THE HAWAII CONJECTURE AND ITS GENERALIZATIONS

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The object of the talk is to present a proof of the following statement which was conjectured by T. Craven, G. Csordas and W. Smith [1] and is known now as the Hawaii conjecture.

Hawaii conjecture. *If a real polynomial p has exactly $2m$ nonreal zeros, counting multiplicities, then its logarithmic derivative has at most $2m$ critical points, counting multiplicities.*

We show that this conjecture is true not only for real polynomials but also for all real entire functions of genus 1^* with finitely many nonreal zeros. A real entire function f is of genus 1^* if $f(z) = e^{-az^2}g(z)$, where $a \geqslant 0$ and g is a real polynomial or a real entire function of genus 0 or 1.

In the talk, we also discuss some open problems concerning generalizations of the Hawaii conjecture and the Newton inequalities. We show that the original Hawaii conjecture is more natural for entire functions rather than for the polynomials.

1. T. Craven, G. Csordas, W. Smith, "The zeros of derivatives of entire functions and the Pólya-Wiman conjecture," Ann. of Math., **125**, No. 2, 405–431 (1987).