DOMINATION AND ROMAN DOMINATION IN SOME PRODUCT GRAPHS

ISMAEL GONZALEZ YERO

Departamento de Matemáticas,
Universidad de Cádiz - Escuela Politécnica Superior de Algeciras,
Av. Ramón Puyol s/n, 11202 Algeciras, España
E-mail: ismael.gonzalez@uca.es

Joint work with D. Kuziak and J. A. Rodríguez-Velázquez

Index

Index

Index

Index

Index

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Domination in graphs

- $G = (V, E)$, a simple graph. $S \subset V$, set of vertices of G.
- S is a dominating set if $N(S) = V$, *i.e.*, every vertex $v \in \overline{S}$ is adjacent to a vertex of S.
- $\gamma(G)$, domination number of G: minimum cardinality of any dominating set in G.

Domination related parameters

- Domination plus conditions on vertices of the dominating set or its complement: Total domination, connected domination, independent domination, etc.
- Conditions over the style of domination: k-domination, distance domination, etc.
- Dominating functions: Roman domination, signed domination, minus domination, etc.

Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0, 1, 2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0, 1, 2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0, 1, 2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0, 1, 2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V \,:\, f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0, 1, 2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0, 1, 2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V : f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

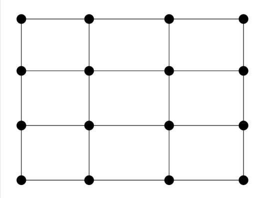
Roman domination

- A map $f : V \to \{0,1,2\}$, Roman dominating function for G, if for every $v \in V$ with $f(v) = 0$ there exists $u \in N(v)$ such that $f(u) = 2$.
- The weight of f if $f(V) = \sum_{v \in V} f(v)$.
- $\gamma_R(G)$, Roman domination number of G: minimum weight of any Roman dominating function for G.
- Every Roman dominating function induces three sets B_0, B_1, B_2 such that $B_i = \{v \in V \,:\, f(v) = i\}$, $i \in \{0,1,2\}$.

- $\gamma(G) \leq \gamma_R(G) \leq 2\gamma(G)$
- $\gamma(G) = \gamma_R(G)$ if and only if $G = \overline{K_n}$.
- G is called a *Roman graph* if $\gamma_R(G) = 2\gamma(G)$.
- There is an open problem related to characterizing all Roman graphs (Roman trees are characterized).

