Hopf Algebras and Tensor Categories

University of Almería

July 4-8, 2011

Abstracts

Sponsored by:

Hopf semialgebras

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In this talk, we introduce and investigate the notions of semibialgebras and Hopf semialgebras over semirings. We also investigate several related categories of Doi-Koppinen semimodules.

Crossed product of Hopf algebras

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The main properties of the crossed product in the category of Hopf algebras are investigated. Let A and H be two Hopf algebras connected by two morphism of coalgebras $\triangleright : H \otimes A \to A$, $f : H \otimes H \to A$. The crossed product $A\#_f^{\triangleright} H$ is a new Hopf algebra containing A as a normal Hopf subalgebra. In fact, we prove that a Hopf algebra E is isomorphic as a Hopf algebra to a crossed product of Hopf algebras $A\#_f^{\triangleright} H$ if and only if E factorizes through a normal Hopf subalgebra A and a subcoalgebra H such that $1_E \in H$. The universality of the construction, the existence of integrals, commutativity or involutivity of the crossed product are studied. The crossed product $A\#_f^{\triangleright} H$ is a semisimple Hopf algebra if and only if both Hopf algebras A and H are semisimple. Looking at the quantum side of the construction we shall give necessary and sufficient conditions for a crossed product to be a coquasitriangular (braided) Hopf algebra. In particular, all braided structures on the monoidal category of $A\#_f^{\triangleright} H$-comodules are explicitly described in terms of their components.

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Hopf algebras and the geometry of real hyperplane arrangements

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The starting point for our considerations is the notion of graded Hopf algebra, particularly those graded over the nonnegative integers. When the latter are replaced by finite sets, one arrives at the notion of Hopf monoid in Joyal's category of species.

The goal of this talk is to go one step further, replacing finite sets by finite real hyperplane arrangements. Geometric considerations allow us to define a generalized notion of "Hopf algebra" in this setting. The key ingredient in this construction is furnished by the projection maps of Tits. Algebraically, we construct a monad and a comonad linked by a mixed distributive law in the sense of Beck.

We will discuss these ideas without assuming familiarity with hyperplane arrangements or distributive laws. The case of finite sets (Hopf monoids in species) is recovered by restricting to braid arrangements. This will be used to illustrate the constructions.

This is joint work in progress with Swapneel Mahajan.

Polynomial identities and graded algebras

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We consider G-graded algebras and their corresponding graded identities. In the lecture, I will present generalizations of some fundamental results in PI theory (originally proved by Kemer for $G = \{e\}$) in the context of G-graded algebras, where G is an arbitrary finite group. The same statements for H-comodule algebras (where H is an arbitrary semisimple Hopf algebra) are open.

A presentation by generators and relations of Nichols algebras of diagonal type

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The Lifting Method of Andruskiewitsch and Schneider is the leading method to classify pointed Hopf algebras [1]. It involves as an inicial step to know for which braided vector spaces their associated Nichols algebra is finite-dimensional; such braided vector spaces were classified by Heckenberger [3].

A second step is the following one: for each of these Nichols algebras, give a nice presentation by generators a relations. In the present talk we give an answer to this question, following [2]. We characterize convex orders on root systems associated to finite Weyl groupoids and use a description of coideal subalgebras of Nichols algebras [4]. We describe then a set of relations using the PBW bases of [5].

We use such presentation to prove that every finite-dimensional pointed Hopf algebra over $\mathbb{C}$, whose group of group-like elements is abelian, is generated by its group-like and skew-primitive elements, a conjecture due to Andruskiewitsch and Schneider.

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Preantipodes for dual-quasi bialgebras

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It is known that a dual quasi-bialgebra with antipode H, i.e. a dual quasi-Hopf algebra, fulfils a fundamental theorem for right dual quasi-Hopf H-bicomodules. The converse in general is not true. We prove that, for a dual quasi-bialgebra H, the structure theorem amounts to the existence of a suitable endomorphism S of H that we call a preantipode of H. This is based on joint work with Alice Pavarin (University of Padova, Italy), see arXiv:1012.1956.

The central elements of the universal enveloping algebra of higher orders and the construction of Knizhik-Zamolodchikov type equations for root systems of types A,D,B

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The talk is based on a joint work with V.A. Goloubeva (Moscow Aviational Institute, Russia).

The talk is devoted to some generalization of the classical Riemann-Hilbert problem of construction of a Pfaffian system of fuchsian type, whose singular set is a collection of reflection hyperplanes defined by a system of roots B_n. Also some new results are obtained for the root systems A and D.

In construction of Knizhnik-Zamolodchikov equations, whose singular set is associated with the root system A_n, the Casimir element of the second order is used. In the case of root system B_n a similar construction was done by A. Leibman in one parameter case, while the corresponding equations must depend on two parameters, as the number of orbits of the corresponding root system equals to two. For other root systems such constructions are not known.

