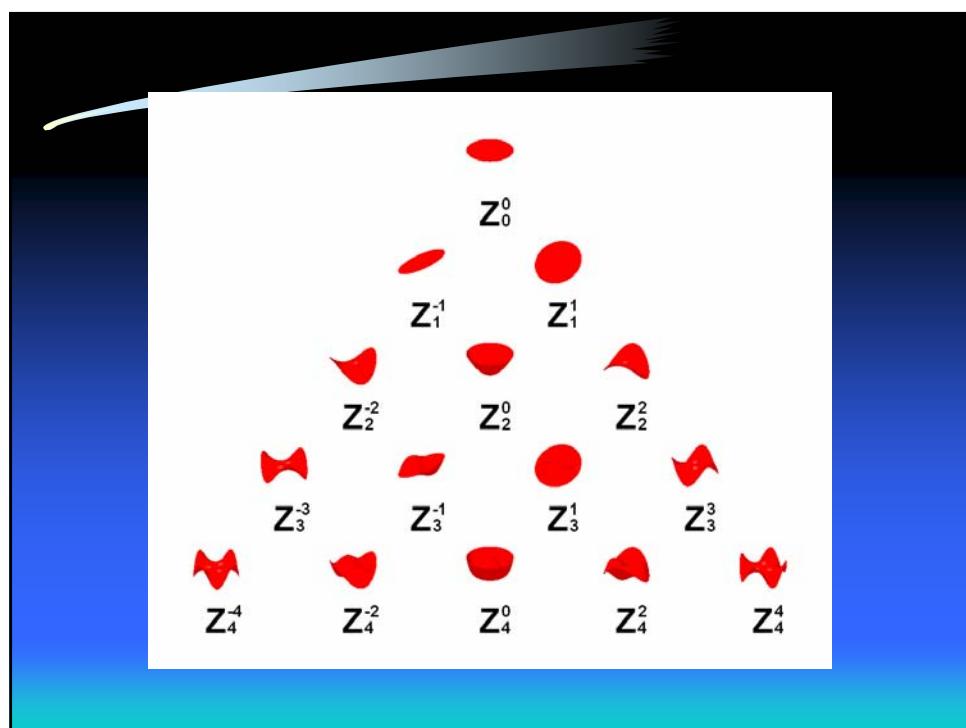
Las **aberraciones de alto orden** vienen representadas por los polinomios de Zernike de orden  $n \geq 3$ . Son de tercer orden el astigmatismo triangular ( $j = 6$  y  $9$ ) así como el coma vertical y el coma horizontal ( $j = 7$  y  $8$ ) mientras que la aberración esférica ( $j=12$ ) es de cuarto orden.

Listado de polinomios de Zernike hasta 6º orden, notación estándar de la OSA

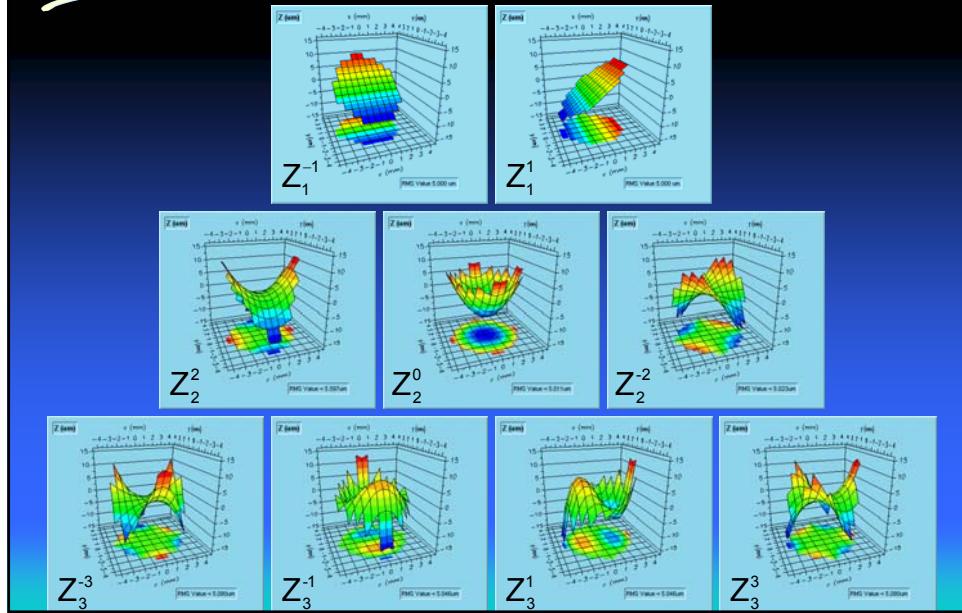
$Z_n^f$     n: radial order  
              f: angular frequency

Zernike coefficient	Radial order	Angular frequency	Aberration
0	0	0	Piston
1	1	-1	Tip, Tilt (Prism)
2	1	1	
3	2	-2	Astigmatism
4	2	0	Defocus
5	2	2	Astigmatism
6	3	-3	Trefoil 3-fold
7	3	-1	Coma (vertical)
8	3	1	Coma (horizontal)
9	3	3	Trefoil 3-fold
10	4	-4	4-fold
11	4	-2	Astigmatism
12	4	0	Spherical Aberration
13	4	2	Astigmatism
14	4	4	4-fold
15	5	-5	5-fold
16	5	-3	3-fold
17	5	-1	Coma (vertical)
18	5	1	Coma (horizontal)
19	5	3	3-fold
20	5	5	5-fold
21	6	-6	6-fold
22	6	-4	4-fold
23	6	-2	Astigmatism
24	6	0	Spherical aberration
25	6	2	Astigmatism
26	6	4	4-fold
27	6	6	6-fold