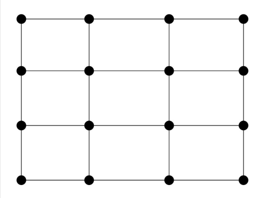
Product graphs

Cartesian product graphs, $G \square H$

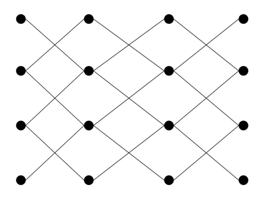


Product graphs

Cartesian product graphs, $G\Box H$

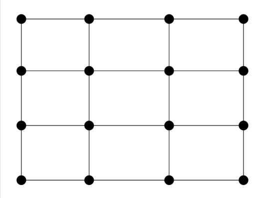


Direct product graphs, $G \times H$

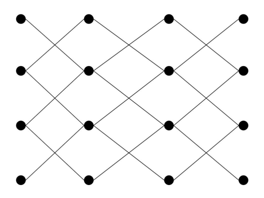


Product graphs

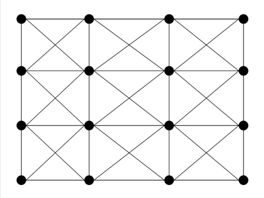
Cartesian product graphs, $G\Box H$



Direct product graphs, $G \times H$

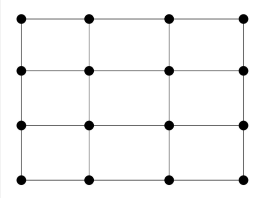


Strong product graphs, $G \boxtimes H$

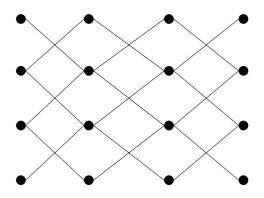


Product graphs

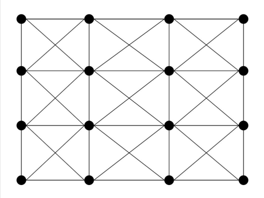
Cartesian product graphs, $G \square H$



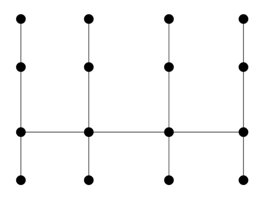
Direct product graphs, $G \times H$



Strong product graphs, $G \boxtimes H$



Rooted product graphs, $G \circ H$



Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G\square H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\square H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\square H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\square H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G \square H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G \square H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G \square H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G \square H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

- One of the most important problems about domination in graphs: $\gamma(G \Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G \Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G \Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G \Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

Domination versus product graphs

Vizing's conjecture

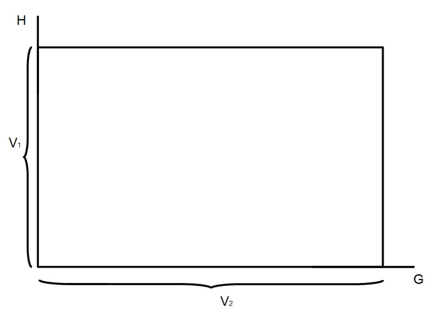
- One of the most important problems about domination in graphs: $\gamma(G\Box H) \geq \gamma(G)\gamma(H)$.
- Several Vizing-like results for other domination (also not domination related) parameters.
- $\Gamma(G\Box H) \geq \Gamma(G)\Gamma(H)$, $\gamma(G \times H) \leq 3\gamma(G)\gamma(H)$, $\gamma(G \boxtimes H) \leq \gamma(G)\gamma(H)$, etc.
- The best approximation to Vizing's conjecture: $2\gamma(G\Box H) \geq \gamma(G)\gamma(H)$ (Clark and Suen).

Roman domination

- $\gamma_R(G\Box H) \geq \gamma(G)\gamma(H)$.
- There were no more results in this topic.
- So, we did it.

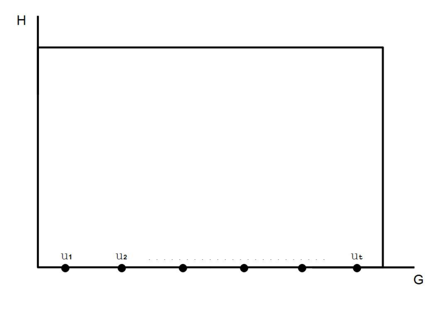
Index

A general bound



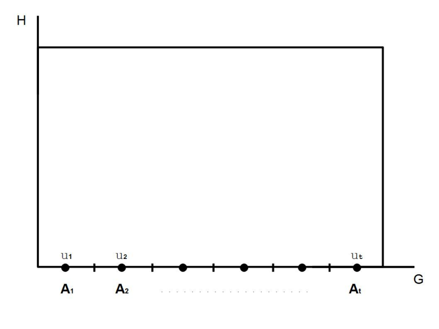
- V_1 and V_2, the vertex sets of G and H, respectively.

A general bound



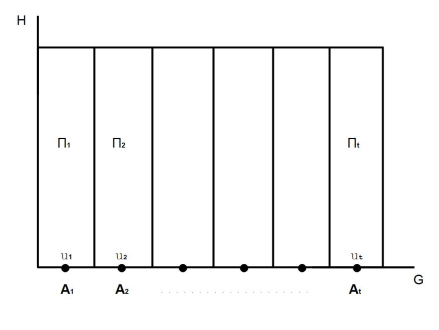
- V_1 and V_2, the vertex sets of G and H, respectively.
- $S = \{u_1, ..., u_t\}$,a dominating set for G, $t = \gamma(G)$.

