Metric Dimension, Upper Dimension and Resolving Number of Graphs

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University of Seville

joint work with D. Garijo and A. Márquez

VIII Jornadas de Matemática Discreta y Algorítmica **Almería, 11-13 de Julio de 2012**

Resolving Sets and Metric Dimension

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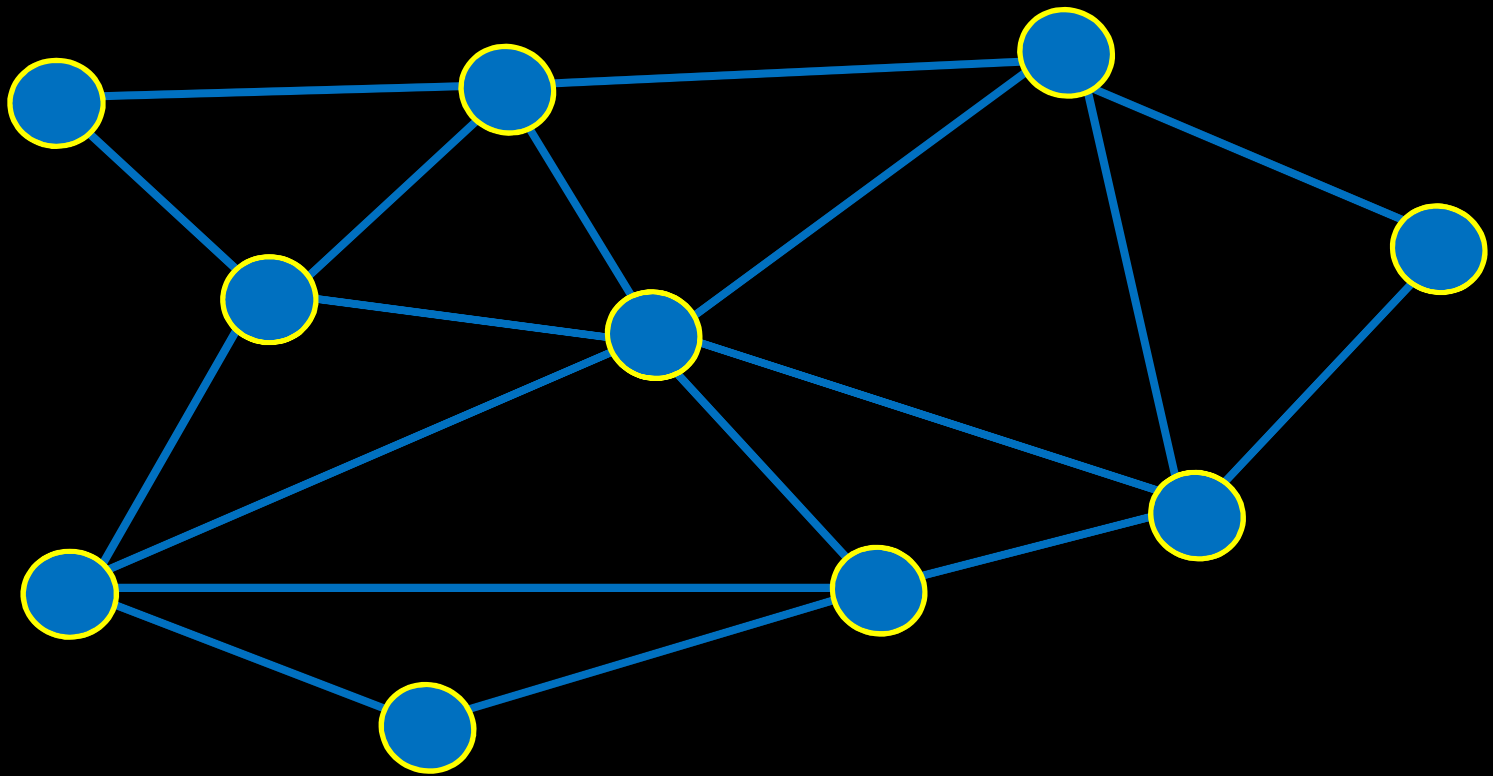

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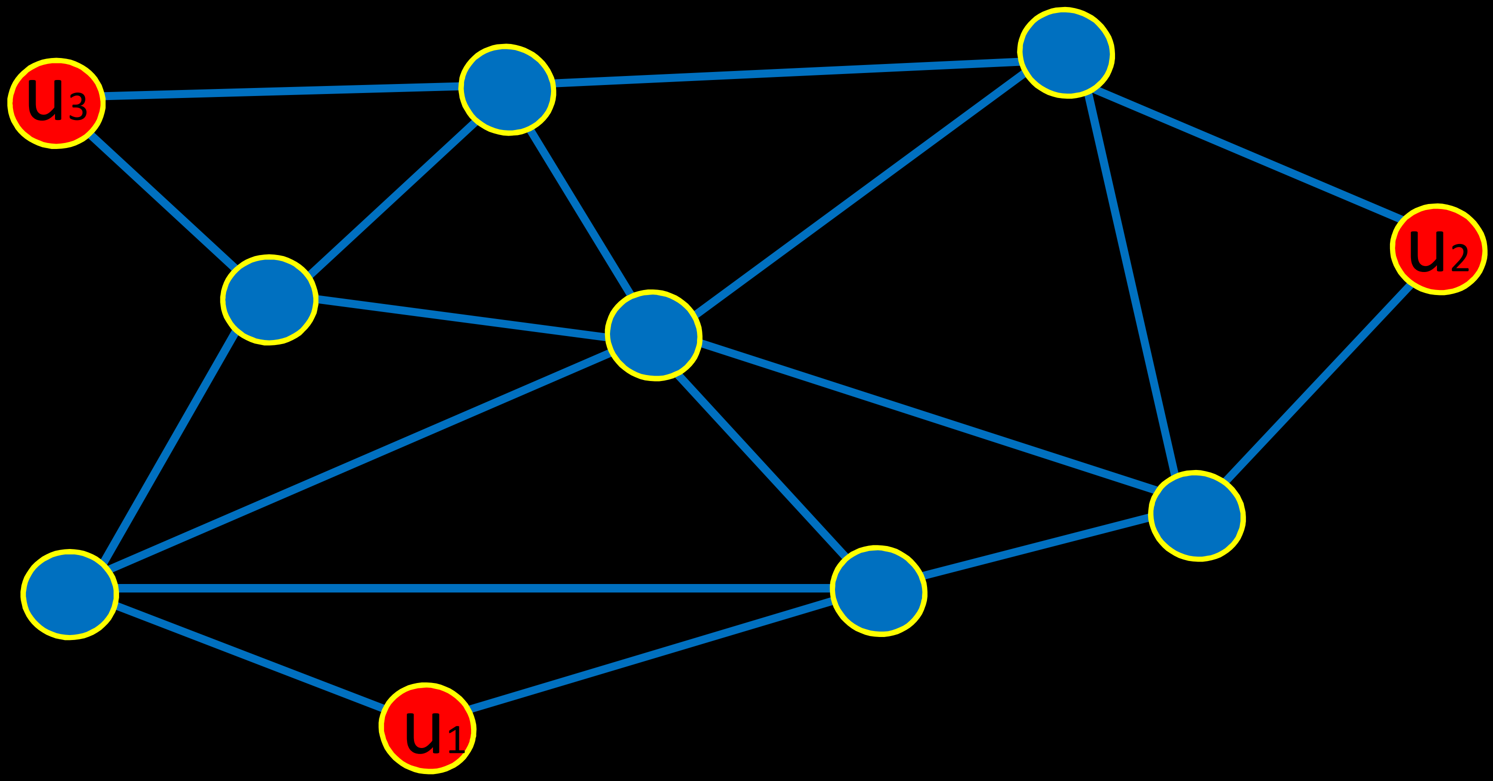

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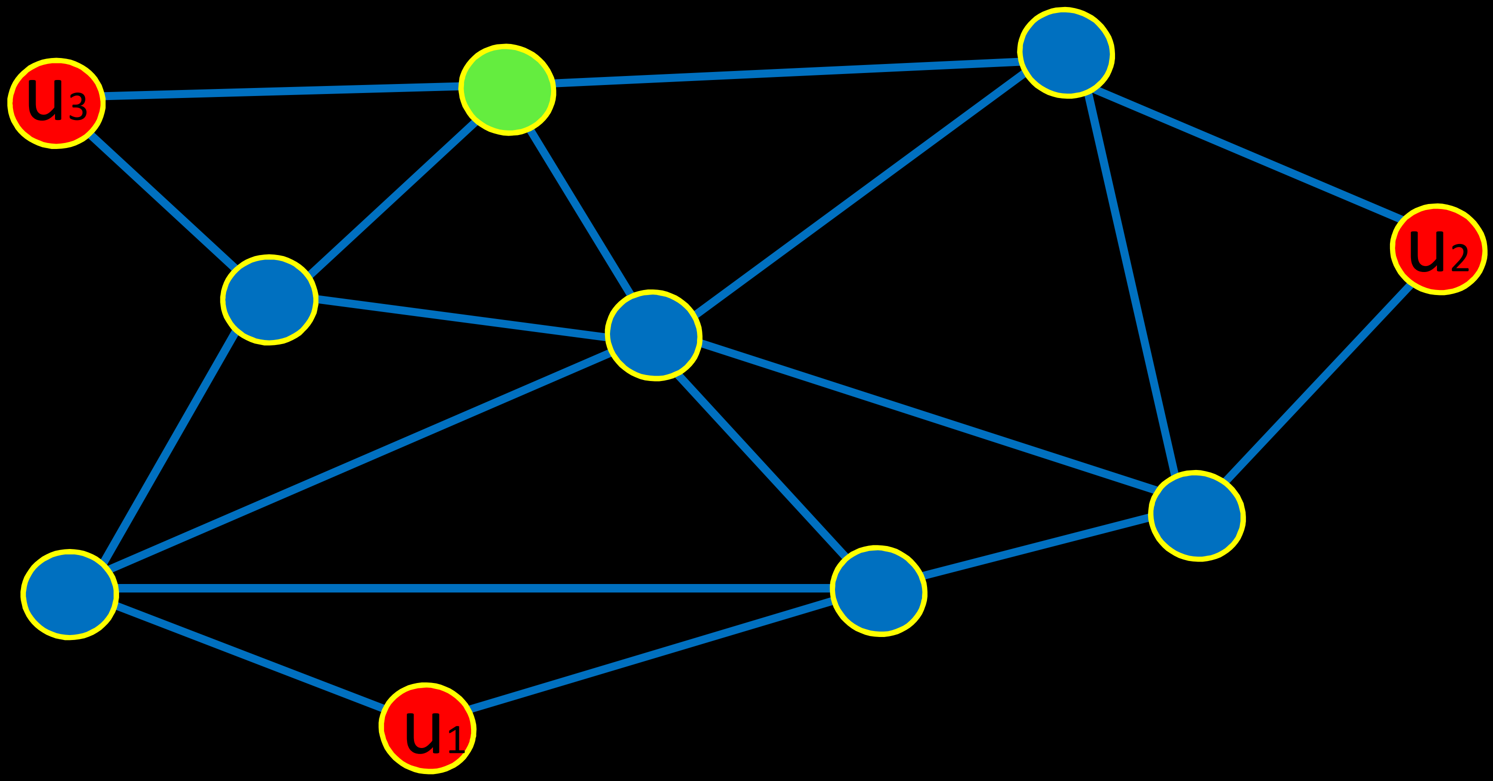