## Zernike Polynomials

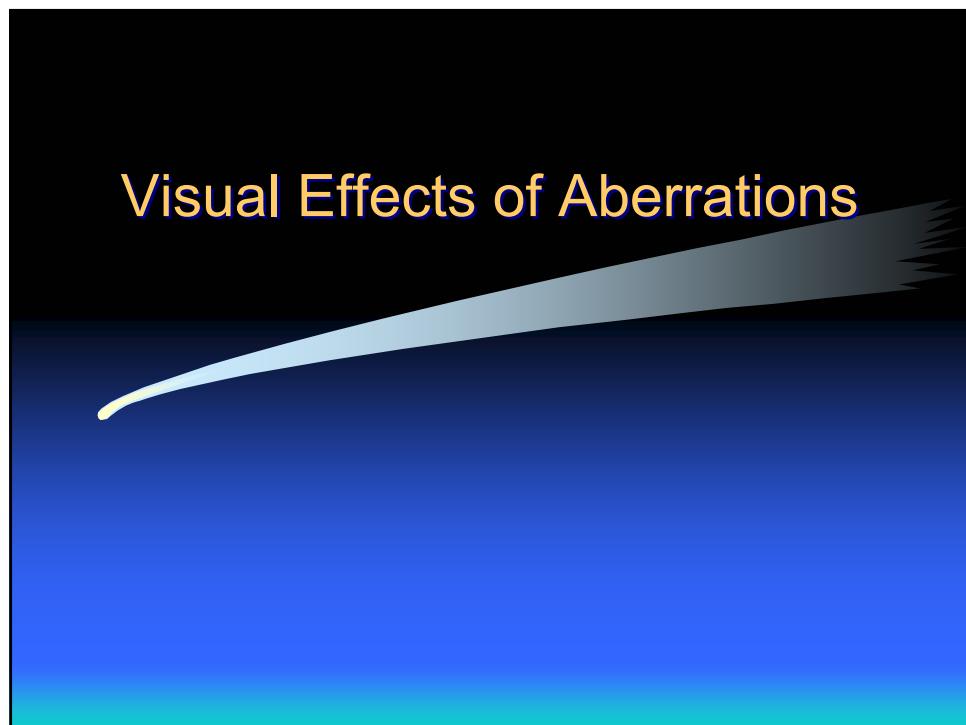


## The Root-Mean-Square (RMS) Wavefront Error

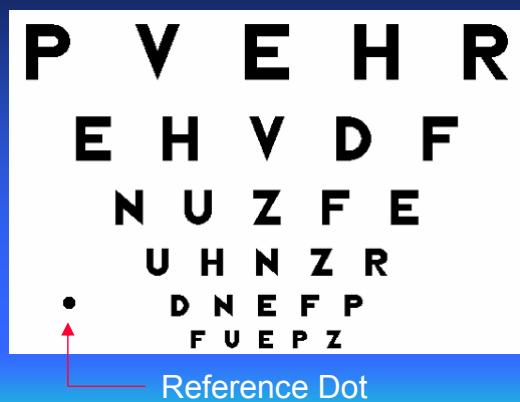
The Root Mean Square Error (RMS) is a measure of the difference between the measured and ideal wavefronts.



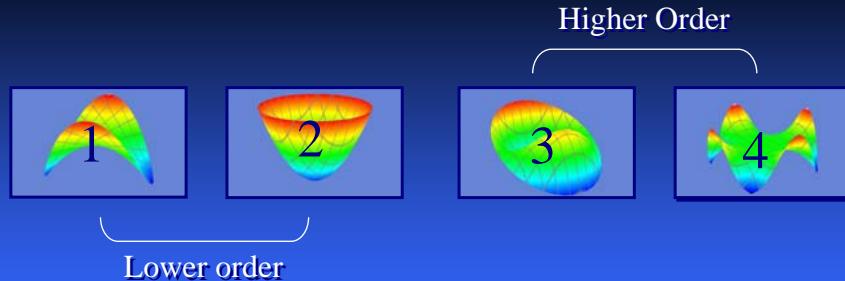
## Visual Effects of Aberrations



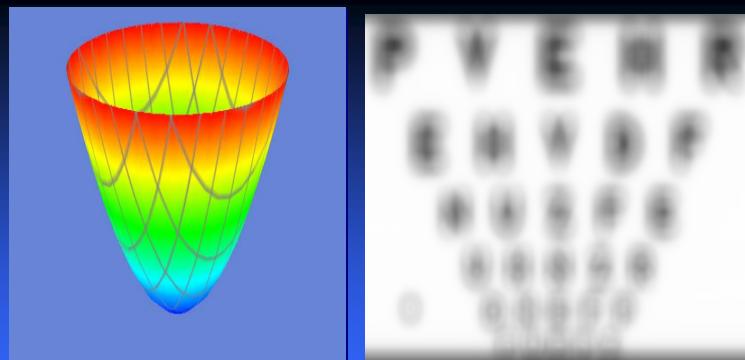
Visual Acuity Chart Image  
Used in Vision Simulation



## What Are The Visual Effects of Under Correcting Aberrations?



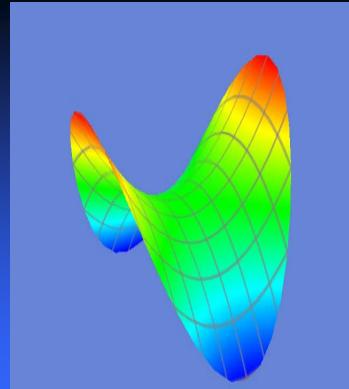
## Wavefront Error and Simulated Visual Function



