Clifford theory for semisimple Hopf algebras

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The classical Clifford correspondence for normal subgroups is considered in the setting of semisimple Hopf algebras. We prove that this correspondence still holds if the extension determined by the normal Hopf subalgebra is cocentral. Other particular situations where Clifford theory also works will be discussed. This talk is based on the paper "Clifford theory for cocentral extensions" Israel J. Math, 181, 2011, (1), 111-123 and some work in progress of the author.

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Hopf algebras and Tensor categories,
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Plan of the talk

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Based on Isr. J. Math, 2011 and some work in progress.

Rieffel's work on semisimple normal extensions

Definition of normal subrings:

Let $B \subset A$ an extension of semisimple rings. The extension is called normal if $A(I \cap B) = (I \cap B)A$ for any maximal ideal I of A.

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Normal extensions of Hopf algebras

More generally the same thing is true for semisimple Hopf algebras.

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The stabilizer always exists but is not unique; there might be more then one stabilizers for a given module W.

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If $A = kG$ and $B = kH$ for a normal subgroup H then $d = g \in G$ and ${}^d M$ coincides with the conjugate module ${}^g M$ introduced by Clifford in [1].

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for all $x \in B$ (see Proposition 5.3 of [10]).

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The set $\{d \in \mathrm{Irr}(A^*) \mid {}^d\alpha = \epsilon(d)\alpha\}$ is closed under multiplication and " $*$ ". Thus it generates a Hopf subalgebra Z_α of A that contains B.

- Z_α is called the stabilizer of α in A.
- If $A = kG$ and $B = kN$ for a normal subgroup N then the stabilizer Z_α coincides with the stabilizer Z_M of M introduced by Clifford in [1].

Clifford correspondence for normal Hopf algebras

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If $A = kG$ and $B = kN$ for a normal subgroup N then the above inequality is equality. It states that the number of conjugate modules of α is the index of the stabilizer of α in G. (Orbit formula)

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If $A = kG^*$ then $K(A) = A$ and the previous theorem gives the well known description of the irreducible modules over $D(G)$ in terms of the centralizers $C_G(g)$.

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s	g^2	g^3	g
s^2	g^3	g	g^2
st	g^3	g^2	g
ts	g^2	g	g^3

Table: The right action of $\mathbb{S}_3$ on $\mathbb{C}_4$

$\mathbb{C}_4 \lhd \mathbb{S}_3$	g	g^2	g^3
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