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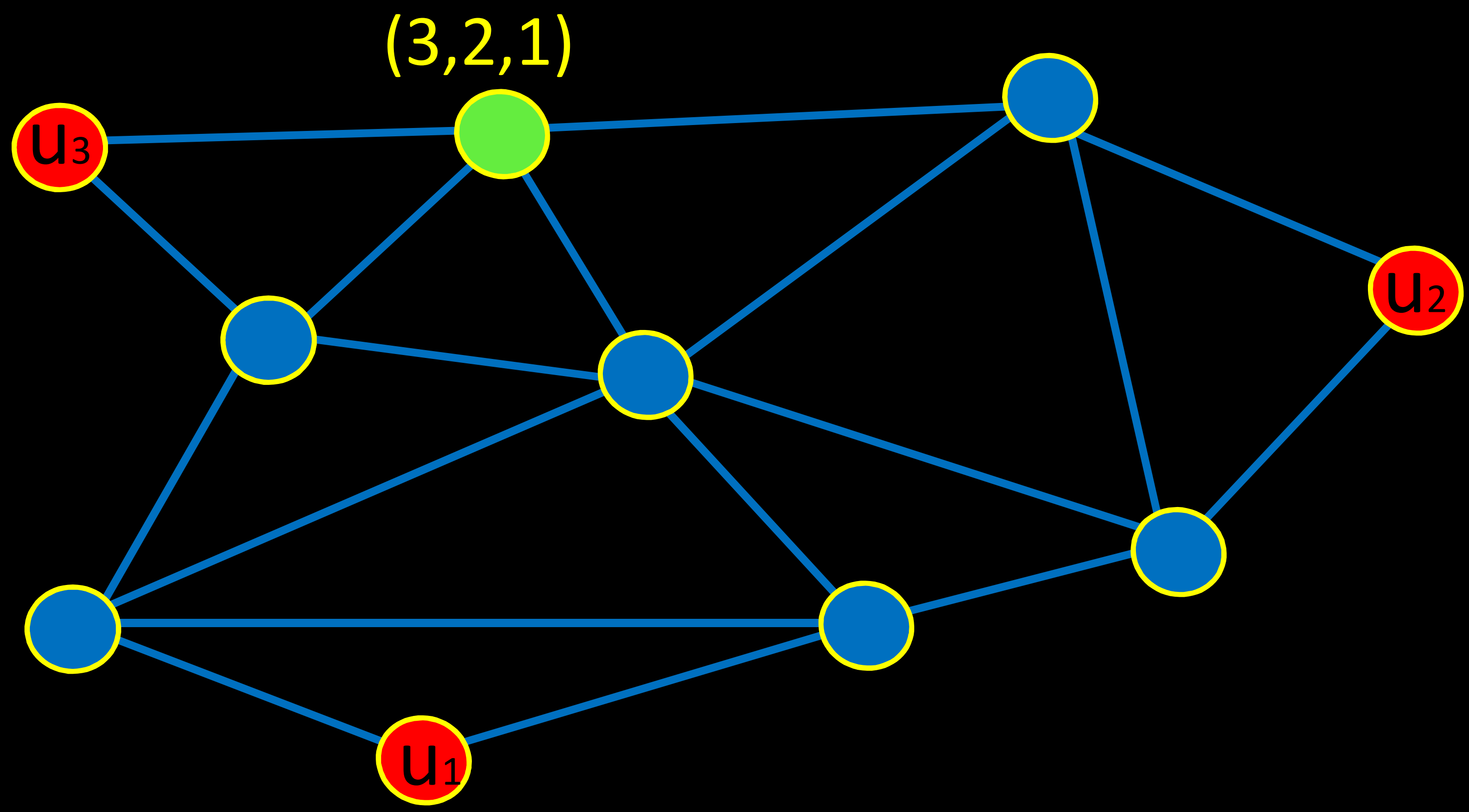

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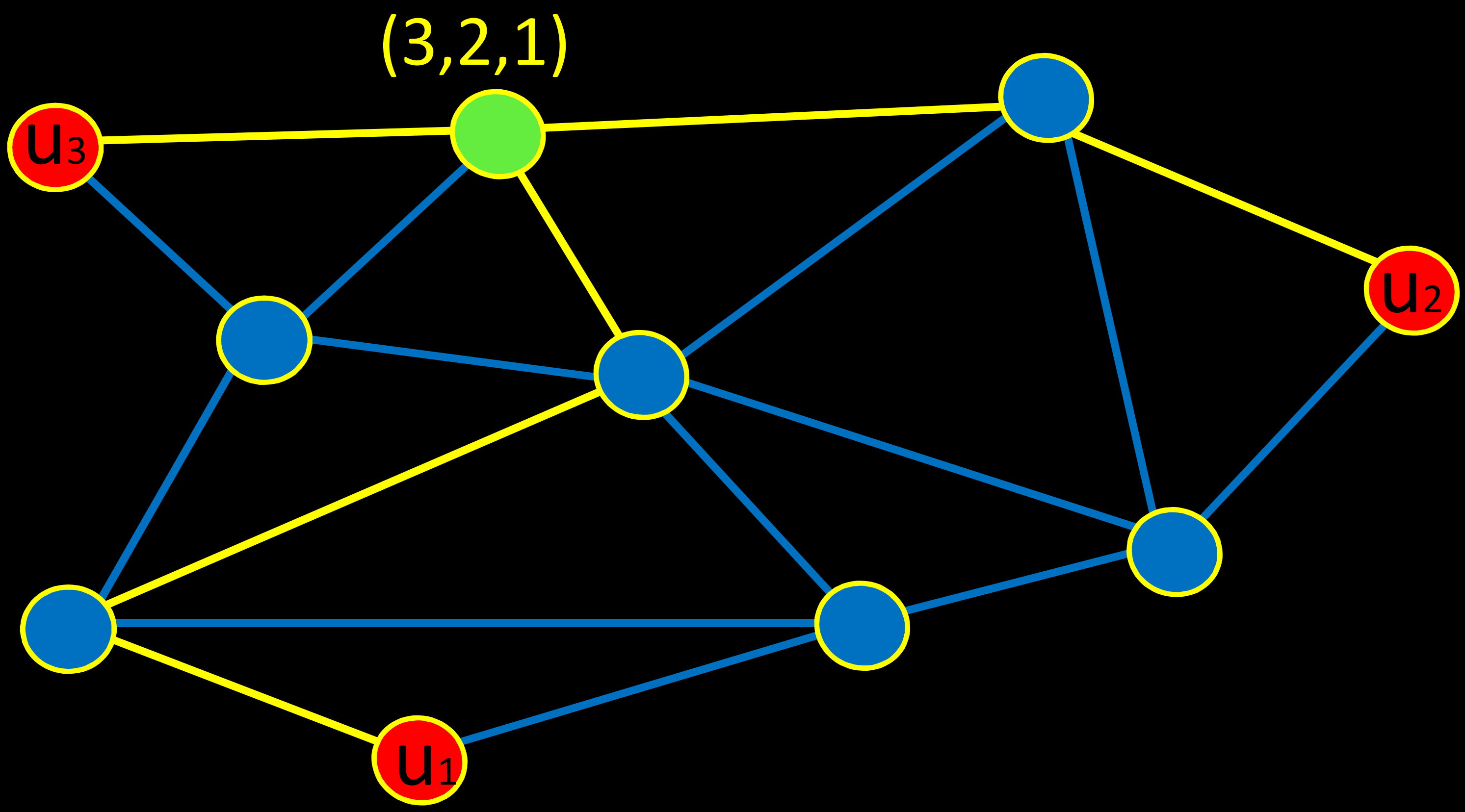

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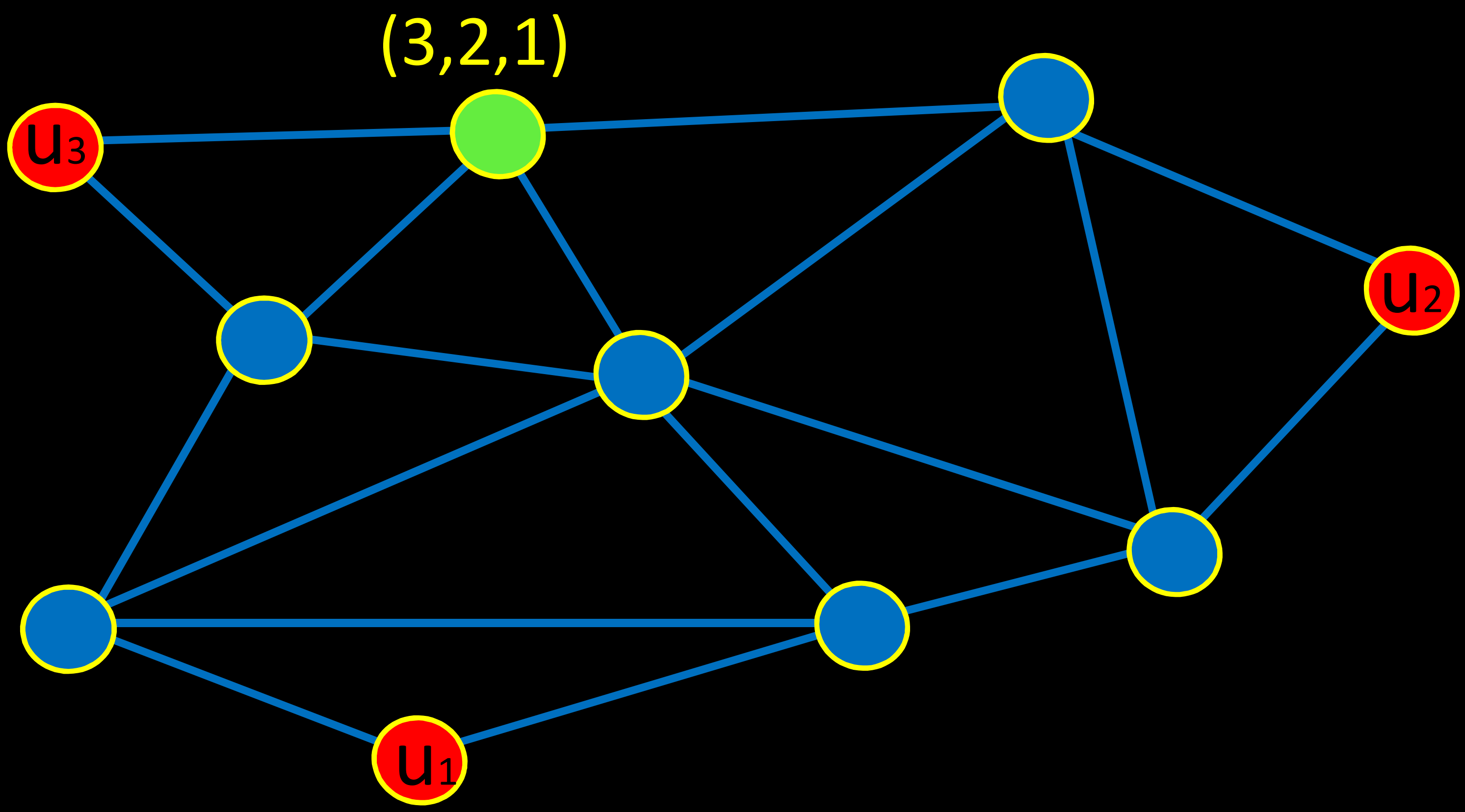