The Knizhnik-Zamolodchikov equations associated with the root system B have the following form:

$$dy = \lambda\Big(\sum_{i<j} \frac{\tau_{ij}}{x_i - x_j} d(x_i - x_j) + \frac{\mu_{ij}}{x_i + x_j} d(x_i + x_j) + \sum_i \frac{\nu_i}{x_i} dx_i\Big) y, \ \ i, j = 1, ..., n.$$

The Frobenius condition of integrability is equivalent to a system of commutation relations on the coefficients $\tau_{ij}, \mu_{ij}, \nu_i$. Some nontrivial solutions of this system of commutation relations is found in the Hopf algebra $U(U(o_N)) \otimes ... \otimes U(U(o_N))$, where the tensor product is taken n times and N is arbitrary. The key role in this construction is played by central elements of $U(o_N)$, which can have order higher than 2.

Semisimple Hopf algebras

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Suppose that H is a finite dimensional semisimple Hopf algebra over an algebraically closed field whose characteristic does not divide the dimension of H. We shall assume that for any positive integer $d > 1$ any two irreducible H-modules of dimension d are isomorphic. The category of left H-modules ${}_H\mathcal{M}$ is a monoidal category. In the talk we shall discuss Clebsch-Gordan coefficients in decompositions in ${}_H\mathcal{M}$ of tensor products of irreducible H-modules. Some classifications results are obtained in the case when there exists up to an isomorphism a unique irreducible H-module of dimension greater than 1.

Twisted partial Hopf actions

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The notion of a twisted partial Hopf action is a natural generalization of both, twisted partial group actions and partial Hopf actions. The twisted partial group actions arise in the context of graded algebras, allowing them to be classified as crossed products. The partial actions and coactions of Hopf algebras were originally used to put partial Galois extensions of commutative algebras in a broader context of Galois corings. In this work, we define a twisted partial action of a Hopf algebra on a unital algebra, construct partial crossed products and relate them with partially cleft extensions. The globalization theorem for twisted partial Hopf actions is also discussed.

Classifying Hopf algebras of a given dimension

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Over an algebraically closed field, the problem of classifying all Hopf algebras even for some given small dimension, such as 16 or 32, or for a class of dimensions, such as p, pq , pq^2, etc, for p, q prime, is a difficult one. Some recent techniques using the coradical filtration are due to D. Fukuda; he applied these to dimensions 18 and 30. Cheng and Ng have recently investigated Hopf algebras of dimension p in the Yetter-Drinfeld category over the 4-dimensional Sweedler Hopf algebra and used these results to study dimension $4p$. They show that Hopf algebras of dimensions $20, 28$, or 44 are either semisimple, pointed or copointed.

In this talk some more techniques will be mentioned with applications to dimension p^3 in mind. Hopf algebras of dimension 27 will be completely described.

This is joint work with G.A. García.

Finite quantum groups and quantum permutation groups

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This talk reports on joint work with Teodor Banica and Sonia Natale (arXiv:1104. 1400).

A quantum permutation algebra is a Hopf algebra having the diagonal algebra k^n as a faithful comodule algebra. The corresponding quantum group acts faithfully on a finite classical space and is called a quantum permutation group. Several unexpected Hopf algebras appear as quantum permutation algebras and so it is natural to ask if any finite-dimensional semisimple Hopf algebra is a quantum permutation algebra, i.e., if a Cayley theorem holds for finite quantum groups. We show, by considering bicrossed products associated to exact factorizations of finite groups, the existence of a semisimple Hopf algebra of dimension 24 that is not a quantum permutation algebra. This example is minimal since on the other hand, we show that any semisimple Hopf algebra of dimension less than 23 is a quantum permutation algebra.

Weak bimonads and weak Hopf monads

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An algebra over a commutative ring is known to be a bialgebra if and only if its category of (left or right) modules is monoidal such that the forgetful functor is strong monoidal. By analogy, a monad can be called a "bimonad" whenever its Eilenberg-Moore category is monoidal such that the forgetful functor is strong monoidal. A bimonad in this sense was proved to be the same as an opmonoidal monad, see recent works by Moerdijk, Mc Crudden and others.

More generally, an algebra over a commutative ring is known to be a weak bialgebra if and only if its category of (left or right) modules is monoidal such that the forgetful functor possesses a so-called separable Frobenius monoidal structure. By analogy, we define a "weak bimonad" as a monad with additional structures that are equivalent to the monoidality of its Eilenberg-Moore category such that the forgetful functor is separable Frobenius monoidal. Whenever in the base category idempotent morphisms split, a simple set of axioms is provided, that characterizes the monoidal structure of the Eilenberg-Moore category as a weak lifting of the monoidal structure of the base category. The relation to bimonads, and the relation to weak bialgebras in a braided monoidal category are revealed. We also discuss antipodes, obtaining the notion of weak Hopf monad.

The talk is based on the paper [G. Böhm, S. Lack and R. Street, *Weak bimonads and weak Hopf monads.* J. Algebra **328** (2011), 1-30].

Galois-Grothendieck duality, Tannaka duality and Hopf (co)monads

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In SGA 1, Alexandre Grothendieck defines the algebraic fundamental group of an scheme S. As is his wont, Grothendieck adopts a very general setting: given an abstract category C (think of it as the category of étale coverings of S) endowed with a functor ω to finite sets, satisfying certain conditions, Grothendieck constructs a profinite group G, the group of automorphisms of ω, and shows that C is equivalent to the category of continuous finite G-sets.