A general bound



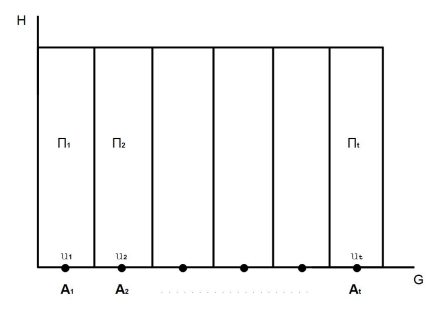
- V_1 and V_2, the vertex sets of G and H, respectively.
- $S = \{u_1, ..., u_t\}$,a dominating set for G, $t = \gamma(G)$.
- $\Pi = \{A_1, A_2, ..., A_{\gamma(G)}\}$, a vertex partition of G such that $u_i \in A_i$ and $A_i \subseteq N[u_i]$

A general bound



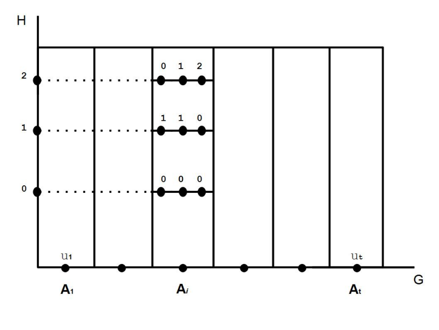
- $\{\Pi_1, \Pi_2, ..., \Pi_{\gamma(G)}\}$, a vertex partition of $G \Box H$, such that $\Pi_i = A_i \times V_2$ for every $i \in \{1, ..., \gamma(G)\}$

A general bound



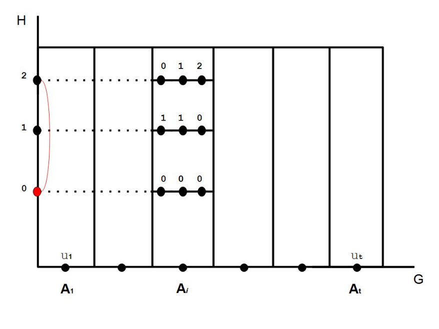
- $\{\Pi_1, \Pi_2, ..., \Pi_{\gamma(G)}\}$, a vertex partition of $G\Box H$, such that $\Pi_i = A_i \times V_2$ for every $i \in \{1, ..., \gamma(G)\}$
- $f = (B_0, B_1, B_2)$, a $\gamma_R(G\Box H)$-function

A general bound



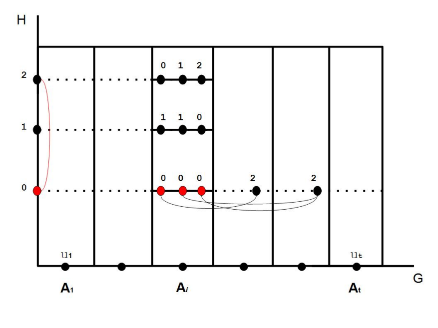
- $\{\Pi_1, \Pi_2, ..., \Pi_{\gamma(G)}\}$, a vertex partition of $G\Box H$, such that $\Pi_i = A_i \times V_2$ for every $i \in \{1, ..., \gamma(G)\}$
- $f = (B_0, B_1, B_2)$, a $\gamma_R(G\Box H)$-function
- For every $i \in \{1, ..., \gamma(G)\}$, $f_i : V_2 \to \{0, 1, 2\}$, a function on H such that $f_i(v) = \text{máx}\{f(u, v) \ : \ u \in A_i\}$.

A general bound



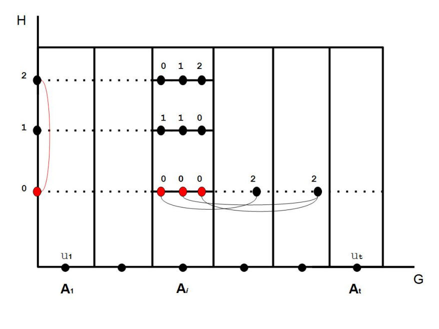
- $f_i = (X_0^{(i)}, X_1^{(i)}, X_2^{(i)})$, not a Roman dominating function for H, there is a vertex $v \in \overline{X_0^{(i)}}$, $N(v) \cap X_2^{(i)} = \emptyset$.

A general bound



- $f_i = (X_0^{(i)}, X_1^{(i)}, X_2^{(i)})$, not a Roman dominating function for H, there is a vertex $v \in \overline{X_0^{(i)}}$, $N(v) \cap X_2^{(i)} = \emptyset$.