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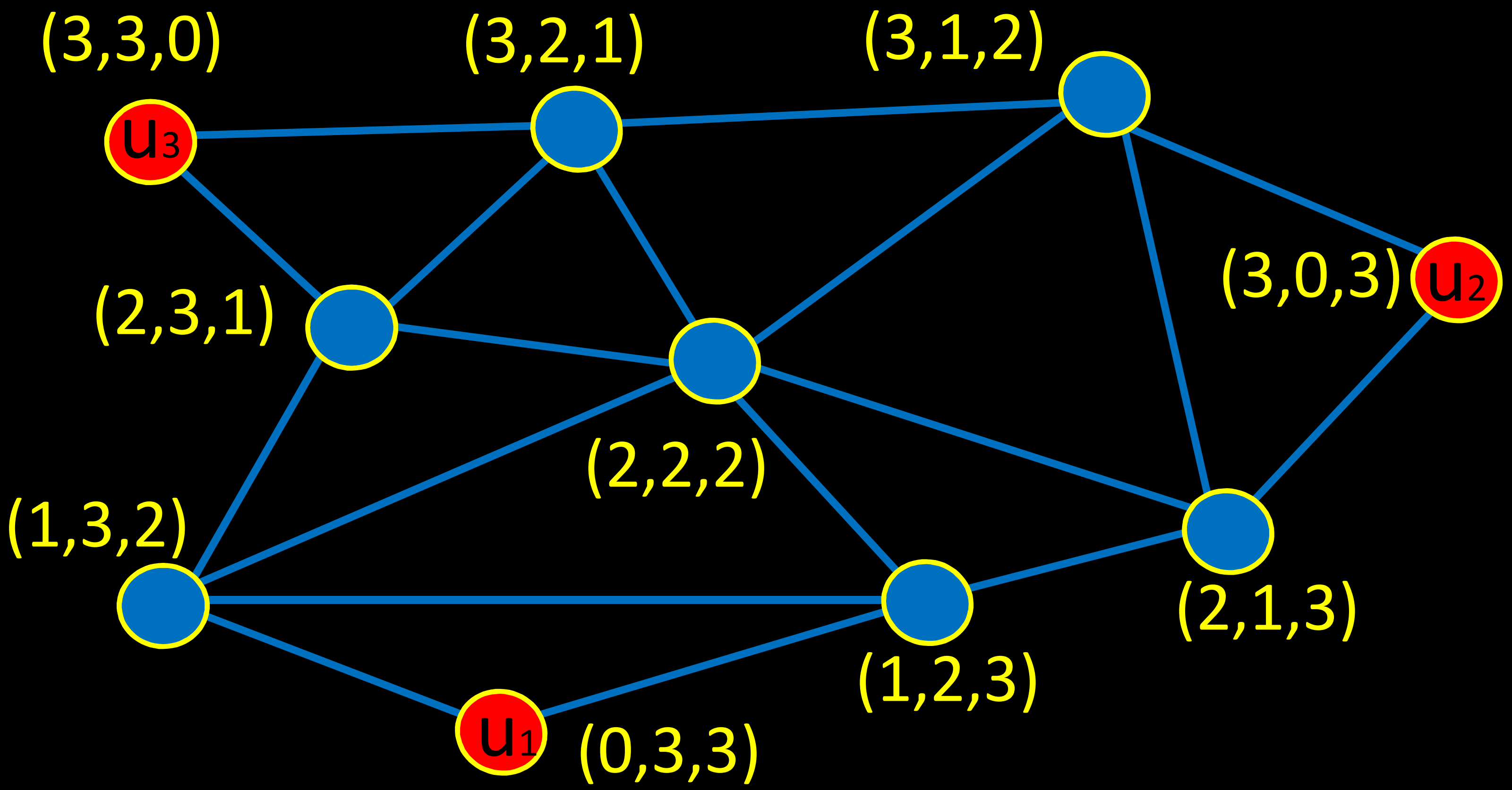

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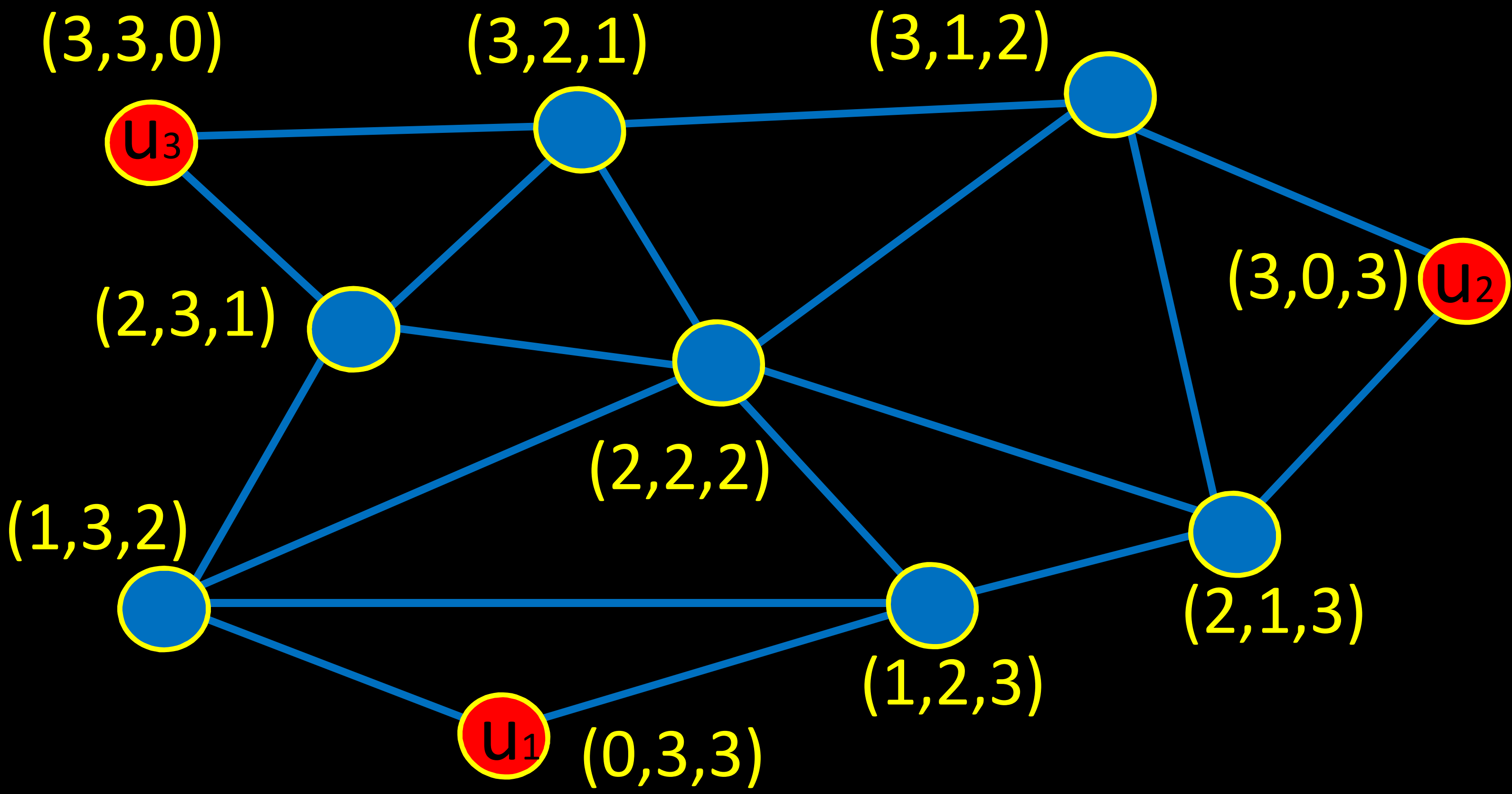

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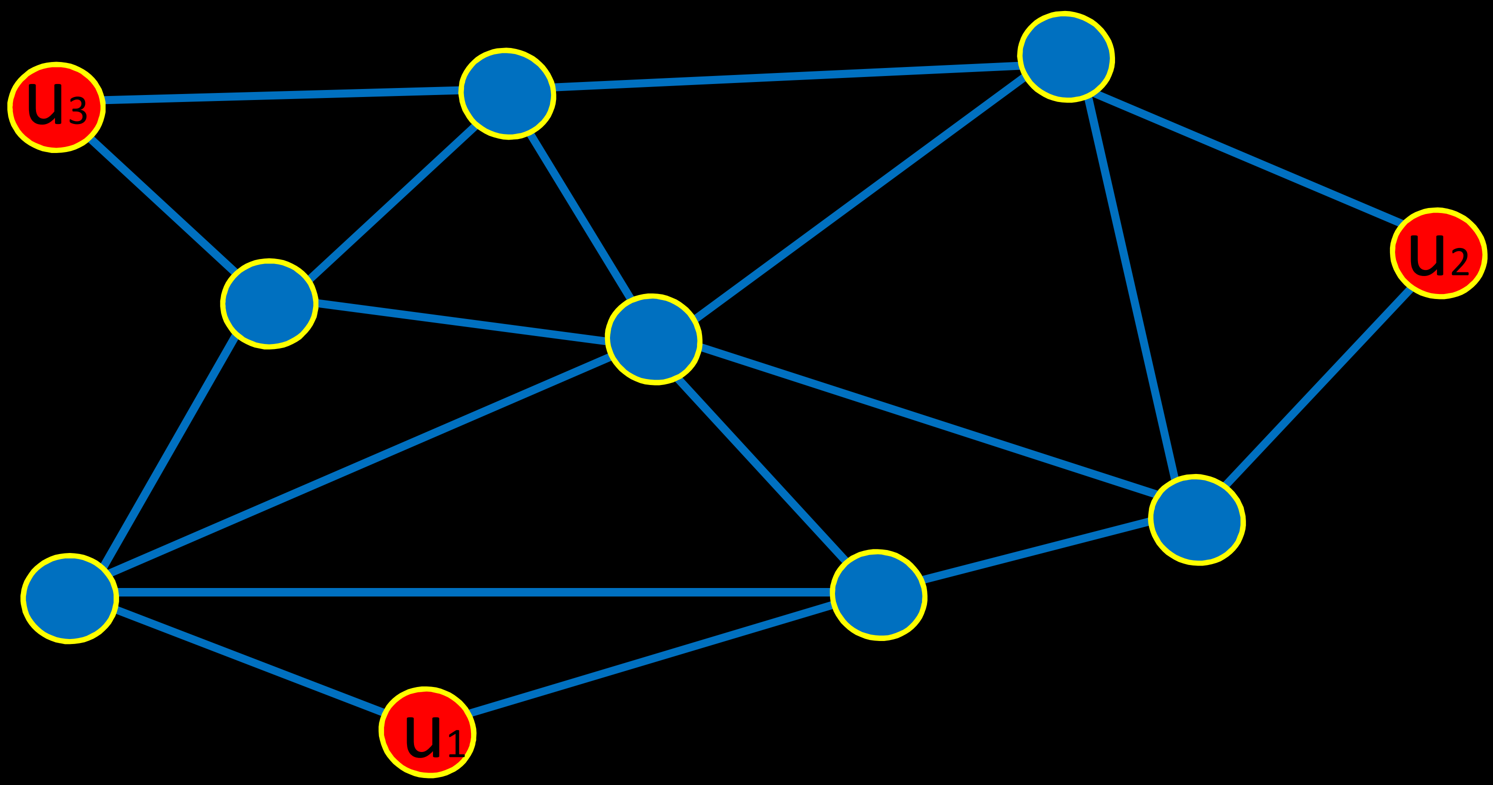