Similarly, Tannaka duality (in the larger sense) associates with a tensor category C endowed with a fiber functor ω a sort of group G (affine group, gerbe in the commutative setting, Hopf algebra, Hopf algebroid in the non-commutative setting) in such a way that C is equivalent to the category of finite dimensional representations of G.

We will propose a general setting which encompasses these two analogous situations; given a monoidal functor $\omega : C \to B$, we will show that, under general conditions on C, B and ω, there exists a Hopf monad T on B such that the category of ind-objects of C is equivalent to the category of 'representations' of T, and C itself, to the category of representations 'of finite type' of T.

Hopf monads generalize groups and Hopf algebras in a non-braided setting.

We will also explain how this result yields Galois-Grothendieck duality as well as Tannaka duality, and other results on tensor functors.

On cross product Hopf algebras

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For A, B algebras and coalgebras but not necessarily bialgebras in a braided monoidal category $\mathcal{C}$ we give necessary and sufficient conditions for which a cross product algebra and a cross coproduct coalgebra structure afford on $A \otimes B$ a bialgebra structure in $\mathcal{C}$. We also find sufficient conditions for which such a cross product bialgebra is a Hopf algebra in $\mathcal{C}$. We then describe those cross product Hopf algebras that are a double cross (co)product, a biproduct or, more generally, a smash (co)product Hopf algebra, respectively, and to each of them we associate the appropriate Hopf algebra projection context.

Clifford theory for semisimple Hopf algebras

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The classical Clifford correspondence for normal subgroups is considered in the setting of semisimple Hopf algebras. We prove that this correspondence still holds if the extension determined by the normal Hopf subalgebra is cocentral. Other particular situations where Clifford theory also works will be discussed. This talk is based on the paper [*Clifford theory for cocentral extension.* Israel J. Math **181** (2011), 111-123] and some work in progress of the author.

Monoidal structures on the category of relative Hopf modules

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This talk is based on a joint work with Daniel Bulacu.

Let B be a bialgebra, and A a left B-comodule algebra in a braided monoidal category $\mathcal{C}$, and assume that A is also a coalgebra, with a not-necessarily associative or unital left B-action. Then we can define a right A-action on the tensor product of two relative Hopf modules, and this defines a monoidal structure on the category of relative Hopf modules if and only if A is a bialgebra in the category of left Yetter-Drinfeld modules over B.

On sheets of conjugacy classes in reductive algebraic groups

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The representation theory of $U_e(\mathfrak{g})$, the non-restricted quantized enveloping algebra of a semisimple Lie algebra $\mathfrak{g}$ at the roos of 1, is strictly related to the conjugacy classes in the (adjoint) algebraic group G whose Lie algebra is $\mathfrak{g}$. De Concini, Kac and Procesi constructed a map associating a conjugacy class C of G to each simple $U_e(\mathfrak{g})$-module V and conjectured in the early 90's a relation between the dimensions of V and C. Motivated by this connection, we classify certain families of conjugacy classes in G, called "sheets", and we describe sheets explicitly. After reporting on the state of the art of the subject, we show how our results can be used in order to refine the conjecture. This talk is based on joint work with Francesco Esposito (Padova).

Interrelations between Hopf algebras and their duals - proposed new directions

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An essential property of finite-dimensional Hopf algebras is that they are Frobenius algebras and hence are strongly related to their duals. The Frobenius map enabled us to define the concept of "conjugacy classes" for semisimple Hopf algebras H. With additional structural properties on H, the map was also used to define a Fourier transform on H. We shall review the basic ideas involved and suggest new directions in which to generalize some theorems from group theory to Hopf algebras. We shall also discuss some aspects of the theory when H is no longer semisimple.

Computing of the combinatorial rank of quantum groups

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In general an intersection of two biideals is not a biideal. By this reason one may not define a biideal generated by a set of elements, and the bialgebras do not admit a usual combinatorial representation by generators and relations. Heyneman-Radford theorem implies that each nonzero biideal of a pointed bialgebra has a nonzero skew primitive element. Each ideal generated by skew primitive elements is a biideal, but certainly a biideal in general is not generared as an ideal by its skew primitive elemens. The Heyneman-Radford theorem allows one to define a combinatorial representation over the coradical in the following form:

$$\mathfrak{A} = C\langle X \,||F_1 = 0|F_2 = 0|\ldots|F_\kappa = 0|\rangle,$$

where X is a set of generators, F_1 is a set of skew primitive relations, F_i, $1 < i \leq \kappa$ is a set of relations that are skew primitive in $C\langle X \,||F_1 = 0|F_2 = 0| \dots |F_{i-1} = 0|\rangle$. The minimal number κ is called the *combinatorial rank* of $\mathfrak{A}$. We prove that the combinatorial rank of the multiparameter version of the Lusztig small quantum group $u_q(\mathfrak{so}_{2n+1})$, or equivalently of the Frobenius-Lusztig kernel of type B_n, equals $\lfloor \log_2(n-1) \rfloor + 2$ provided that q has a finite multiplicative order $t > 4$. In the case A_n the combinatorial rank equals $\lfloor \log_2 n \rfloor + 1$, see [1].

