

# ON THE EXACT MARKOV INEQUALITY FOR $k$ -MONOTONE POLYNOMIALS IN UNIFORM AND $L_1$ -NORMS

ANDRÁS KROÓ

## Abstract

We consider the classical extremal problem of estimating norms of higher order derivatives of algebraic polynomials when their norms are given. The corresponding extremal problem for general polynomials in uniform norm was solved by A. A. Markov, while Bernstein found the exact constant in the Markov inequality for *monotone* polynomials. In this note we give Markov-type inequalities for higher order derivatives in the general class of *k-monotone* polynomials. In particular, in case of first derivative we find the exact solution of this extremal problem in both uniform and  $L_1$ -norms. This exact solution is given in terms of the largest zeros of certain Jacobi polynomials.

**Keywords:** Markov inequality,  $k$ -convex polynomial, Jacobi polynomial, Bessel function

**AMS Classification:** 41A17.

András Kroó  
Alfred Rényi Institute of Mathematics  
Hungarian Academy of Sciences  
Budapest, Hungary.  
email: kroo@renyi.hu