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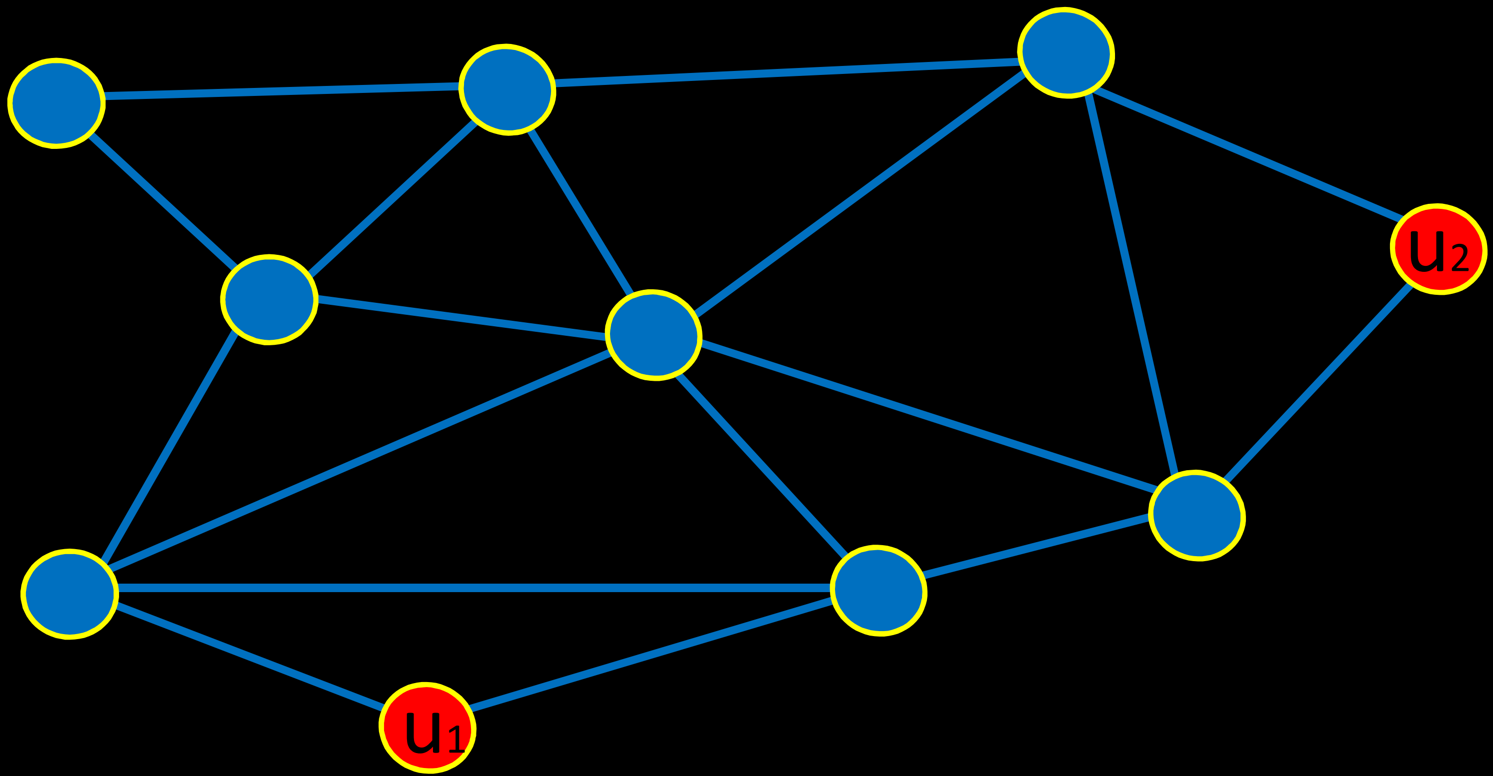

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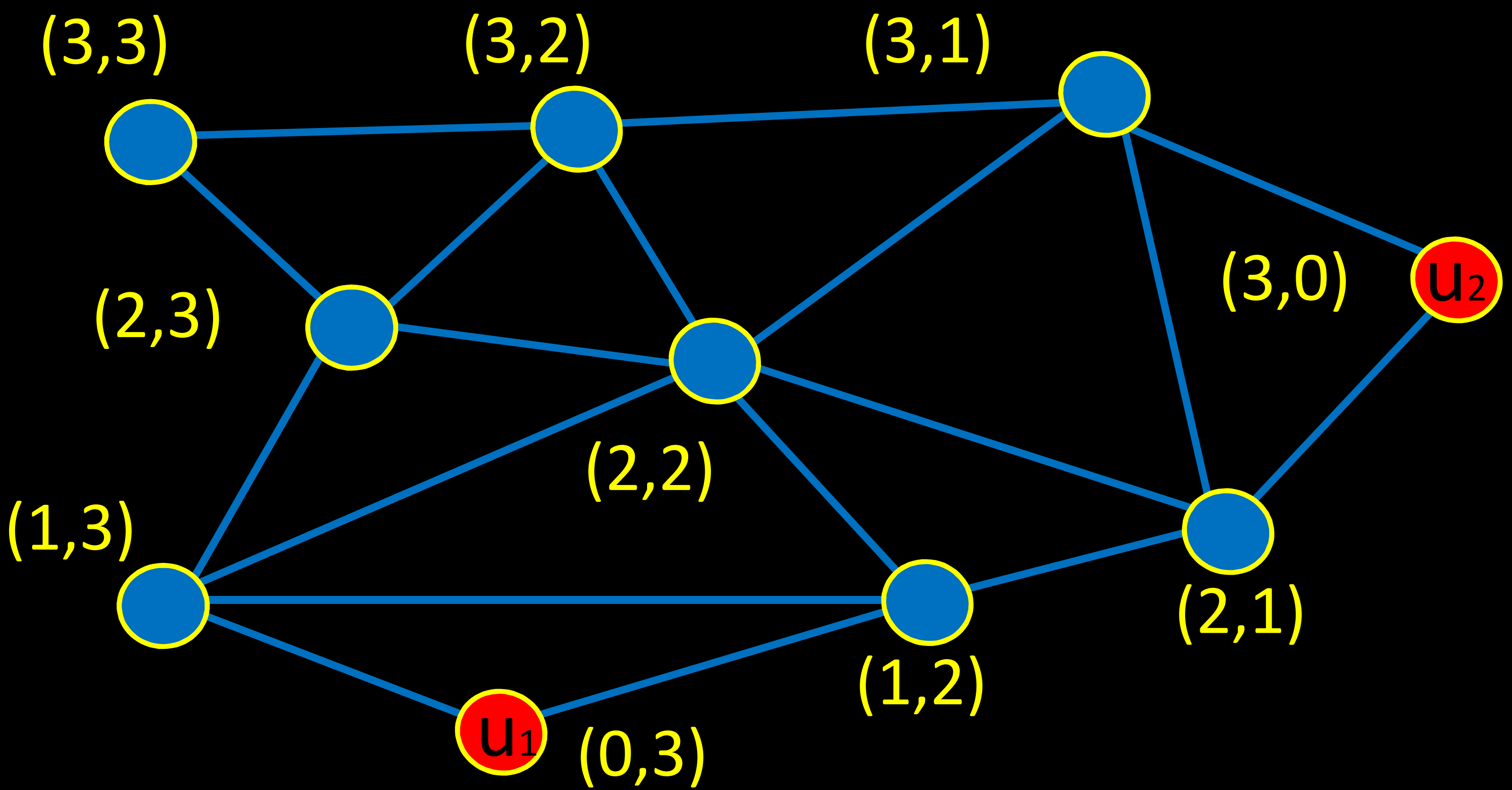

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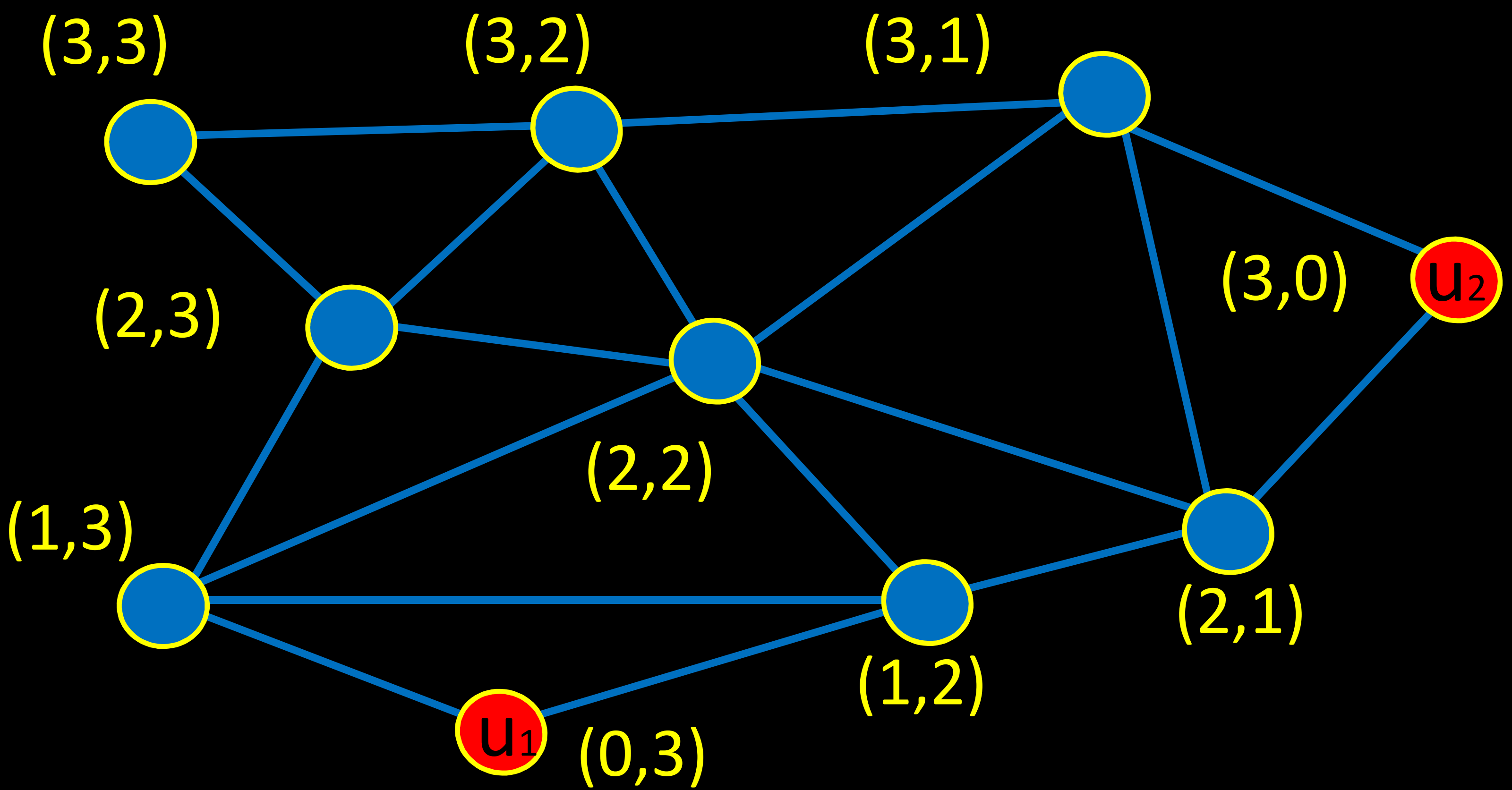
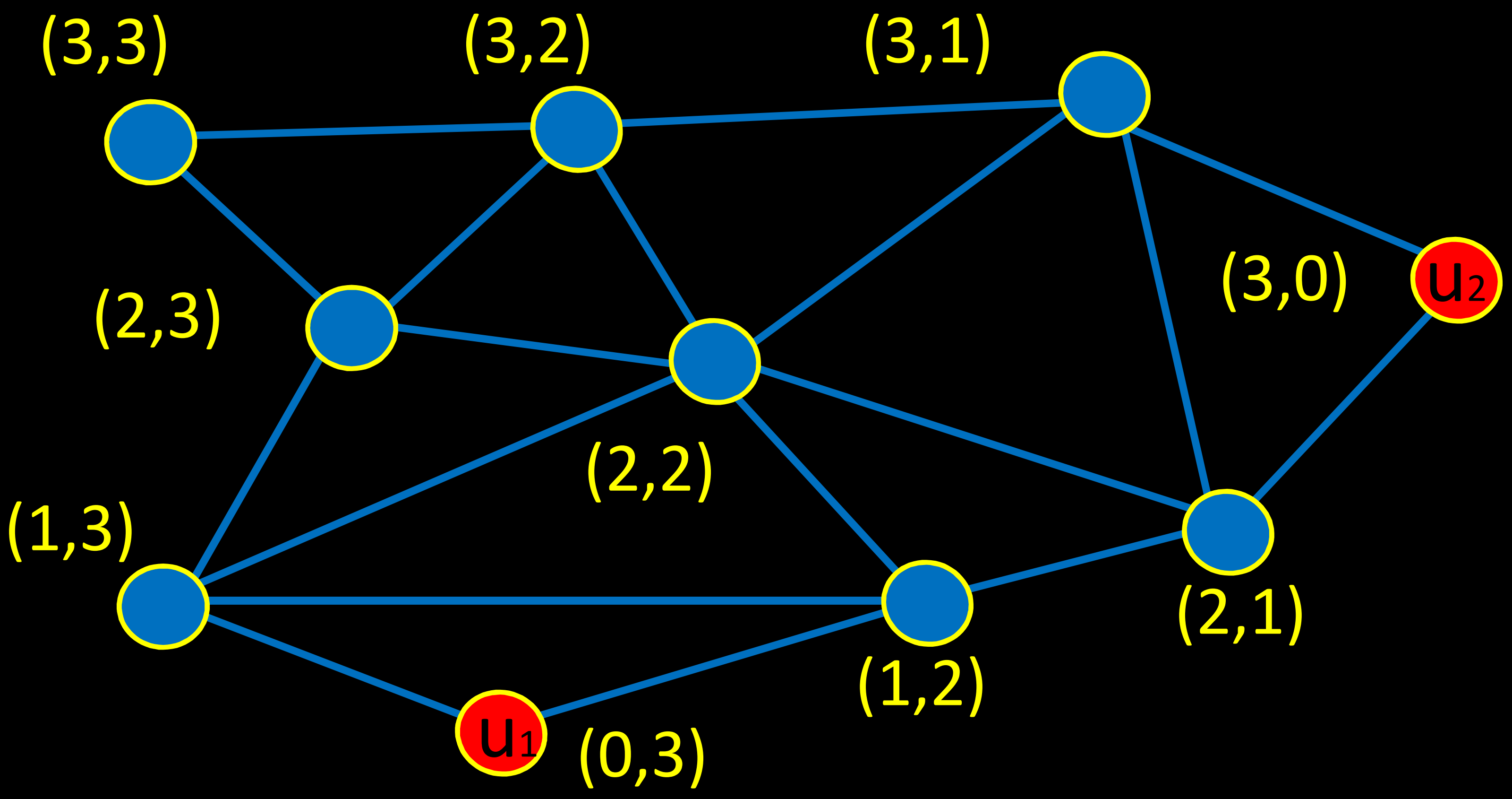