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Twisted homogeneous racks of type D

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The problem of classifying finite-dimensional pointed Hopf algebras over non-abelian finite groups reduces in many cases to a question on conjugacy classes or, more generally, on (twisted homogeneous) racks and 2-cocycles. The racks of type D are a distinguished family of racks since they give arise to Nichols algebras of dimension infinite for any cocycle.

In this talk, we present some techniques to check when a twisted homogeneous rack (THR) is of type D and present a list of known THR of type D for alternating and sporadic groups.

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The Hopf automorphism group and the quantum Brauer group

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We lift a known exact sequence for the quantum Brauer group of a Hopf algebra over a commutative ring to the level of a braided monoidal category. This permits one to get new relations that describe the quantum Brauer group of a Hopf algebra H over a field k. Let B be a Hopf algebra in $\mathcal{C} = {}^H_H\mathcal{Y}D$, the category of Yetter-Drinfel'd modules over H. We consider the quantum Brauer group $\mathrm{BQ}(\mathcal{C}; B)$ of B in $\mathcal{C}$, which is isomorphic to the usual quantum Brauer group $\mathrm{BQ}(k; B \rtimes H)$ of the Radford biproduct Hopf algebra $B \rtimes H$. We find that under a certain symmetricity condition on the braiding in $\mathcal{C}$, there is an inner action of the Hopf automorphism group of B on the former. We use this fact to generate a new subgroup of the quantum Brauer group for a family of Radford biproduct Hopf algebras $B \rtimes H$. Applying our recent results on the subgroup $\mathrm{BM}(k; B \rtimes H)$ - the Brauer group of module algebras over $B \rtimes H$, - we obtain new estimations of the respective quantum Brauer group. In particular, we get new information on the quantum Brauer group of some known Hopf algebras.

Compact coalgebras, daggers and Tannakian reconstruction

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From the viewpoint of Tannakian reconstruction, and with the idea of understanding the reconstruction of compact quantum groups, we consider the simple case of coalgebras. We consider the concept of compactness for o-coalgebras and characterize the notion of compactness in terms of the extension of the dagger functor from $Vect$ to the category of C-comodules.

This is joint work with Ignacio López Franco (University of Cambridge, England).

Clifford theory for graded fusion categories

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I will report on progress towards the classification of module categories over graded fusion categories. We develop a categorical analogue over graded fusion categories of Clifford theory for strongly graded rings. We describe module categories over a fusion category $\mathcal{C}$ graded by a group G as induced from module categories over fusion subcategories associated with the subgroups of G. We define invariant

$\mathcal{C}_e$-module categories and extensions of $\mathcal{C}_e$-module categories. The construction of module categories over $\mathcal{C}$ is reduced to determine invariant module categories for subgroups of G and the indecomposable extensions of this modules categories. We associate a G-crossed product fusion category to each G-invariant $\mathcal{C}_e$-module category and give a criterion for a graded fusion category to be a group-theoretical fusion category. We give necessary and sufficient conditions for an indecomposable module category to be extended.

On pointed Hopf algebras over dihedral groups

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Let k be an algebraically closed field of characteristic 0 and let D_m be the dihedral group of order $2m$ with $m = 4t$; $t \geq 3$. This talk will be based on a joint work with Fernando Fantino [2], where we classify all finite-dimensional Nichols algebras over D_m and all finite-dimensional pointed Hopf algebras whose group of group-likes is D_m, by means of the Lifting Method. As a byproduct we obtain new examples of finite-dimensional pointed Hopf algebras. In particular, we give an infinite family of non-abelian groups with non-trivial examples of pointed Hopf algebras over them and where the classification is completed. The difference with the case of the symmetric groups S_3 y S_4, see [1] and [3], respectively, is that each dihedral group provide an infinite family of new examples.

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Representations of the category of modules over pointed Hopf algebras over $\mathbb{S}_3$ and $\mathbb{S}_4$

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This is a joint work with Martín Mombelli. It will appear in Pacific Journal of Mathematics. A preprint is available at `arXiv:1006.1857v1[math.QA]`.

We will recall the basic results on module categories over finite-dimensional Hopf algebras [2], [4] and the classification of finite-dimensional Hopf algebras with coradical $\Bbbk\mathbb{S}_3$ or $\Bbbk\mathbb{S}_4$ from [1], [3], respectively.

Using these results, we will show that if $n = 3, 4$ and $\mathcal{M}$ is an exact indecomposable module category over $\mathrm{Rep}(\mathfrak{B}(X, q)\#\Bbbk\mathbb{S}_n)$, then there exist

- a subgroup $F < \mathbb{S}_n$ and a 2-cocycle $\psi \in Z^2(F, \Bbbk^\times)$,
- a subset $Y \subseteq X$ invariant under the action of F,
- a family of scalars $\{\xi_C\}$ *compatible* with (F, ψ, Y),

such that $\mathcal{M} \simeq {}_{\mathcal{B}(Y,F,\psi,\xi)}\mathcal{M}$, where $\mathcal{B}(Y, F, \psi, \xi)$ is a left $\mathfrak{B}(X, q)\#\Bbbk\mathbb{S}_n$-comodule algebra constructed from data (Y, F, ψ, ξ). We also show a classification of connected homogeneous left coideal subalgebras $\mathcal{B}(Y, F, \psi, \xi)$ of $\mathrm{gr}\, H$ and together with a presentation by generators and relations.