A general bound



- $f_i = (X_0^{(i)}, X_1^{(i)}, X_2^{(i)})$, not a Roman dominating function for H, there is a vertex $v \in \overline{X_0^{(i)}}$, $N(v) \cap X_2^{(i)} = \emptyset$.
- For every $u \in A_i$, (u, v) is adjacent to some vertex not in Π_i

A general bound

- For every $i \in \{1, ..., \gamma(G)\}$, we count the number of vertices of H satisfying that they are not adjacent to any vertex with label two (2).

A general bound

- For every $i \in \{1, ..., \gamma(G)\}$, we count the number of vertices of H satisfying that they are not adjacent to any vertex with label two (2).
- For every vertex v of H we count the G-cells satisfying that all their vertices are not adjacent to any vertex with label two (2) in the same "column".

A general bound

- For every $i \in \{1, ..., \gamma(G)\}$, we count the number of vertices of H satisfying that they are not adjacent to any vertex with label two (2).
- For every vertex v of H we count the G-cells satisfying that all their vertices are not adjacent to any vertex with label two (2) in the same "column".
- By doing a double sum we get that

$$\gamma_R(G \square H) \geq \frac{2}{3}\gamma(G)\gamma_R(H)$$

The general bound

For any graphs G and H,

$$\gamma_R(G \square H) \geq \frac{2}{3}\gamma(G)\gamma_R(H).$$

The general bound

For any graphs G and H,

$$\gamma_R(G \square H) \geq \frac{2}{3}\gamma(G)\gamma_R(H).$$

If $\gamma_R(H) > \frac{3\gamma(H)}{2}$, then

$$\gamma(G \square H) \geq \frac{\gamma(G)\gamma(H)}{2} + \frac{\gamma(G)}{3}.$$

The general bound

For any graphs G and H,

$$\gamma_R(G\Box H) \geq \frac{2}{3}\gamma(G)\gamma_R(H).$$

If $\gamma_R(H) > \frac{3\gamma(H)}{2}$, then

$$\gamma(G\Box H) \geq \frac{\gamma(G)\gamma(H)}{2} + \frac{\gamma(G)}{3}.$$

For any graph G and any Roman graph H,

- $\gamma_R(G\Box H) \geq \frac{4}{3}\gamma(G)\gamma(H)$.
- $\gamma(G\Box H) \geq \frac{2}{3}\gamma(G)\gamma(H)$.

Index

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,

$$\gamma_R(G \boxtimes H) \leq \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,

$$\gamma_R(G \boxtimes H) \leq \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Idea of the proof

f on $G \boxtimes H$ defined as

$$f(u,v) = \begin{cases} 2, & (u,v) \in (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2), \\ 1, & (u,v) \in A_1 \times B_1, \\ 0, & \text{otherwise.} \end{cases}$$

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,
$$\gamma_R(G \boxtimes H) \le \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Idea of the proof

f on $G \boxtimes H$ defined as

$$f(u,v) = \begin{cases} 2, & (u,v) \in (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2), \\ 1, & (u,v) \in A_1 \times B_1, \\ 0, & \text{otherwise.} \end{cases}$$

- $(A_0 \times B_0) \cup (A_0 \times B_2) \cup (A_2 \times B_0)$ is dominated by $A_2 \times B_2$,

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,

$$\gamma_R(G \boxtimes H) \leq \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Idea of the proof

f on $G \boxtimes H$ defined as

$$f(u,v) = \begin{cases} 2, & (u,v) \in (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2), \\ 1, & (u,v) \in A_1 \times B_1, \\ 0, & \text{otherwise.} \end{cases}$$

- $(A_0 \times B_0) \cup (A_0 \times B_2) \cup (A_2 \times B_0)$ is dominated by $A_2 \times B_2$,
- $A_1 \times B_0$ is dominated by $A_1 \times B_2$ and

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,
$$\gamma_R(G \boxtimes H) \leq \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Idea of the proof

f on $G \boxtimes H$ defined as

$$f(u,v) = \begin{cases} 2, & (u,v) \in (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2), \\ 1, & (u,v) \in A_1 \times B_1, \\ 0, & \text{otherwise.} \end{cases}$$

- $(A_0 \times B_0) \cup (A_0 \times B_2) \cup (A_2 \times B_0)$ is dominated by $A_2 \times B_2$,
- $A_1 \times B_0$ is dominated by $A_1 \times B_2$ and
- $A_0 \times B_1$ is dominated by $A_2 \times B_1$.