Resolving Sets and Metric Dimension

dim(G) = size of a minimum resolving set



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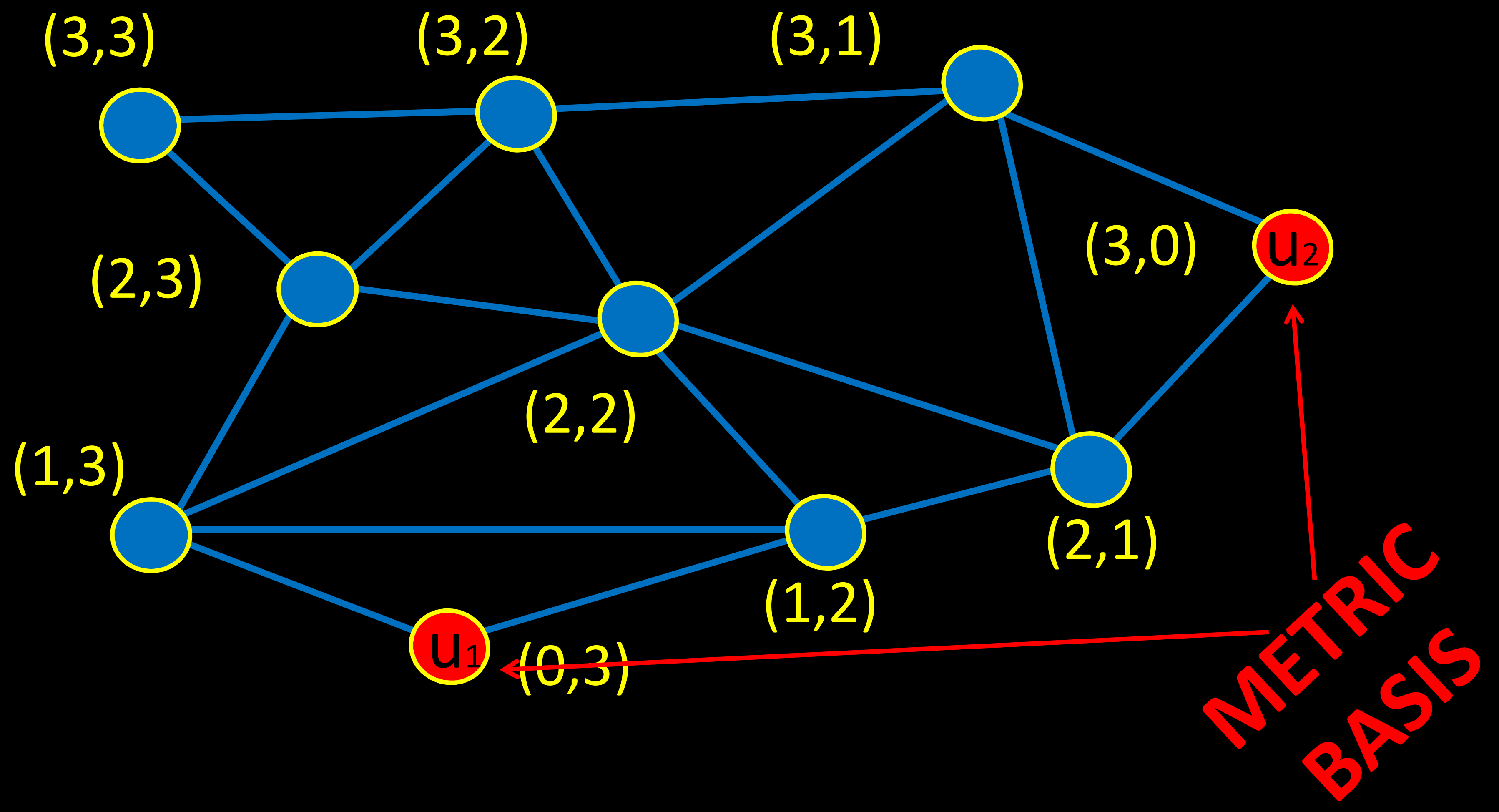


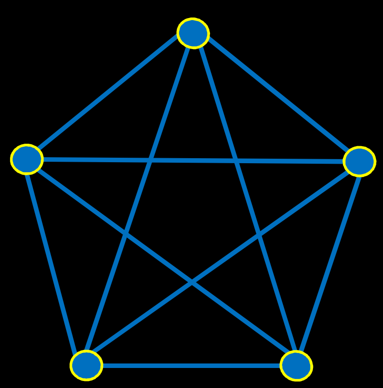
Resolving Sets and Metric Dimension

$\dim(G)$ = size of a minimum resolving set

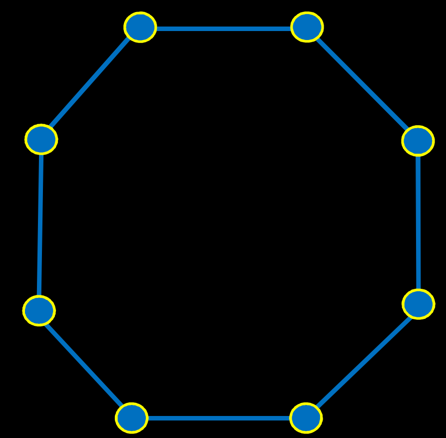
Resolving Sets and Metric Dimension

$\dim(G)$ = size of a minimum resolving set

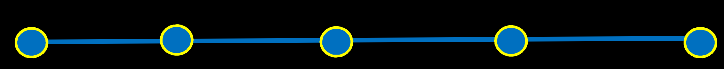
$\dim(K_n) = n-1$



$\dim(C_n) = 2$



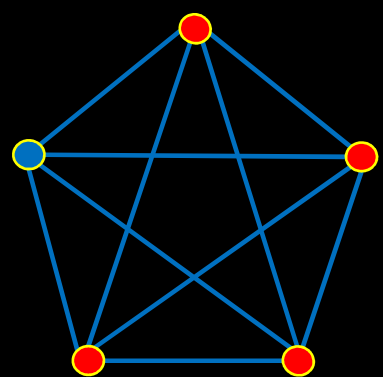
$\dim(P_n) = 1$



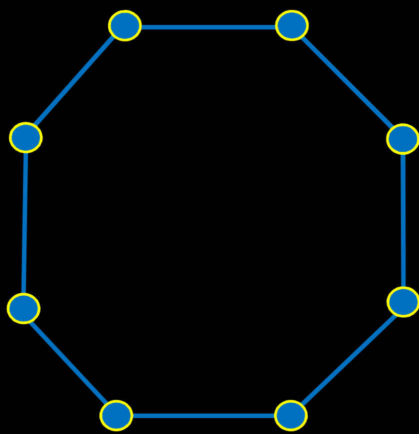
Resolving Sets and Metric Dimension

dim(G) = size of a minimum resolving set

dim(K_n) = n-1



dim(C_n) = 2



dim(P_n) = 1

Resolving Sets and Metric Dimension

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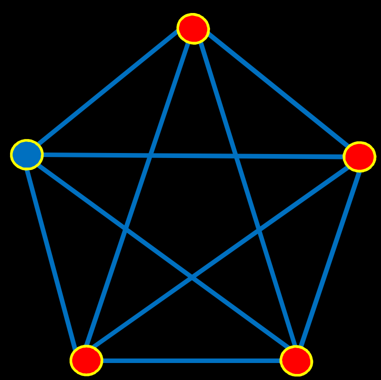

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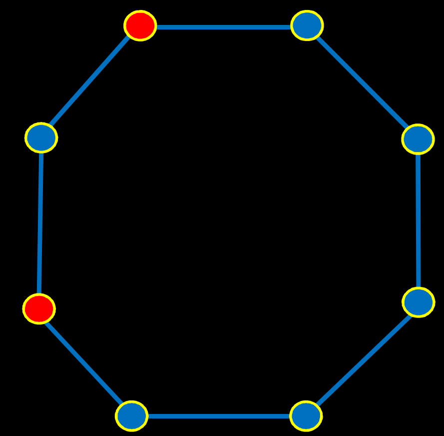

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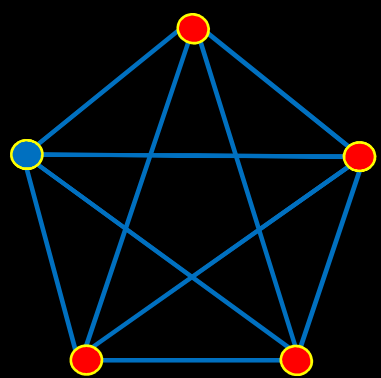

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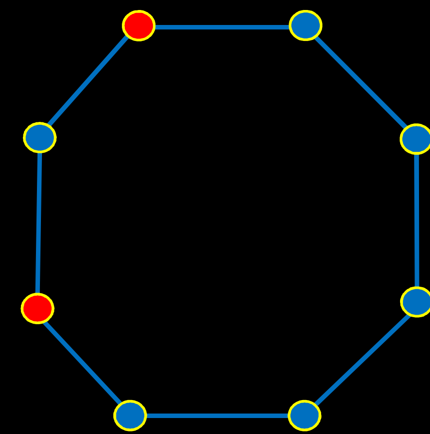

dim(P_n) = 1

Upper Dimension and Resolving Number

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$\dim^+(G)$= maximum size of a minimal resolving set

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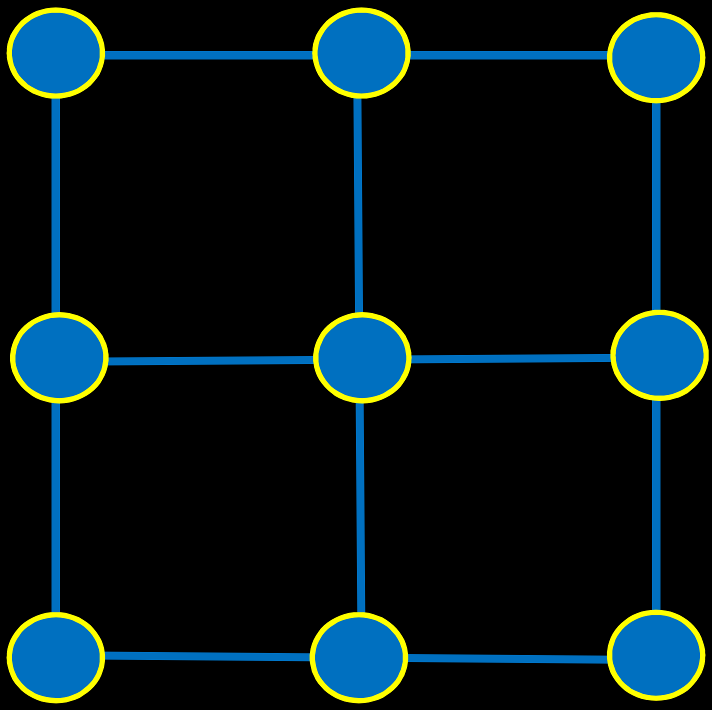