Finally we prove that if H is a finite-dimensional Hopf algebra with coradical $\Bbbk\mathbb{S}_3$ or $\Bbbk\mathbb{S}_4$ then H and $\mathrm{gr}\, H$ are cocycle deformations of each other, a result analogous to a theorem of Masuoka for abelian groups. This implies that there is a bijective correspondence between module categories over $\mathrm{Rep}(H)$ and $\mathrm{Rep}(\mathrm{gr}\, H)$.

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Weak factorizations

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We will look to a weak factorization theorem for monads in a general bicategory, with an eye on weak bialgebras. Most of the results are obtained in collaboration with Gabriella Böhm.

The Miyashita-Ulbrich action for weak Hopf algebras

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In this talk we show that for every weak Galois extension $B \hookrightarrow A$ associated to a weak Hopf algebra in a symmetric closed category with equalizers and coequalizers, the centralizer of B in A, denoted by $C_A(B)$, is a H-module algebra and a Yetter-Drinfeld module via the so-called Miyashita-Ulbrich action. We also prove that there exists a lax monoidal functor $C_{-}(B): {}_A\mathcal{M}_A \to \mathcal{M}_H$ with factorization through the category ${}_{C_A(B)}(\mathcal{M}_H)$ (i.e., the category of relative Hopf modules) and we obtain explicitly the form of the Miyashita-Ulbrich action when we work with weak H-cleft extensions. If the functor $-\otimes H$ preservers equalizers, we show that there exists a lax monoidal functor $YC_{-}(B): {}_A(\mathcal{M}^H)_A \to \mathcal{YD}_H^H$ generalizing the result obtained by Peter Schauenburg in [4]. As a consequence, if the antipode of H is an isomorphism, using the results proved in [2] we obtain that every weak H-Galois extension gives rise a non-trivial weak Yang-Baxter operator (see [2] and [1] for the definition and properties). Finally, we prove that $C_A(B)$ is a commutative algebra in $\mathcal{YD}_H^H$ and the functor $YC_{-}(B)$ factors through the category ${}_{C_A(B)}(\mathcal{YD}_H^H)$.

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On the duality of generalized Hopf and Lie algebras

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Michaelis [1] introduced the notion of a Lie coalgebra and proved [2] the following duality property for any Hopf algebra H:

$$P(H^\circ) \cong Q(H)^*.$$

Here P is the functor that assigns to every Hopf algebra the Lie algebra of primitive elements and Q is the dual functor, introduced by Michaelis, that assigns to every

Hopf algebra the Lie coalgebra of indecomposables. H° is the Sweedler dual of H and $(-)^*$ is our notation for the vector space dual, that in particular turns a Lie coalgebra structure into a Lie algebra structure.

Since the work of Michaelis, the notion of a Hopf algebra has known several generalizations and different kinds of dualities on the category of (generalized) Hopf algebras have been introduced, in particular to cover the apparent dualities in the theory of quantum groups. The aim of the work that is presented here, is to lift these dualities for Hopf algebras to the Lie algebra level, in the same spirit as Michaelis' theorem.

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Deformations of a class of graded Hopf algebras with quadratic relations

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We consider a special class of graded Hopf algebras, which are finitely generated quadratic algebras with anti-symmetric generating relations. We discuss the automorphism group and Calabi-Yau property of a PBW-deformation of such a Hopf algebra. We show that the Calabi-Yau property of a PBW-deformation of such a Hopf algebra is equivalent to that of the corresponding augmented PBW-deformation under some mild conditions.

Nichols algebras with many cubic relations

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The talk is based on a joint work with A. Lochmann and L. Vendramin. We classify Nichols algebras of irreducible Yetter-Drinfeld modules over groups under the assumption that the underlying rack is braided and the homogeneous component of degree three of the Nichols algebra satisfies a given inequality. This assumption turns out to be equivalent to a factorization assumption on the Hilbert series. Besides the known Nichols algebras, a new example is obtained. The proof is based on a combinatorial invariant of the Hurwitz orbits with respect to the action of the braid group on three strands.

Serial and co-Frobenius coalgebras, infinite abelian groups, and a class of quantum groups

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We present some recent classification results of co-Frobenius coalgebra structures on monomial subcoalgebras of path coalgebras, and of an associated class of quantum groups. These coalgebras are also serial coalgebras; we show how some of these results follow alternatively from a classification of left serial coalgebras. We answer several open questions regarding co-representations of such coalgebras, and show how methods of the theory of infinite abelian groups apply to this context. Finally, we look at a conjecture of Andruskiewitsch and Dăscălescu (which states that a Hopf algebra with nonzero integral must have finite coradical filtration); we examine this from the point of view of tensor categories but also from the perspective of such combinatorial examples of co-Frobenius coalgebras, and investigate the question of when an indecomposable co-Frobenius coalgebra must have finite coradical filtration.