2nd Order Defocus

Simulated Chart Image

## Wavefront Error and Simulated Visual Function

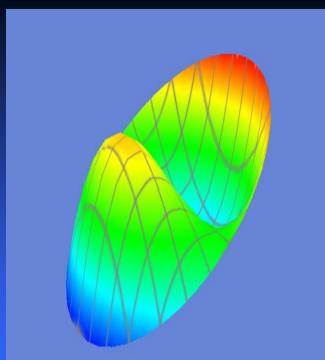


2nd Order  
Mixed Astigmatism

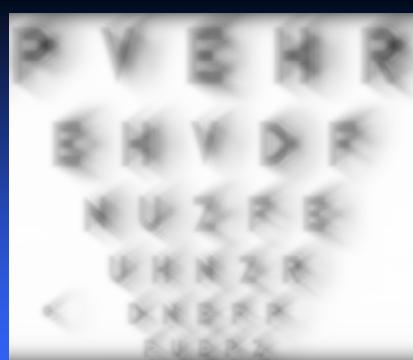


Simulated Chart Image

## Wavefront Error and Simulated Visual Function

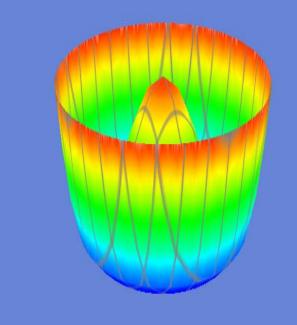


3rd Order Coma



Simulated Chart Image

## Wavefront Error and Simulated Visual Function

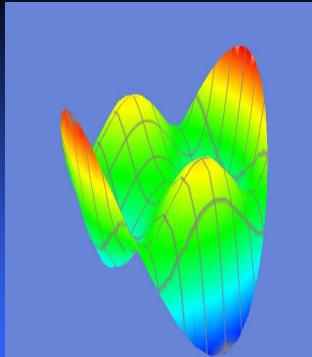


4th Order Spherical Aberration

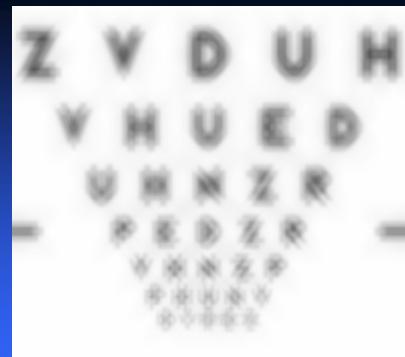


Simulated Chart Image

## Wavefront Error and Simulated Visual Function

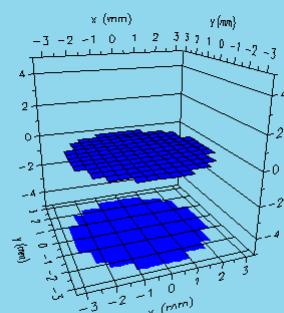


4th Order Secondary Astigmatism



Simulated Chart Image

# Wavefront Error and Simulated Visual Function

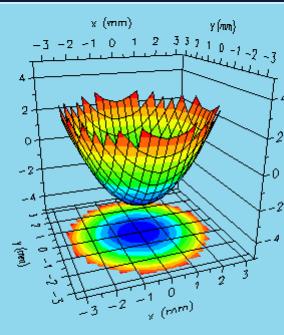


Flat Wavefront

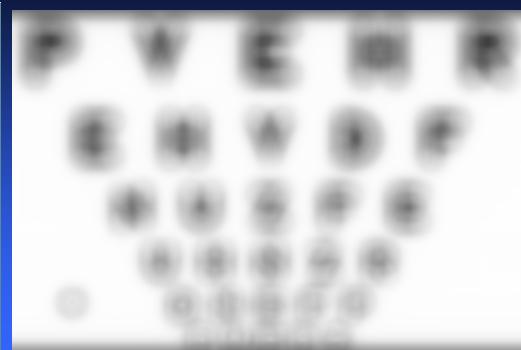


Simulated Chart Image

# Wavefront Error and Simulated Visual Function

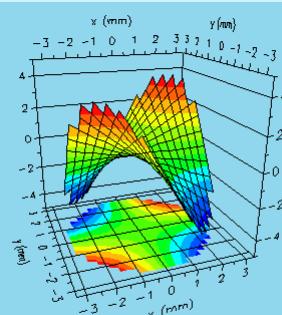


Defocus Error



Simulated Chart Image

# Wavefront Error and Simulated Visual Function

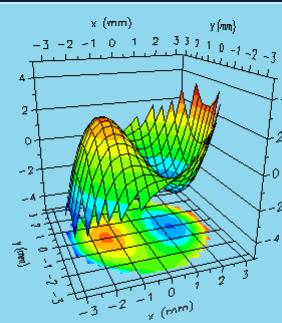