Roman domination

- $f_1 = (A_0, A_1, A_2)$, $\gamma_R(G)$-function. $f_2 = (B_0, B_1, B_2)$, $\gamma_R(H)$-function. Then,

$$\gamma_R(G \boxtimes H) \leq \gamma_R(G)\gamma_R(H) - 2|A_2||B_2|.$$

Idea of the proof

f on $G \boxtimes H$ defined as

$$f(u,v) = \begin{cases} 2, & (u,v) \in (A_1 \times B_2) \cup (A_2 \times B_1) \cup (A_2 \times B_2), \\ 1, & (u,v) \in A_1 \times B_1, \\ 0, & \text{otherwise.} \end{cases}$$

- $(A_0 \times B_0) \cup (A_0 \times B_2) \cup (A_2 \times B_0)$ is dominated by $A_2 \times B_2$,
- $A_1 \times B_0$ is dominated by $A_1 \times B_2$ and
- $A_0 \times B_1$ is dominated by $A_2 \times B_1$.
- f is a Roman dominating function on $G \boxtimes H$.

Index

Domination

- G, graph of order $n \geq 2$. H, graph with root v and at least two vertices. If v does not belong to any $\gamma(H)$-set or v belongs to every $\gamma(H)$-set, then
$$\gamma(G \circ H) = n\gamma(H).$$

Domination

- G, graph of order $n \geq 2$. H, graph with root v and at least two vertices. If v does not belong to any $\gamma(H)$-set or v belongs to every $\gamma(H)$-set, then
$$\gamma(G \circ H) = n\gamma(H).$$

- G, graph of order $n \geq 2$. Then for any graph H with root v and at least two vertices,
$$\gamma(G \circ H) \in \{n\gamma(H), n(\gamma(H) - 1) + \gamma(G)\}.$$

Roman domination

- G, graph of order $n \geq 2$. Then for any graph H with root v and at least two vertices,

$$n(\gamma_R(H) - 1) + \gamma(G) \leq \gamma_R(G \circ H) \leq n\gamma_R(H).$$

Roman domination

- G, graph of order $n \geq 2$. Then for any graph H with root v and at least two vertices,

$$n(\gamma_R(H) - 1) + \gamma(G) \leq \gamma_R(G \circ H) \leq n\gamma_R(H).$$

Tightness of the bounds

- If for every $\gamma_R(H)$-function $f = (B_0, B_1, B_2)$ is satisfied that $f(v) = 0$, then

$$\gamma_R(G \circ H) = n\gamma_R(H).$$

Roman domination

- G, graph of order $n \geq 2$. Then for any graph H with root v and at least two vertices,

$$n(\gamma_R(H) - 1) + \gamma(G) \leq \gamma_R(G \circ H) \leq n\gamma_R(H).$$

Tightness of the bounds

- If for every $\gamma_R(H)$-function $f = (B_0, B_1, B_2)$ is satisfied that $f(v) = 0$, then

$$\gamma_R(G \circ H) = n\gamma_R(H).$$

- If there exist two $\gamma_R(H)$-functions $h = (B_0, B_1, B_2)$ and $h' = (B_0', B_1', B_2')$ such that $h(v) = 1$ and $h'(v) = 2$, then

$$\gamma_R(G \circ H) = n(\gamma_R(H) - 1) + \gamma(G).$$

- G, graph of order $n \geq 2$ and H, graph with root v and at least two vertices.

- G, graph of order $n \geq 2$ and H, graph with root v and at least two vertices.
- If for every $\gamma_R(H)$-function f is satisfied that $f(v) = 1$, then

$$\gamma_R(G \circ H) = n(\gamma_R(H) - 1) + \gamma_R(G).$$

THANKS!!!