Combinatorial representation of quantum groups and related problems

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This talk is based on the joint work with A.V. Lara Sagahon and J.L. Garza Rivera. We use a super computer KanBalam of the UNAM in order to find the total number r_n of the homogeneous right coideal subalgebras containing all group-like elements for the multiparameter versions of the quantum groups $U_q(\mathfrak{so}_{2n+1})$, $q^t \neq 1$ and $u_q(\mathfrak{so}_{2n+1})$, $q^t = 1$, $t > 4$ for small n :

$$r_2 = 38;\ r_3 = 546;\ r_4 = 10696;\ r_5 = 233216;$$

$$r_6 = 6257254;\ r_7 = 178413634.$$

The numerical experiments allow us to conjecture that $n!4^n < r_n < n!n4^n$ for big n. The similar numbers for $U_q(\mathfrak{sl}_{n+1})$ were found in [1]:

$$r_2 = 26;\ r_3 = 252;\ r_4 = 3368;\ r_5 = 58810;$$

$$r_6 = 1290930;\ r_7 = 34604844.$$

Additionally, in the present work, we get $r_8 = 1,107,490,596$. Recall that, in the G_2 case we have $r_2 = 60$; see [2]. For the other types, C, D, E, F, it is already known from a theorem of Heckenberger and Schneider [3] that the similar numbers r_n^{Borel} related to the Borel subalgebras coincide with the order of the corresponding Weyl group W. This implies $r_n < |W|^2$, see [4].

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The lazy cohomology of the Hopf algebra of functions on a finite group

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Lazy cohomology is a natural extension of Sweedler's cohomology to any Hopf algebra, not just cocommutative ones. I will report on joint work with Pierre Guillot (arXiv:0903.2807, published in IRMN 2010 no. 10, 1894–1939) in which we compute the lazy cohomology of the Hopf algebra of functions on a finite group G. The answer involves the group of class-preserving outer automorphisms of G as well as all abelian normal subgroups of G of central type. As a consequence, there exist Hopf algebras whose second lazy cohomology group is not abelian.

Hopf algebras with triality

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In this joint work with G. Benkart and J.M. Pérez-Izquierdo, we revisit and extend the constructions of Glauberman and Doro on groups with triality and Moufang loops to Hopf algebras. We prove that the universal enveloping algebra of any Lie algebra with triality is a Hopf algebra with triality. This allows a new construction of the universal enveloping algebras of Malcev algebras. Our work relays on the approach of Grishkov and Zavarnitsine to groups with triality.

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Generalized Hopf algebras by deforming identities

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The purpose of my talk is to summarize present recent developments and provide some key constructions of Hom-associative and Hom-Hopf algebraic structures. The main feature of Hom-algebras is that the classical identities are twisted by a homomorphism.

The Hom-Lie algebras arise naturally in discretizations and deformations of vector fields and differential calculus, to describe the structures on some q-deformations of the Witt and the Virasoro algebras. They were developed in a general framework by Larsson and Silvestrov. The Hom-associative algebras, Hom-coassociative coalgebras and Hom-Hopf algebras were introduced by Silvestrov and myself. Recently, the Hom-type algebras were intensively investigated. A categorical point of view were discussed by Caenepeel and Goyvaerts. Also Yau showed that the enveloping algebra of a Hom-Lie algebra may be endowed with an structure of Hom-bialgebra.

On module categories over graded fusion categories

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This talk is based on a joint work with Evgeny Musicantov. A graded fusion category $\mathcal{D}$ can be thought of as an extension of a given (base) fusion category $\mathcal{C}$ by some finite group G. Etingof, Nikshych and Ostrik gave a cohomological description, in terms of obstructions and solutions, to all the possible extensions of $\mathcal{C}$ by G.

In this talk I will describe a classification of module categories over graded fusion categories in terms of module categories over the base fusion category and the extension data of the category itself. This classification will also be via cohomology, by considering certain obstructions and their solutions. As a result, I will describe the module categories over the Tambara-Yamagami fusion categories.

On functors which fail to be monadic

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A relevant result concerning monads is the so called Beck's monadicity (tripleability) theorem which characterizes right adjoint functors R which are monadic, i.e., such that the Eilenberg-Moore category of algebras (over the canonical monad associated to the adjunction) is equivalent, through the so-called comparison functor, to the domain category of R. In this talk we investigate those right adjoint functors R which fail to be monadic and measure how far they are to fulfil monadicity. To this aim we propose the definition of comparable functor. The obtained results are tested on a series of examples which also involve (braided) Lie theory and Module theory. This is part of a joint research with A. Ardizzoni (University of Ferrara) and J. Gómez-Torrecillas (University of Granada).

Unified products for Hopf algebras

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Let A be a Hopf algebra and H a coalgebra. We shall describe and classify up to an isomorphism all Hopf algebras E that factorize through A and H: that is, E is a Hopf algebra such that A is a Hopf subalgebra of E, H is a subcoalgebra in E with $1_E \in H$ and the multiplication map $A \otimes H \to E$ is bijective. The tool we use is a new product, we call it the unified product, in the construction of which A and H are connected by three coalgebra maps: two actions and a generalized cocycle. Both the crossed product of a Hopf algebra acting on an algebra and the bicrossed product of two Hopf algebras are special cases of the unified product. A Hopf algebra E factorizes through A and H if and only if E is isomorphic to a unified product of A and H. All such Hopf algebras E are classified up to an isomorphism that stabilizes A and H by a Schreier type classification theorem. An equivalent description of the unified product from the extension of Hopf algebras point of view is given. A necessary and sufficient condition for the canonical morphism $i : A \to A \ltimes H$ to be a split monomorphism of Hopf algebras is proved, i.e., a condition for the unified product $A \ltimes H$ to be isomorphic to a Radford biproduct $L * A$, for some bialgebra L in the category ${}_A^A\mathcal{YD}$ of Yetter-Drinfel'd modules.