Mixed Astigmatism

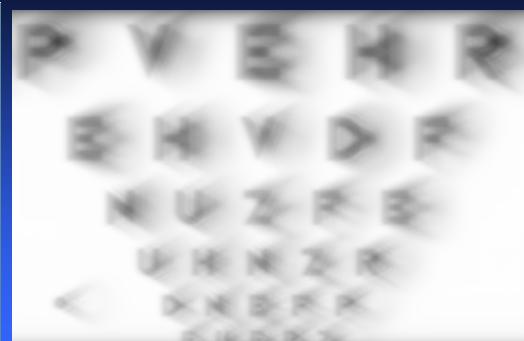


Simulated Chart Image

# Wavefront Error and Simulated Visual Function

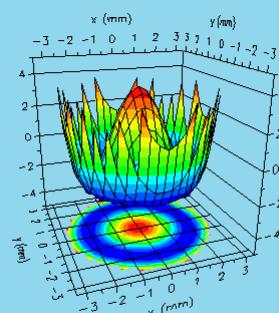


Coma

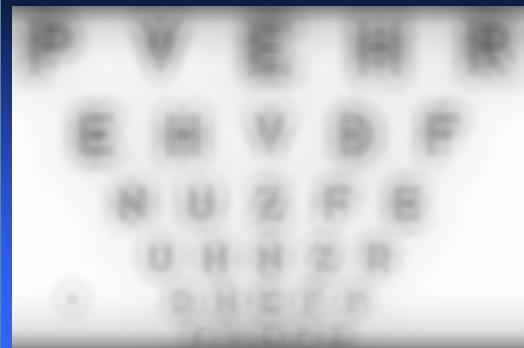


Simulated Chart Image

## Wavefront Error and Simulated Visual Function



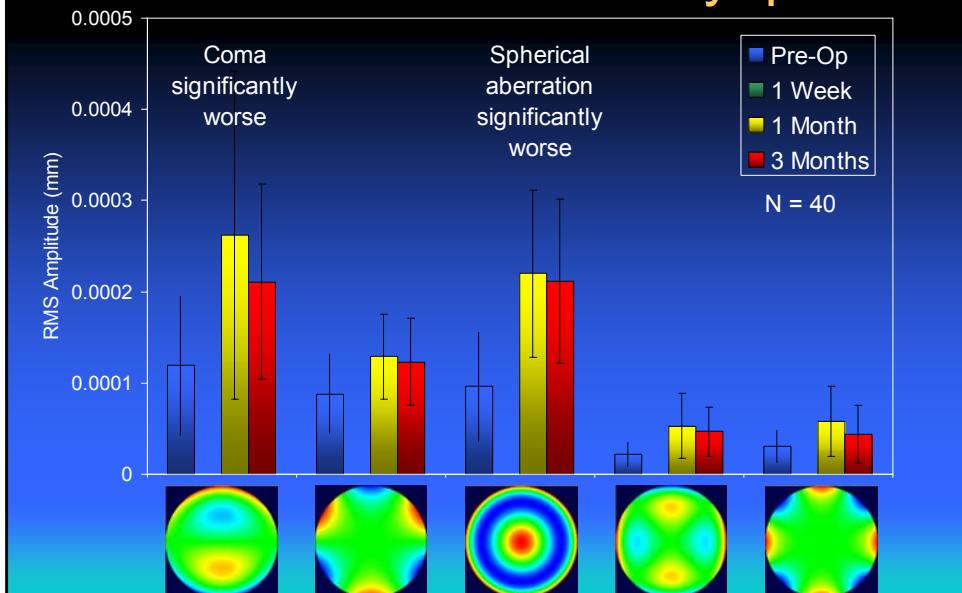
Spherical Aberration



Simulated Chart Image

**6.- How Does Wavefront Sensing Relate to Refractive Surgery?**

## Higher-Order Aberrations: Conventional LASIK Myopes



## CustomCornea: Wavefront Guided Laser Surgery

Measured  
Wavefront

## CustomCornea: Wavefront Guided Laser Surgery

Desired  
Wavefront



## CustomCornea: Wavefront Guided Laser Surgery

Desired  
Wavefront



## CustomCornea: Wavefront Guided Laser Surgery

Conventional  
Treatment

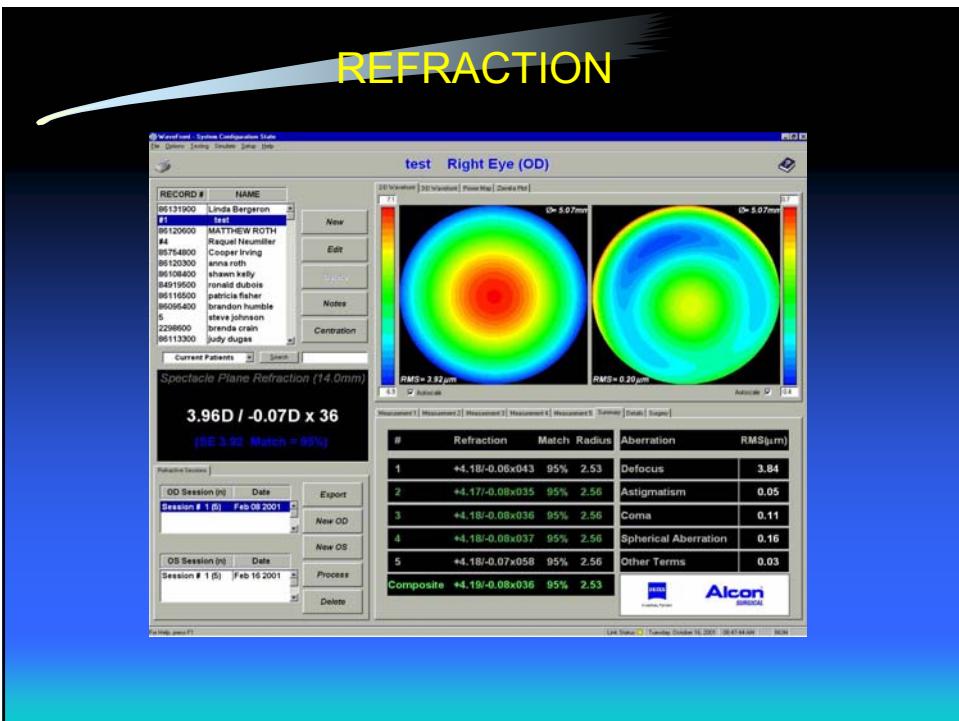


## CustomCornea: Wavefront Guided Laser Surgery

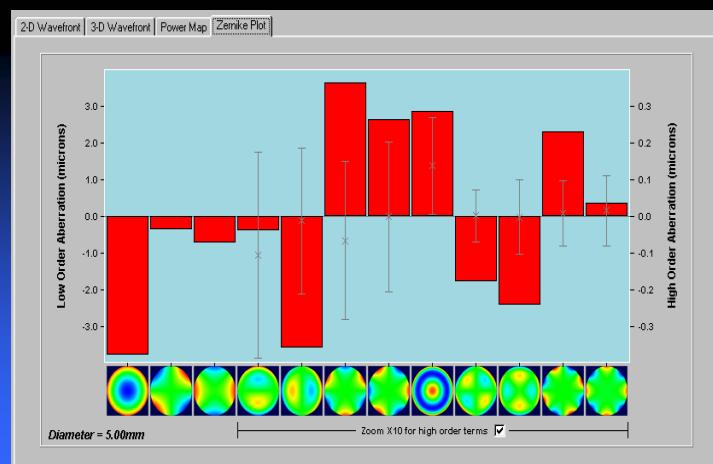
Remove a little  
extra here.

Back off a bit  
here.

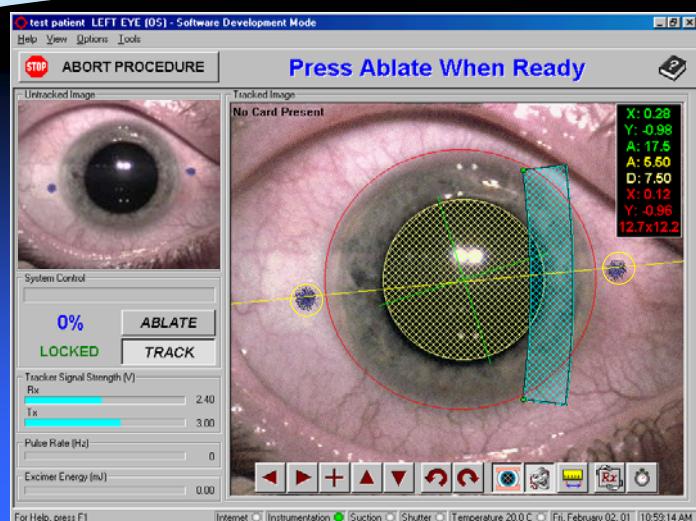


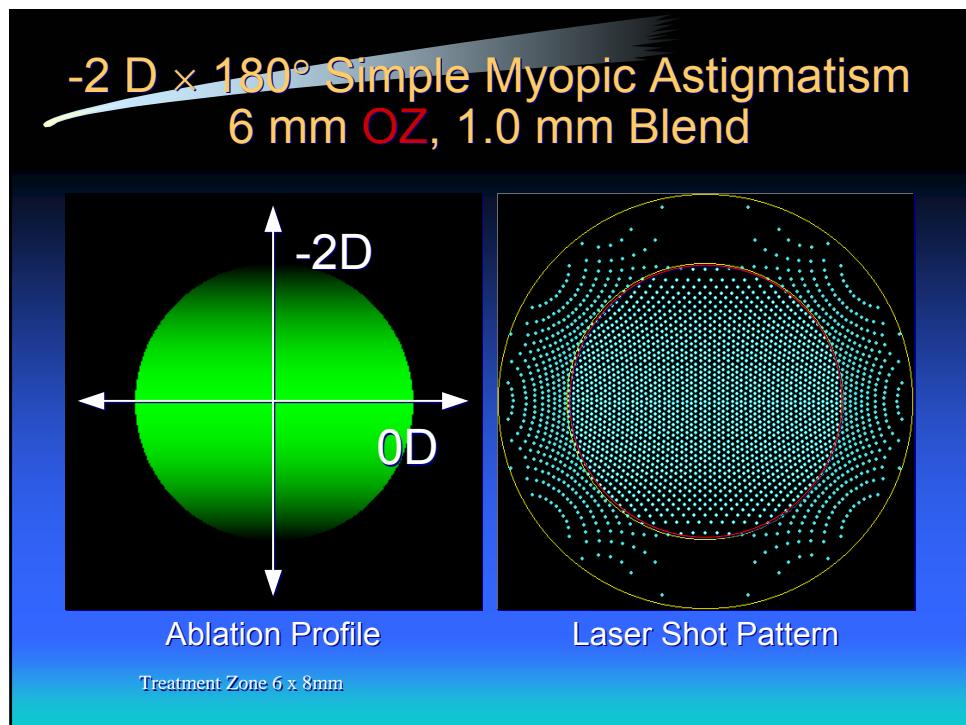
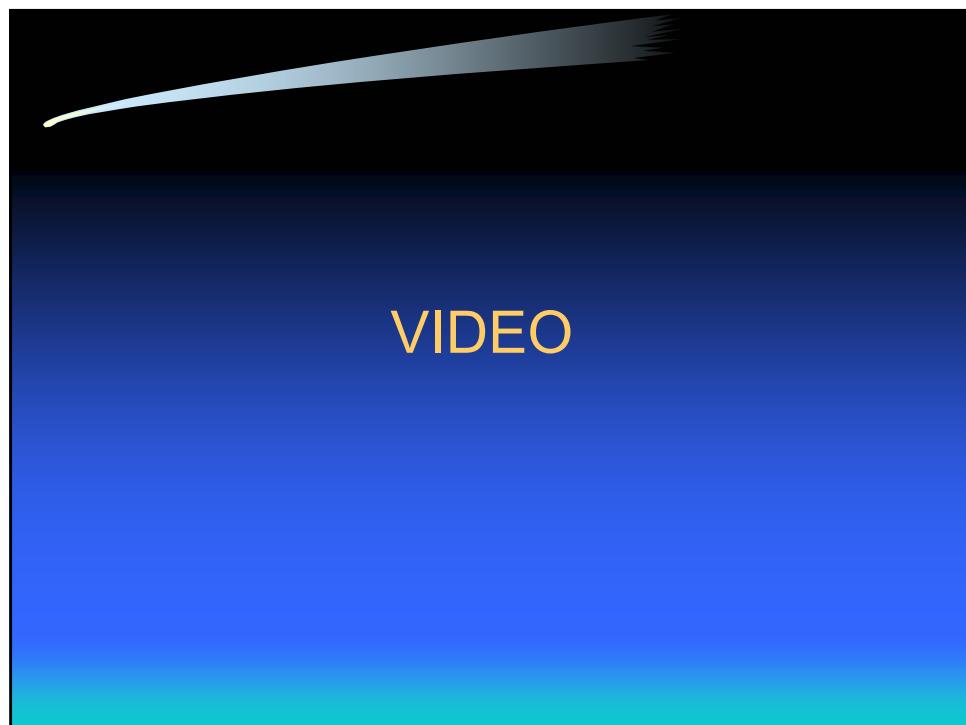


