Asymptotics for Jacobi–Sobolev orthogonal polynomials associated with non–coherent pairs of measures

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We consider the Sobolev inner product

\[ \langle f, g \rangle = \int_{-1}^{1} f(x)g(x)d\psi^{(\alpha,\beta)}(x) + \int f'(x)g'(x)d\psi(x), \]

where \( d\psi^{(\alpha,\beta)}(x) = (1-x)^\alpha(1+x)^\beta \) with \( \alpha, \beta > -1 \), and \( \psi \) is a measure involving a rational modification of a Jacobi weight and with a mass point outside the interval \((-1,1)\). We study the asymptotic behaviour of the polynomials which are orthogonal with respect to this inner product on different regions of the complex plane. In fact, we obtain the outer and inner strong asymptotics for these polynomials as well as the Mehler–Heine asymptotics which allow to obtain the asymptotics of the largest zeros of these polynomials. We also show that in a certain sense the above inner product is equilibrated.