Joint work with A. L. Agore.

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Recent results on Frobenius-Schur indicators for Hopf algebras

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Let H be a semisimple Hopf algebra over $\mathbb{C}$, and let V be an irreducible representation of H. It is known that for each integer n, $1 \leq n \leq Exp(H)$, one may define $\nu_n(V)$, the n^{th} Frobenius-Schur indicator of V, generalizing the facts for representations of finite groups. The indicators are a useful invariant for the category of representations, as they are gauge invariants, and have had nice applications.

Although for $H = \mathbb{C}G$, all values of $\nu_n(V)$ are integers, this is not true in general although they must lie in the ring of n^{th} cyclotomic integers (as is shown by Kashina-Sommerhäuser-Zhu). It was hoped that for nice examples, such as $H = D(G)$, the Drinfel'd double, the values of $\nu_n(V)$ would still be integers.

Recently this has been shown to be true for $D(G)$ in many examples, such as when G is a dihedral group or a "regular" p-group. Computations with GAP show that it is also true for groups with "small" exponent, but that it is false for a group of order 5^6.

We will survey some of these results, due variously to Rebecca Courter, Mio Iovanov, Marc Keilberg, Geoff Mason, Richard Siu-hung Ng, and the speaker.

On fusion categories with few irreducible degrees

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In this talk we shall discuss some results on the structure of a fusion category $\mathcal{C}$ after imposing certain restrictions on the set $c.d.(\mathcal{C})$ of Frobenius-Perron dimensions of its simple objects. The fusion categories that we consider are all integral. In particular, we consider the case where $c.d.(\mathcal{C}) = 1, p$, with p a prime number. We shall treat mostly questions regarding nilpotency and solvability, in the sense introduced by Etingof, Gelaki, Nikshych and Ostrik. The talk is based on joint work with J. Plavnik.

Congruence property and Galois Symmetry of modular categories

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The natural representation of $SL(2, \mathbb{Z})$ associated to a Rational Conformal Field Theory (RCFT) has been conjectured, by Eholzer, to be t-rational and have a congruence kernel. It is further conjectured by Coste and Gannon a Galois symmetry of this representation. Some of these conjectures have been proved mathematically in the context of modular categories via the machinery called generalized Frobenius-Schur indicators. In this talk, I will report recent progress of these conjectures for modular categories.

The Picard crossed module of a braided tensor category

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Let $\mathcal{C}$ be a braided tensor category (e.g., the representation category of a finite dimensional Hopf algebra). One associates to $\mathcal{C}$ two groups: the group G of braided autoequivalences of $\mathcal{C}$ and the group P of invertible $\mathcal{C}$-module categories. The pair (G, P) forms a crossed module (also known as a categorical group). We discuss the structure of this crossed module and explain how it is used in the classification of fusion categories.

This is a report on joint works in progress with A. Davydov and with V. Drinfeld, S. Gelaki, and V. Ostrik.

On the Hennings invariant of a finite-dimensional factorizable Hopf Algebra

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The Hennings invariant of 3-manifolds is defined on the double $D(H)$ of a finite-dimensional Hopf algebra H over a field k in many cases. When H is factorizable, $D(H) = H \otimes H$ as algebras. We explore the implications this equation has for computing the invariant. This work is joint with David De Wit and Louis Kauffman.

Towards Sweedler's cohomology for weak Hopf algebras

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Let H be a cocommutative weak Hopf algebra and B a H-module algebra. We extend to this context some of the notions that arise in the study of Sweedler's cohomology. In particular, we define the concept of 2-cocycle and of cohomologous 2-cocycles. We use this notion to classify weak crossed products of B and H.

We also study H-extensions of B, and we obtain that a normal 2-cocycle induces a weak cleft H-extension of B. As a consequence of the classification of weak crossed products via cohomologous 2-cocycles, we finally obtain that cohomologous 2-cocycles induce isomorphic weak cleft H-extensions.

Localizing braided fusion categories

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In this talk I will introduce and discuss a physically-inspired notion of "localization" for braided fusion categories (BFC), which is reminiscent of fiber functors for fusion categories. Given an object X in a BFC one asks when the associated braid group representations can be "realized" via a braided vector space (R, V) in a certain precise sense (localized). Perhaps surprisingly, integrality of the BFC is not necessary for localizability. Time permitting I will describe an application to quantum computing (joint work with Zhenghan Wang) and some generalized types of localization (joint work with César Galindo and Seung-Moon Hong).