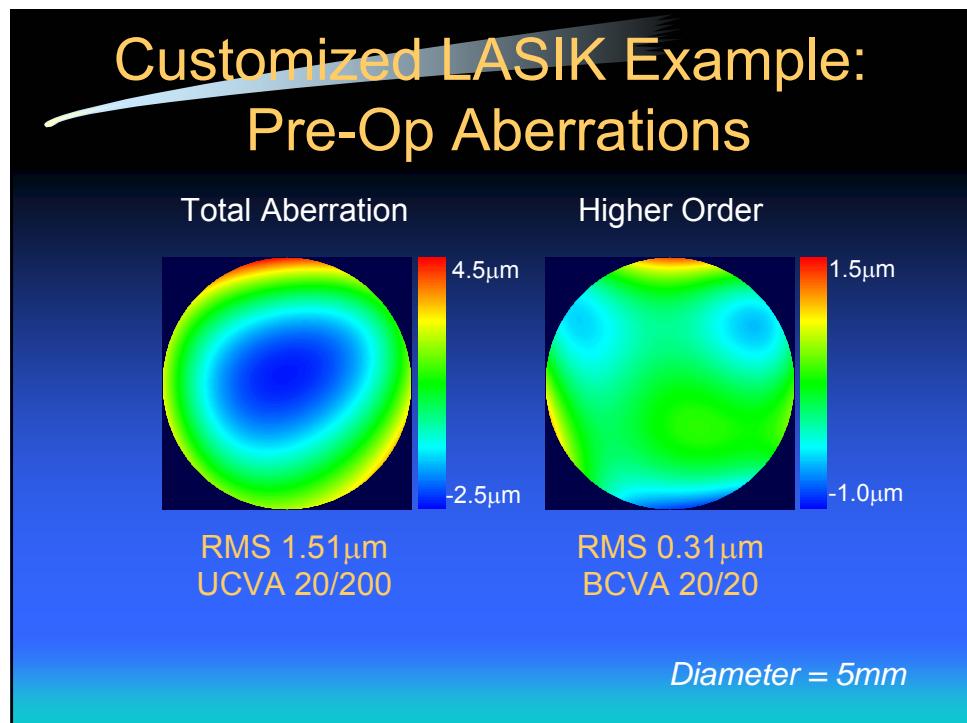
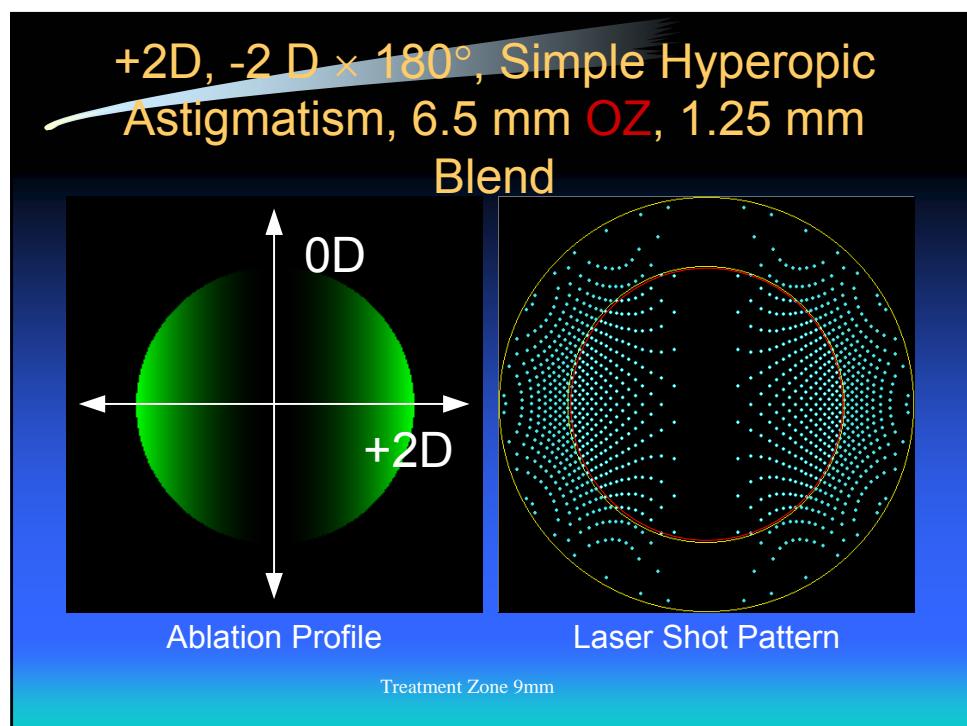
## ZERNIKE DATA