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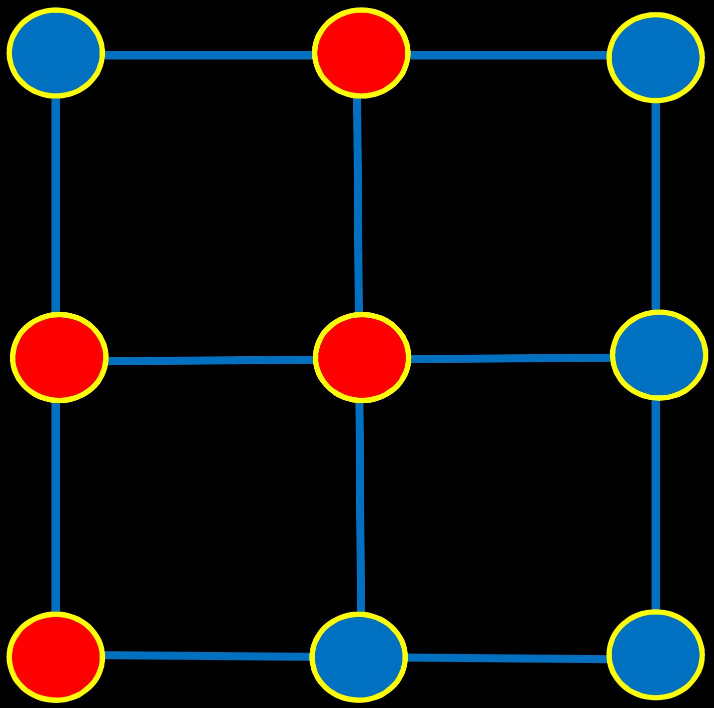

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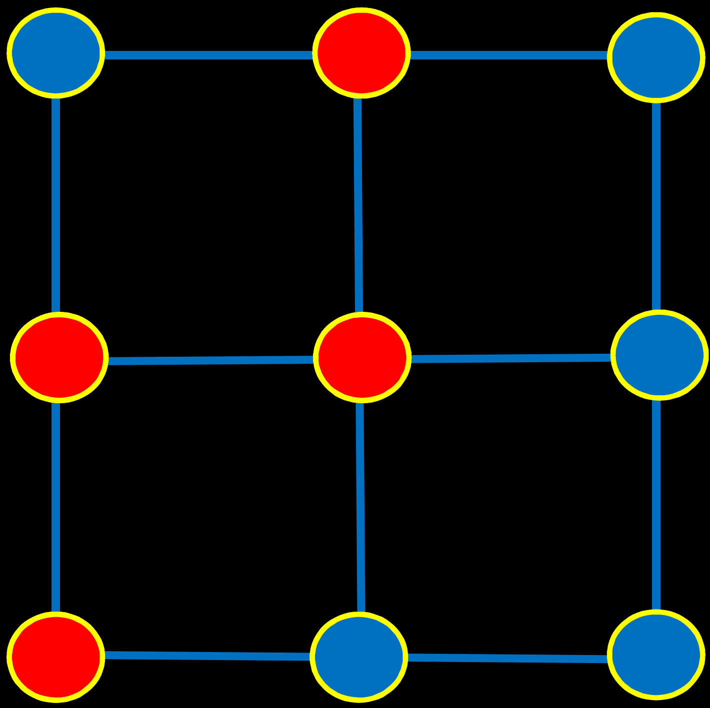
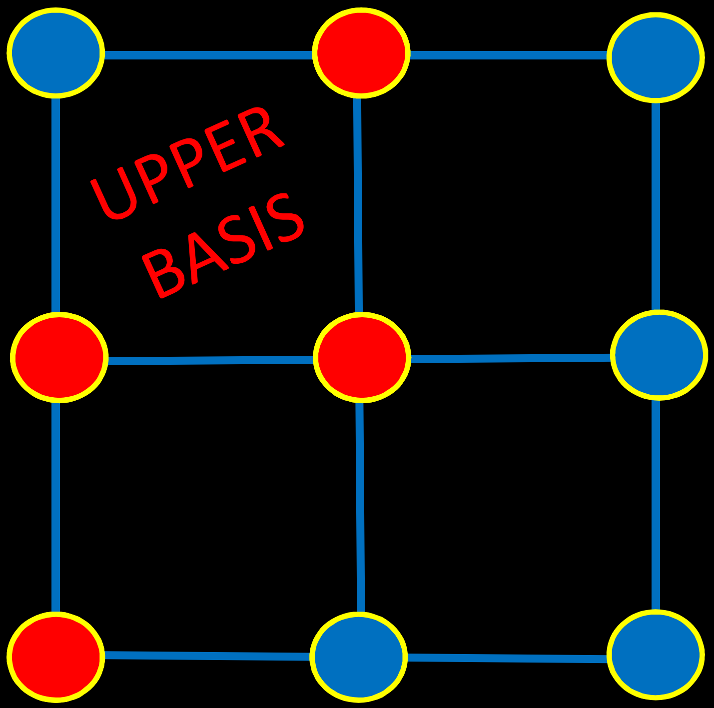


$\dim^+(G) = 4$

Upper Dimension and Resolving Number

$\dim^+(G)$= maximum size of a minimal resolving set

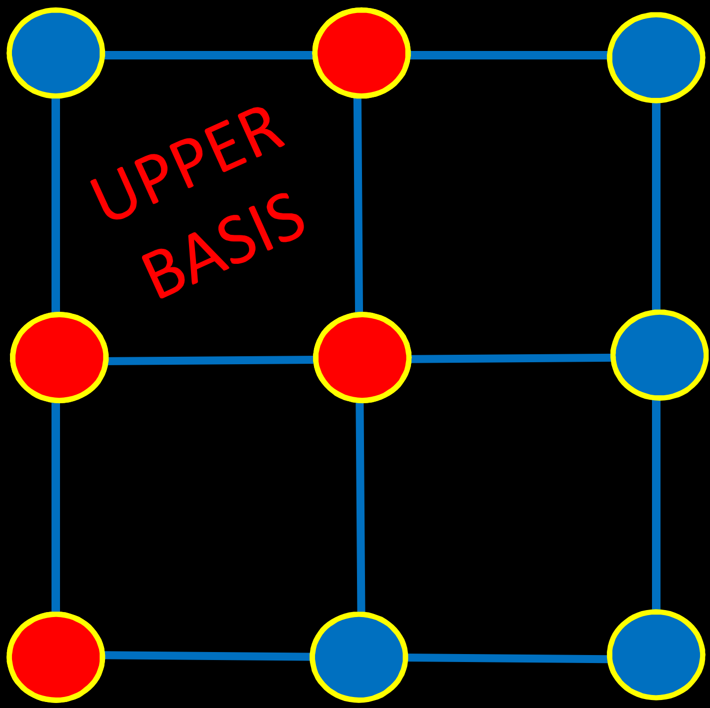


$\dim^+(G) = 4$

Upper Dimension and Resolving Number

$\dim^+(G)$= maximum size of a minimal resolving set

res(G)= minimum k such that every k-subset is a resolving set.



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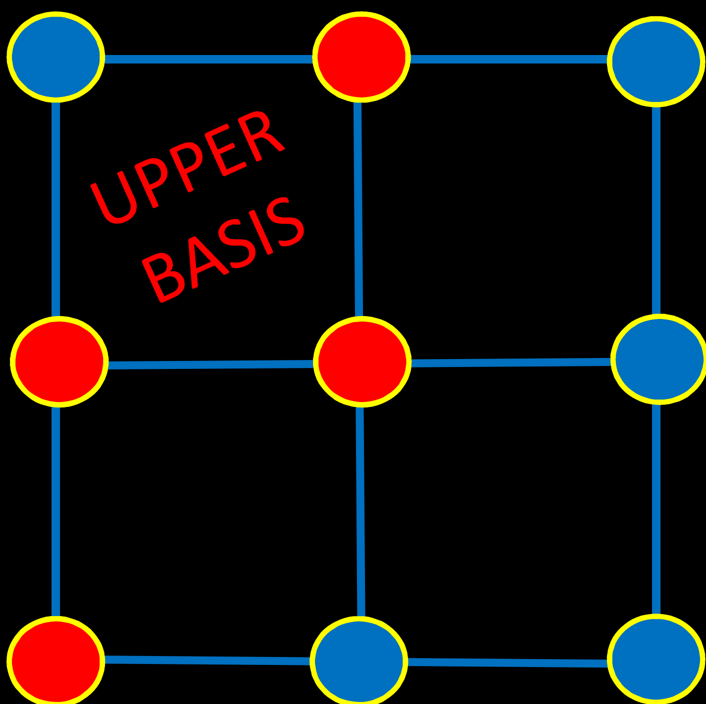

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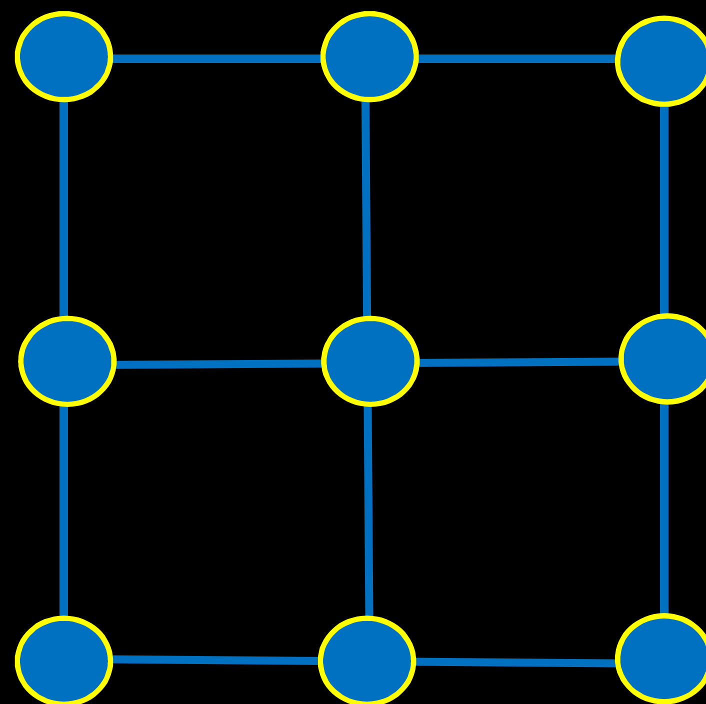
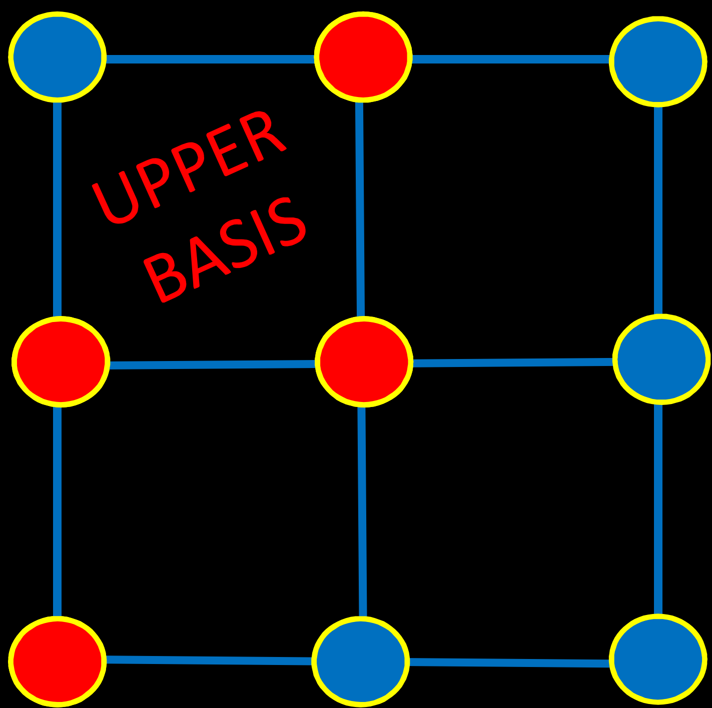