Hom-Hopf algebras and tensor categories

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In this talk new results linking the constructions of twisting products to the theory of Hom-algebras, Hom-coalgebras and Hom-Hopf algebras will be presented. Hom-Hopf algebras, introduced by Makhlouf and Silvestrov in 2007, had been put into broad context of tensor (monoidal) categories in 2009 by Goyvaerts and Caenepeel. In this talk I will review these and other advances in the theory of Hom-Hopf algebras as well as some examples and open problems.

Bilinear forms, Eilenberg-MacLane cocycles, and the central charge

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In joint work with Yongchang Zhu, we recently established that for a semisimple factorizable Hopf algebra, the value of an integral on the Drinfel'd element and the value of this integral on the inverse Drinfel'd element differ only by a fourth root of unity. If the dimension is odd, they only differ by a sign, and this sign is a plus sign if the dimension is one modulo four, but a minus sign if the dimension is three modulo four.

We conjecture that these two integral values always differ only by a sign, even if the dimension is not odd. However, as we explain in the talk, the analogous result for quasi-Hopf algebras is false. This can be seen in a class of quasi-Hopf algebras that are constructed from so-called Eilenberg-MacLane cocycles. If these quasi-Hopf algebras are ordinary Hopf algebras, these Eilenberg-MacLane cocycles are simply bilinear forms. In this case, the conjecture is in fact correct, as we also explain in the talk.

Galois theory for Hopf-Galois extensions

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For a comodule algebra over a Hopf algebra over an arbitrary commutative ring we construct a Galois correspondence between the complete lattices of of subalgebras and the complete lattice of generalised quotients of the structure Hopf algebra. The construction involves techniques of lattice theory and of Galois connections. Such a 'Galois Theory' generalises the classical Galois theory for field extensions, and some important results of S. Chase and M. Sweedler, F. Van Oystaeyen, Y.H. Zhang and P. Schauenburg.

The Grosshans principle for Hopf algebras and the quantum Weitzenböck theorem

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Let k be an algebraically closed field. Given an affine variety W (over k), we denote by $k[W]$ the algebra of regular functions on W. Let G be an affine algebraic group (over k) and let H be a closed subgroup of G. The well-known Grosshans principle says that if G acts rationally on an algebra A, then the algebra of invariants A^H is isomorphic to the algebra $(k[G]^H \otimes A)^G$, where the action of H on the algebra A is given by $(hf)(g) = f(gh)$ for $f \in k[G]$, $h \in H$ and $g \in G$. This principle implies that if G is reductive and the algebra $k[G]^H$ is finitely generated, then the algebra A^H is finitely generated, provided so is A. One of the consequences of this fact is the following classical result.

Theorem (Weitzenböck, 1932). Suppose that $char(k) = 0$ and the additive group $G_a = (k, +)$ acts rationally on a finite dimensional vector space W. Then the algebra of invariants $k[W]^{G_a}$ is finitely generated.

The main objective of my talk is to show that Grosshans's principle works naturally for Hopf algebras. As an application we present a quantum version of the Weitzenböck theorem.

On the duality of generalized Hopf and Lie algebras

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Michaelis [1] introduced the notion of a Lie coalgebra and proved [2] the following duality property for any Hopf algebra H:

$$P(H^\circ) \cong Q(H)^*.$$

Here P is the functor that assigns to every Hopf algebra the Lie algebra of primitive elements and Q is the dual functor, introduced by Michaelis, that assigns to every Hopf algebra the Lie coalgebra of indecomposables. H° is the Sweedler dual of H and $(-)^*$ is our notation for the vector space dual, that in particular turns a Lie coalgebra structure into a Lie algebra structure.

Since the work of Michaelis, the notion of a Hopf algebra has known several generalizations and different kinds of dualities on the category of (generalized) Hopf algebras have been introduced, in particular to cover the apparent dualities in the theory of quantum groups. The aim of the work that is presented here, is to lift these dualities for Hopf algebras to the Lie algebra level, in the same spirit as Michaelis' theorem.

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Conjugacy classes and class sums for Hopf algebras

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We extend the notion of conjugacy classes and class sums from finite groups to semisimple Hopf algebras and show that the conjugacy classes are obtained from the factorization of H as an irreducible left $D(H)$-module. For quasitriangular semisimple Hopf algebras H we prove that the product of two class sums is an integral combination of the class sums up to $1/d^2$ where $d = dim(H)$. We show also that in this case the character table is obtained from the S-matrix associated to $D(H)$.

The Green ring of Taft algebras

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This talk is based on a joint work with Huixiang Chen and F. Van Oystaeyen.

We study the Green ring of Taft algebras $H_n(q)$, where n is a positive integer with $n \geqslant 2$, and q is an n-th root of unity. We first discuss the representations of Taft algebras. There are n^2 non-isomorphic finite dimensional indecomposable modules over $H_n(q)$, and all of them are uniserial. For each $1 \leqslant l \leqslant n$, there are n finite dimensional indecomposable $H_n(q)$-modules $M(l, r)$, $r \in \mathbb{Z}_n$, up to isomorphism. We show that every indecomposable projective $H_n(q)$-module is of dimension n. It turns out that the Green ring $r(H_n(q))$ of the Taft algebra $H_n(q)$ is a commutative ring generated by two elements subject to certain relations. Concrete examples for $n = 2, 3, \ldots, 8$ are given.