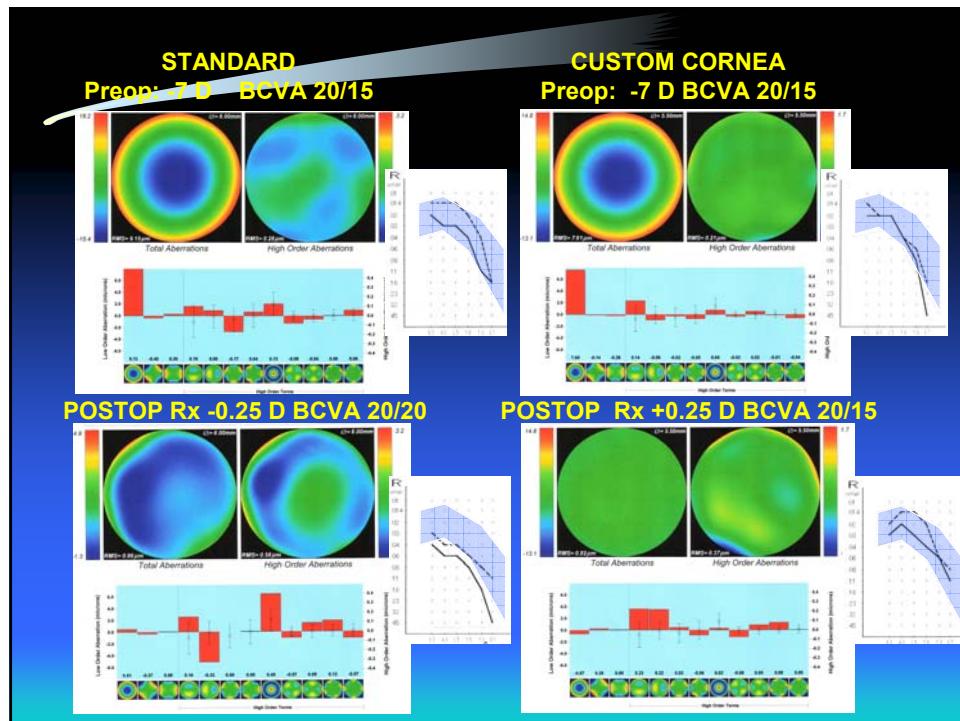
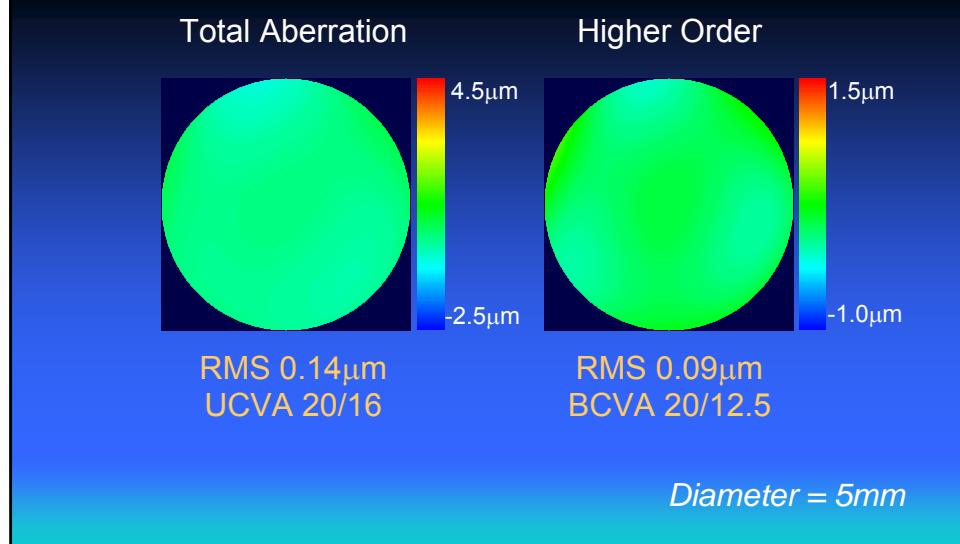
## TREATMENT