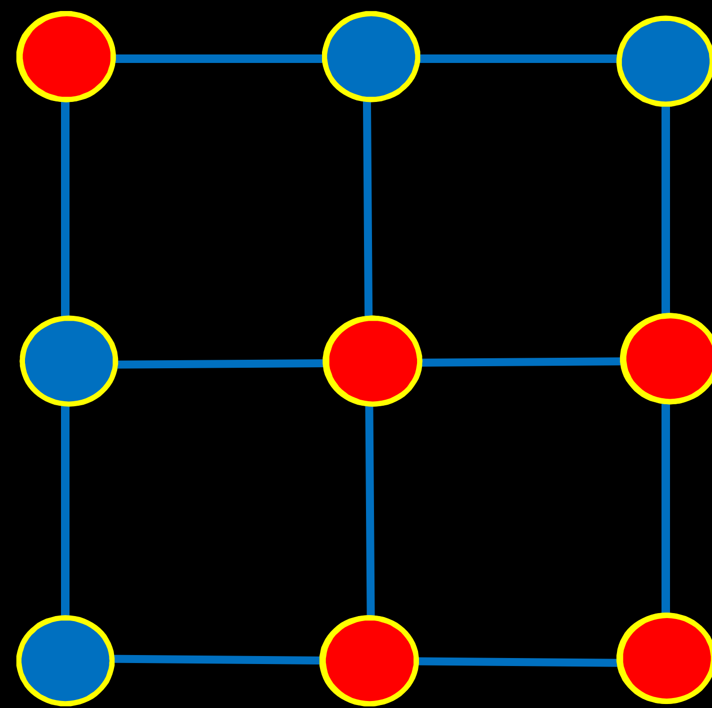
Upper Dimension and Resolving Number

$\dim^+(G)$= maximum size of a minimal resolving set

$\mathrm{res}(G)$= minimum k such that every k-subset is a resolving set.



$\dim^+(G) = 4$



Upper Dimension and Resolving Number

dim$^+$(G)= maximum size of a minimal resolving set

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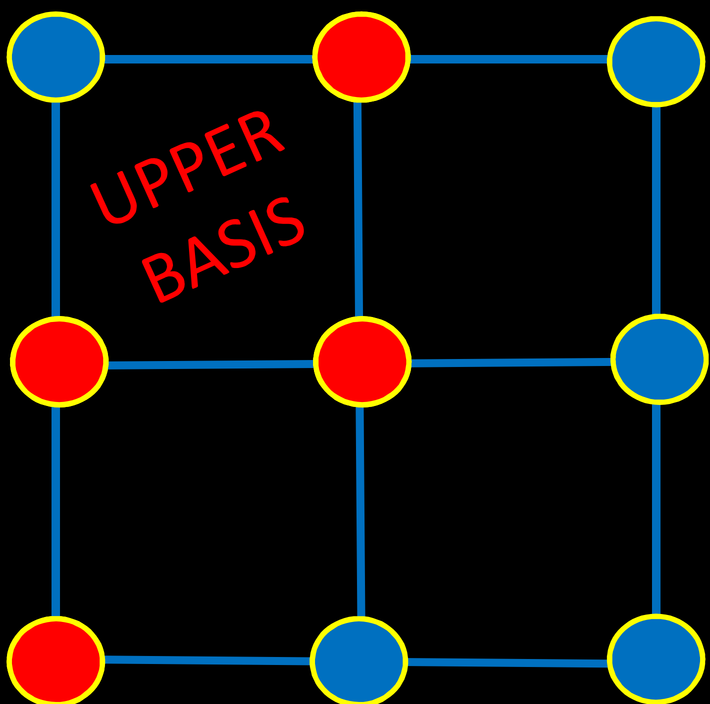

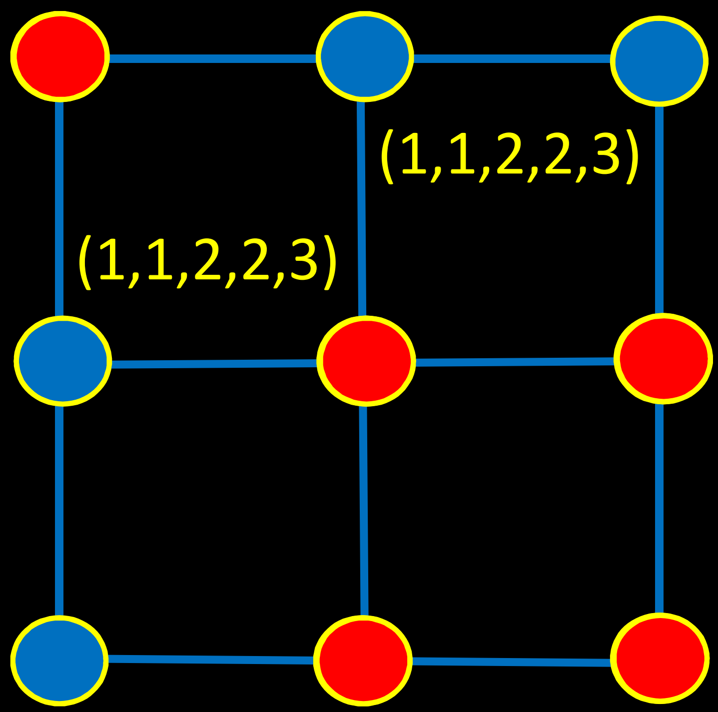

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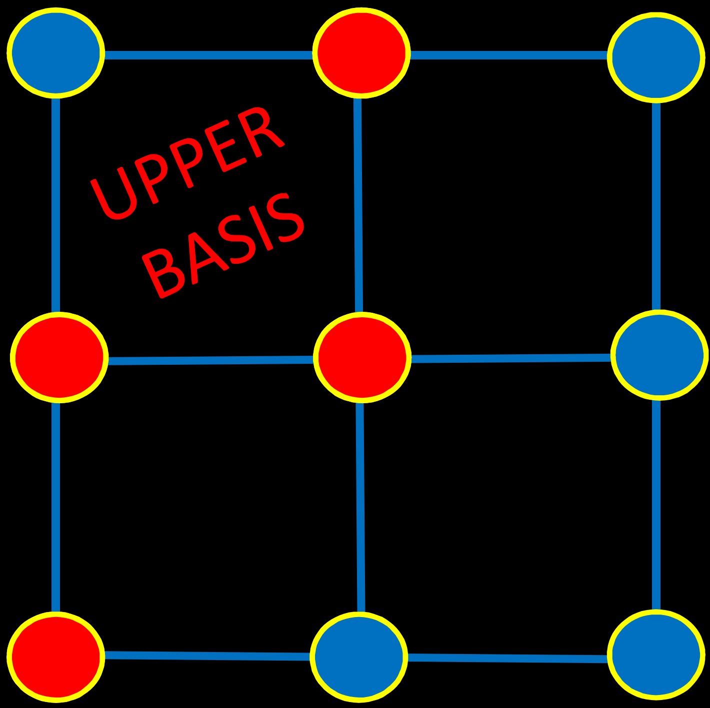

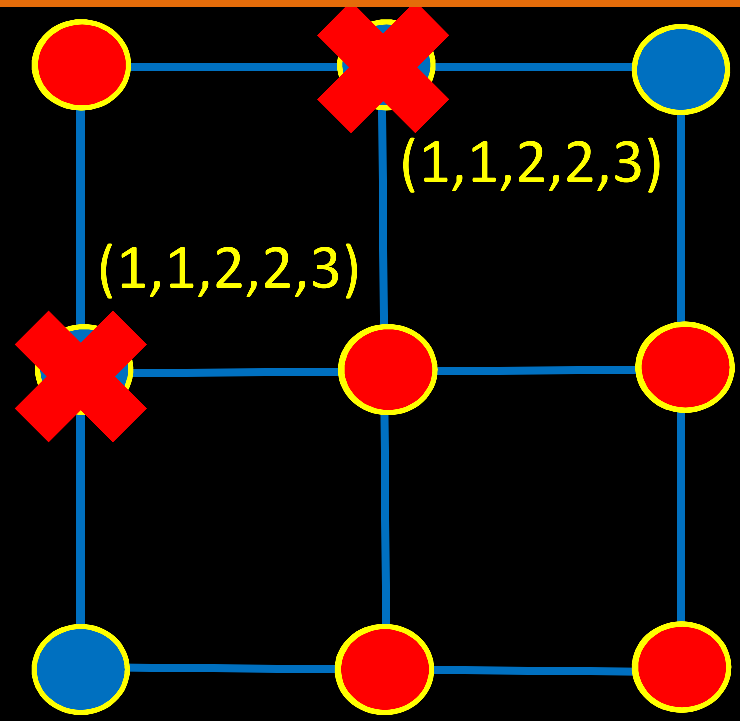

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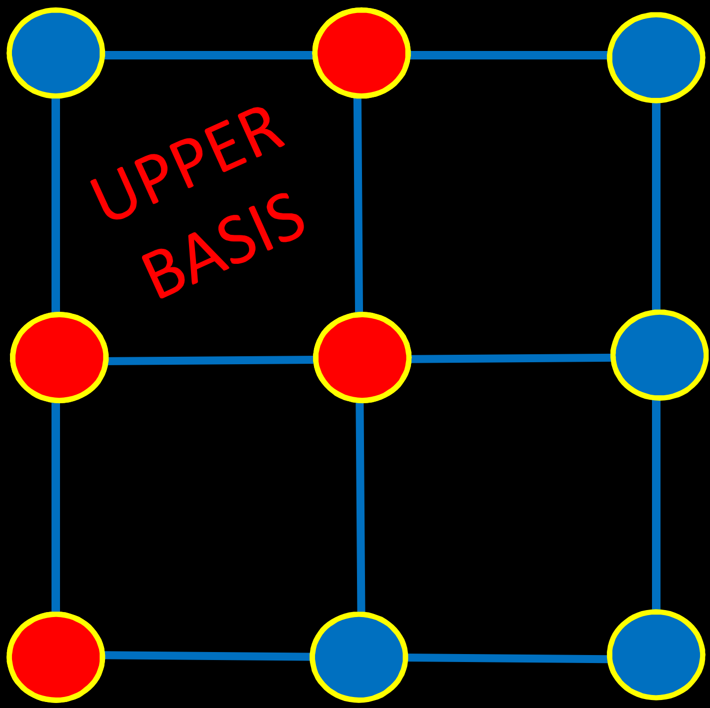
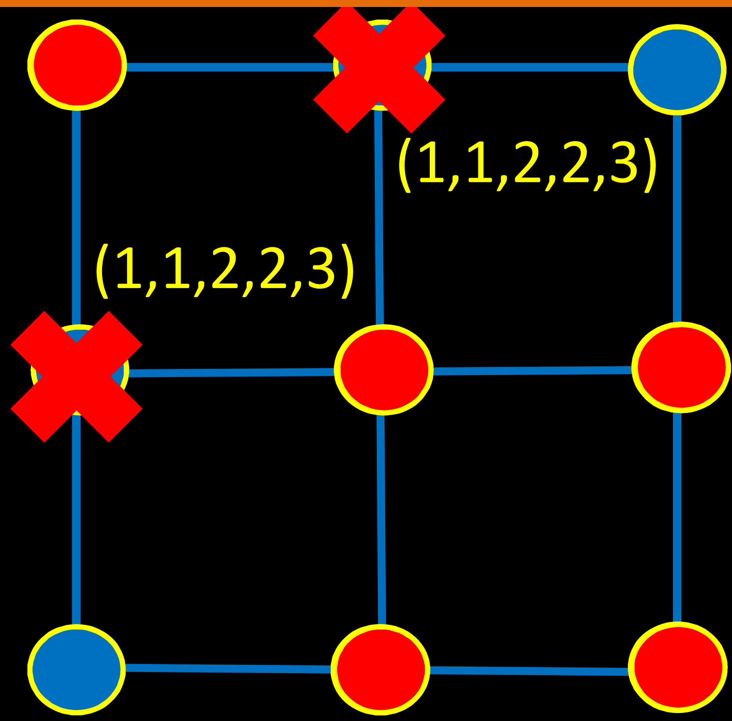