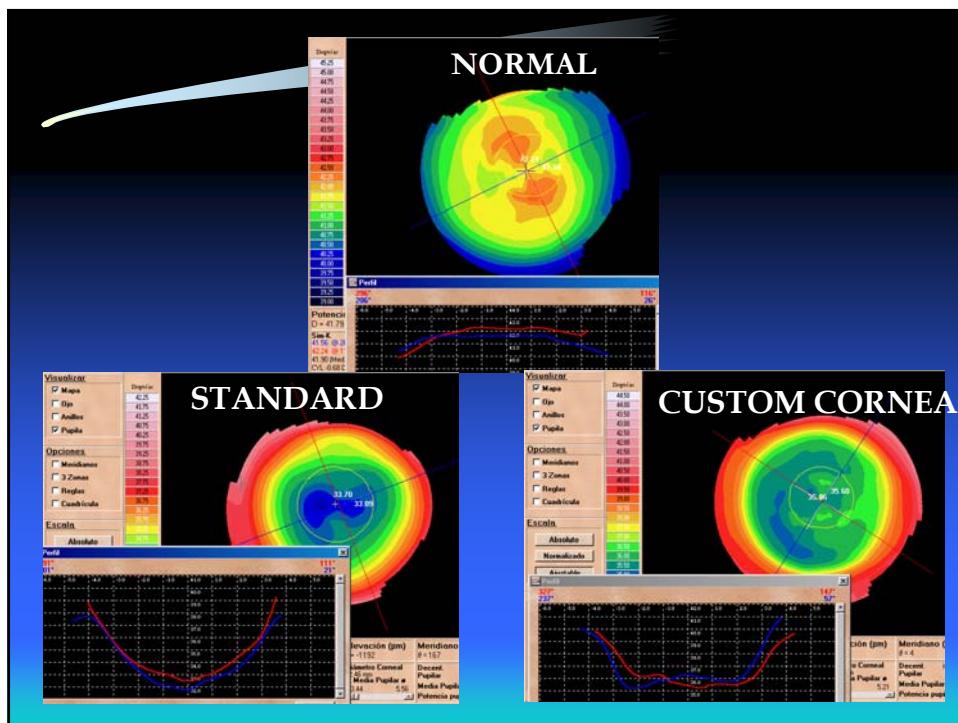






# Customized LASIK Example: Post-Op Aberrations

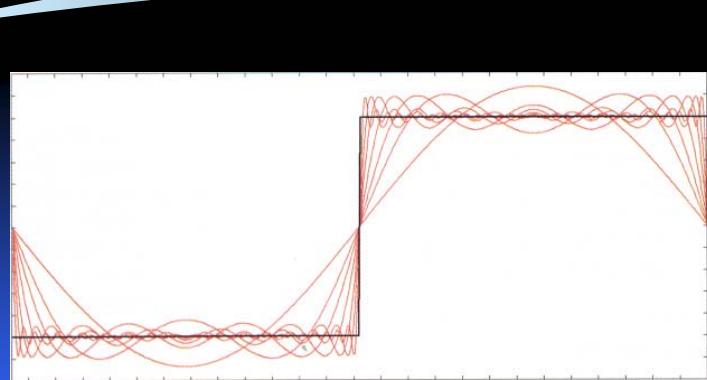




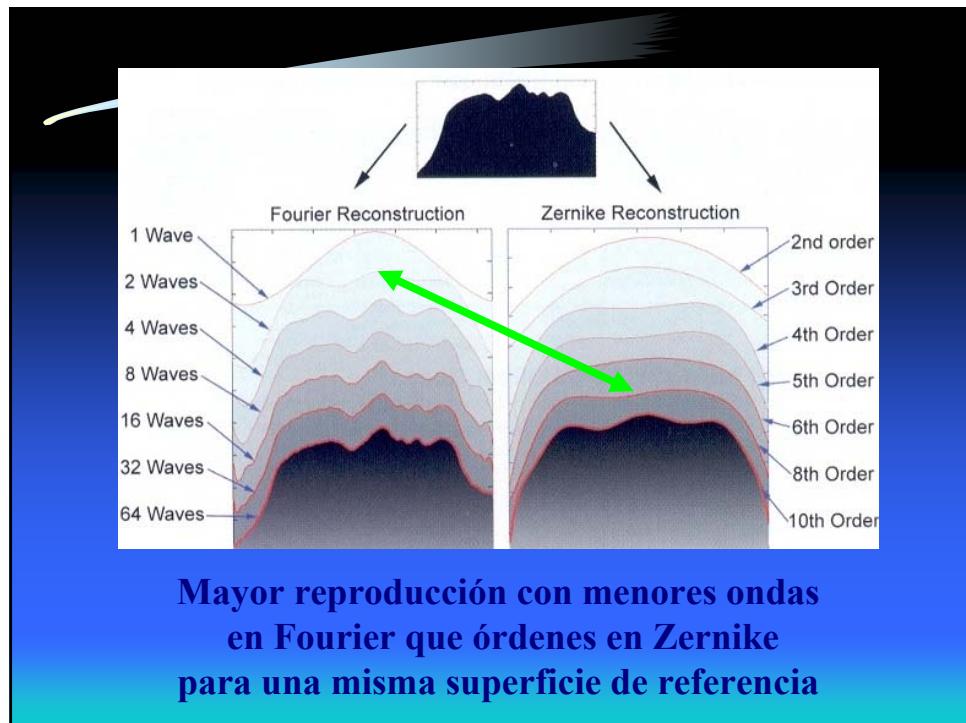
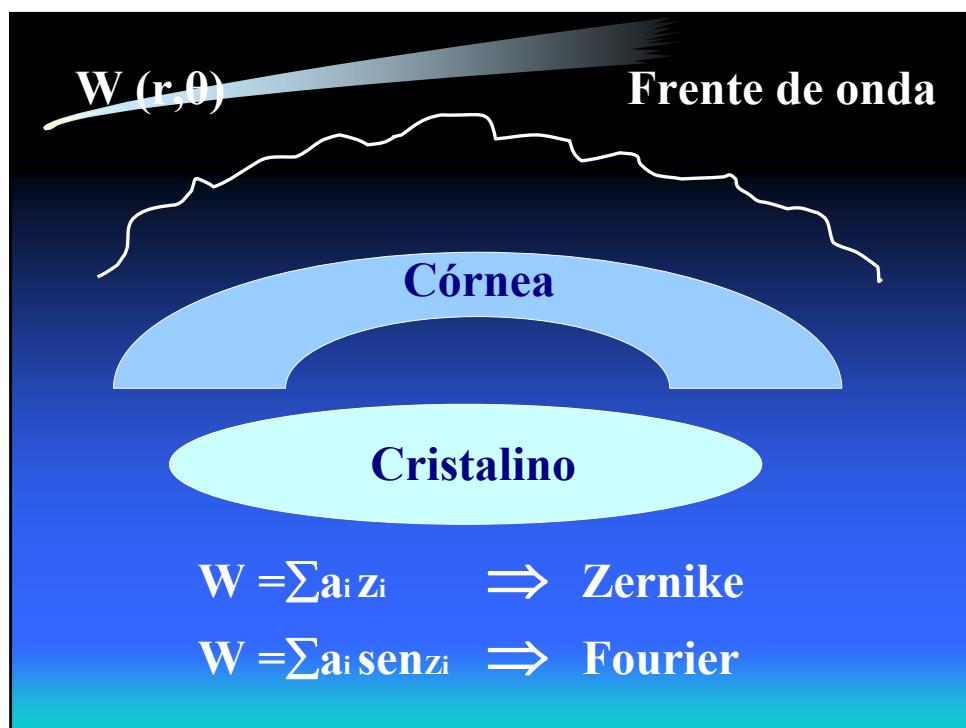
## Summary

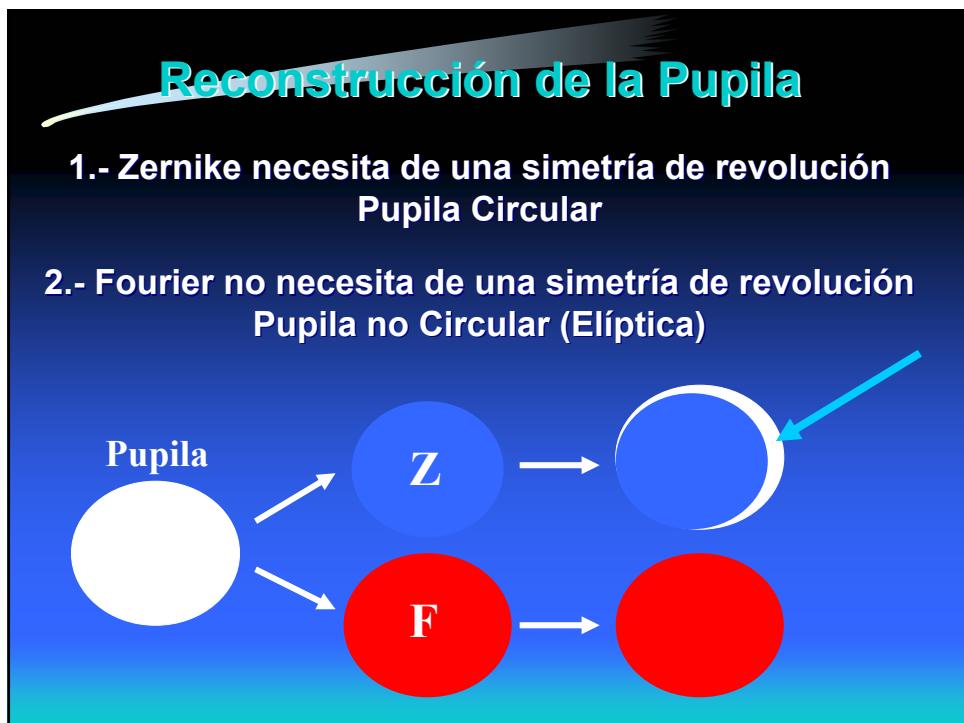
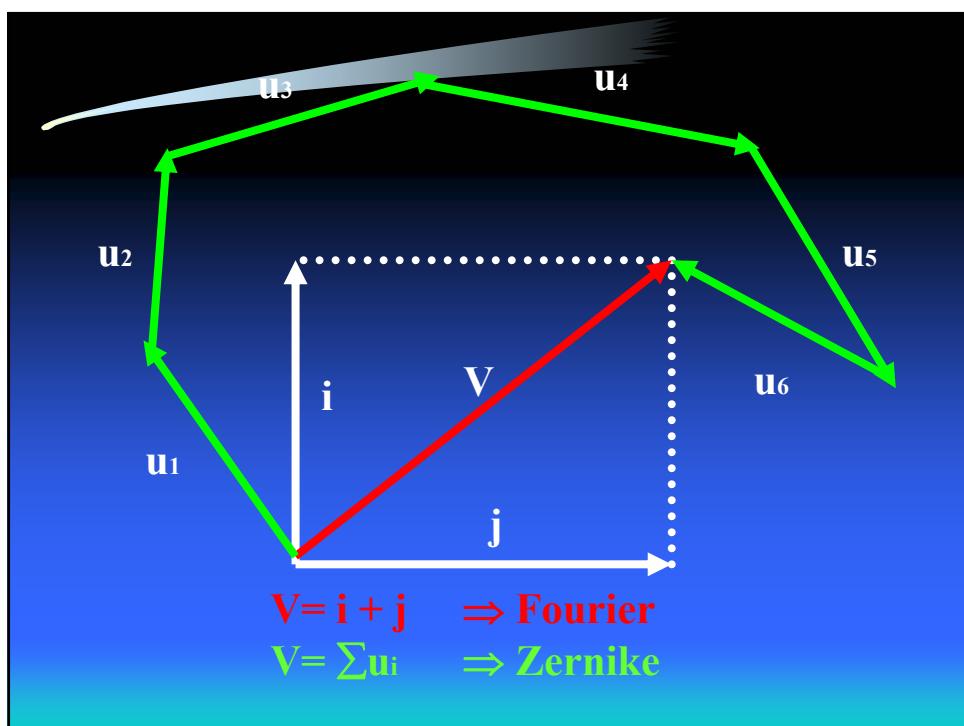
- Wavefront sensing is a powerful tool for understanding the optical functioning of the eye.
- With the right technology, measurement of the wavefront can readily be accomplished in the clinical setting.
- Wavefront data has powerful clinical utility, both in diagnosing visual complaints and in customizing refractive procedures.

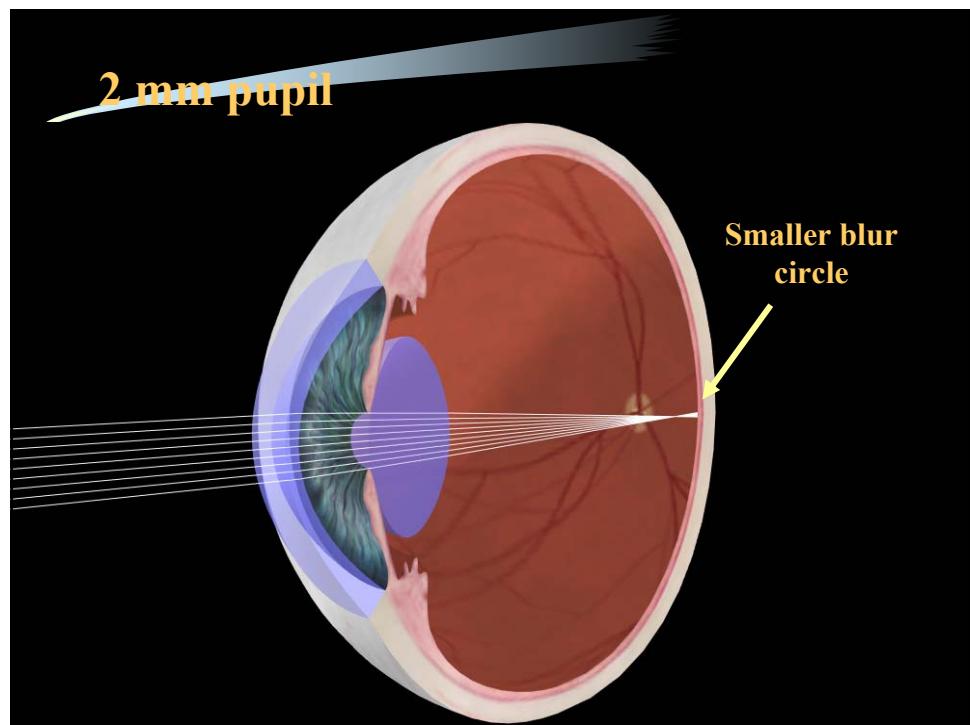
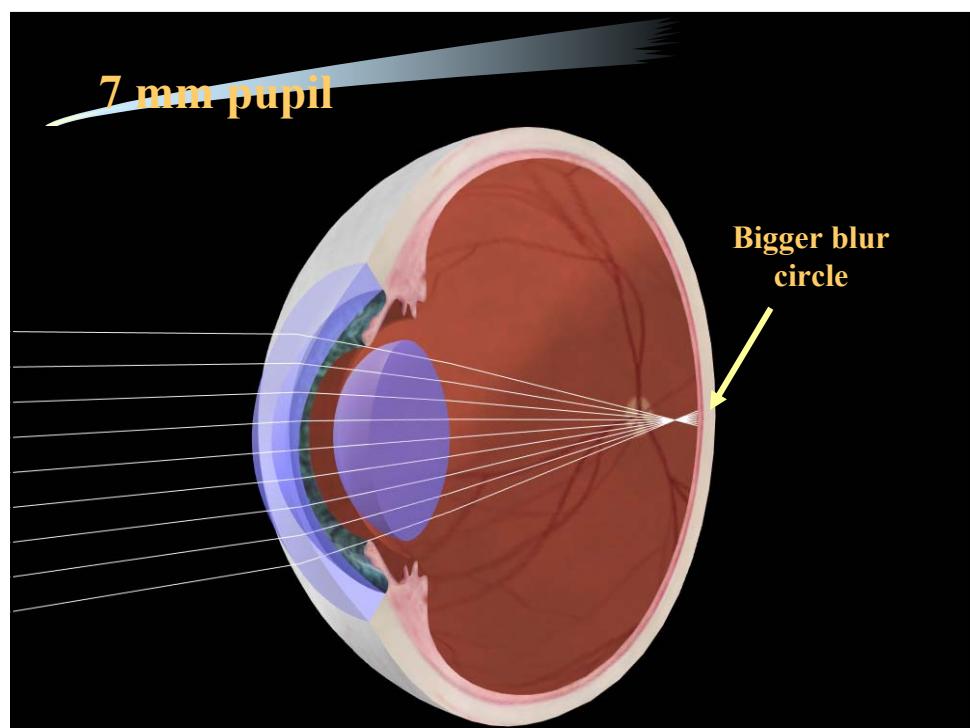
## 7.- J B Joseph Fourier versus Frits Zernike



Mediante un número determinado de ondas sinusoidales podemos describir una onda cuadrada







## Posibles Ventajas Fourier

- 1.- Menos cálculos de computación
- 2.- Mayor resolución con menos órdenes  
(o menor información)
- 3.- Aplicable a pupilas más reales
- 4.- Reconstrucción más real del frente de onda

**Thank you**

Human **Visual Performance**  
Research Group

University of Valencia, Spain

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