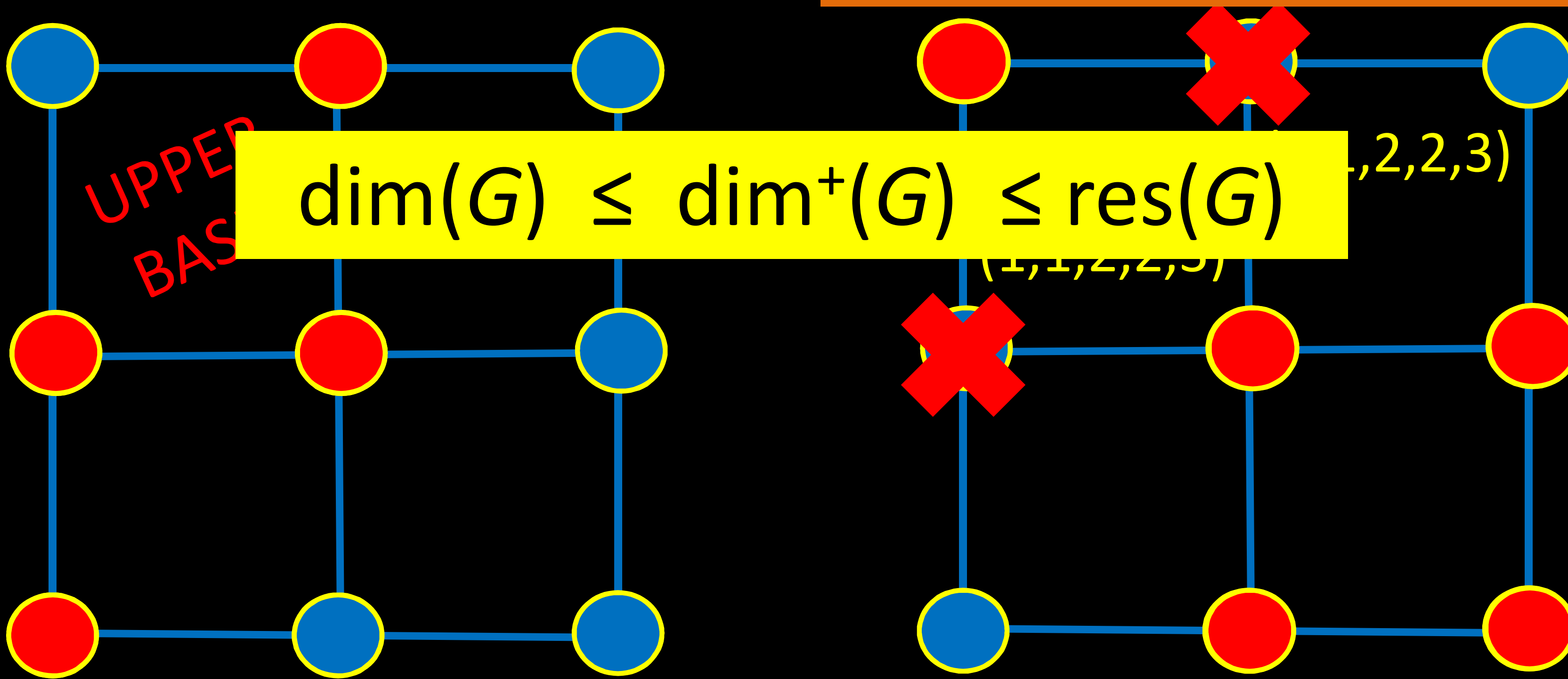
$\dim^+(G) = 4$

$\mathrm{res}(G) = 6$

Upper Dimension and Resolving Number

$\dim^+(G)$ = maximum size of a minimal resolving set

$\mathrm{res}(G)$ = minimum k such that every k-subset is a resolving set.



$$\dim(G) \le \dim^+(G) \le \mathrm{res}(G)$$

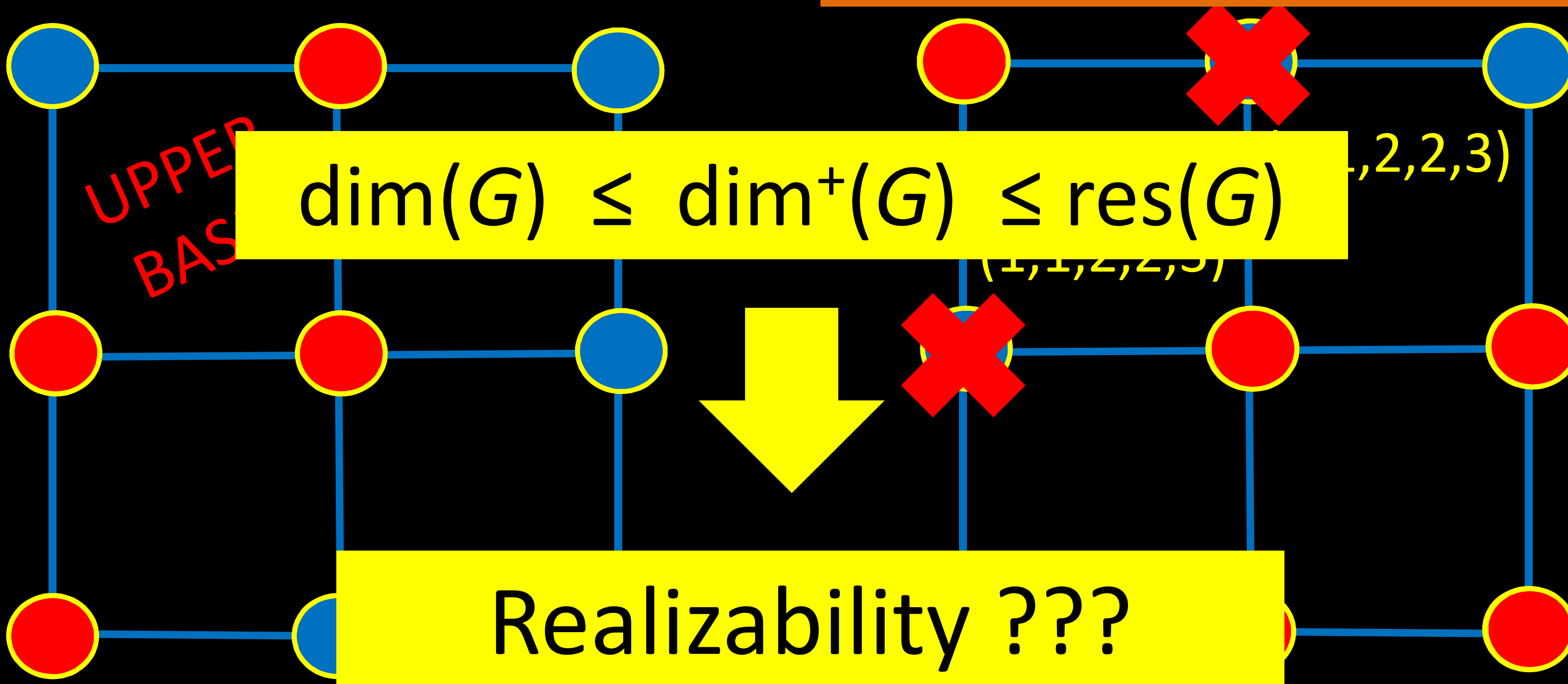
$\dim^+(G) = 4$

$\mathrm{res}(G) = 6$

Upper Dimension and Resolving Number

$\dim^+(G)$= maximum size of a minimal resolving set

$\text{res}(G)$= minimum k such that every k-subset is a resolving set.



$$\dim(G) \le \dim^+(G) \le \text{res}(G)$$

Realizability ???

$\dim^+(G) = 4$

$\text{res}(G) = 6$

$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$

⬇

Realizability ???

Realizability

Realizability

[Chartrand et al.,2000]

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

Realizability ← [Chartrand et al.,2000]

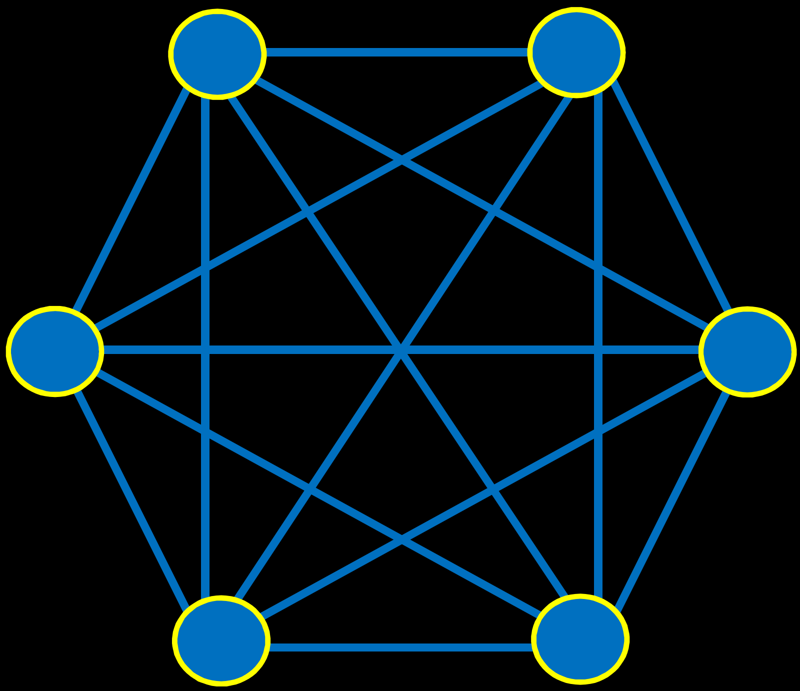
$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

$$\begin{array}{c} \shortparallel \\ a \end{array}$$

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

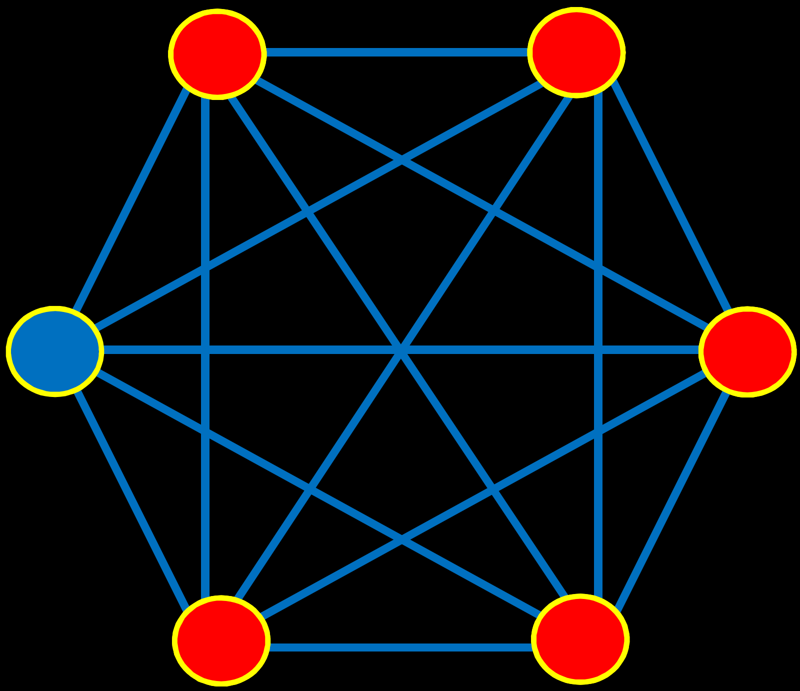
$$\| \quad a$$



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^+(G) \leq \mathrm{res}(G)$$

$$\dim(G) = a$$



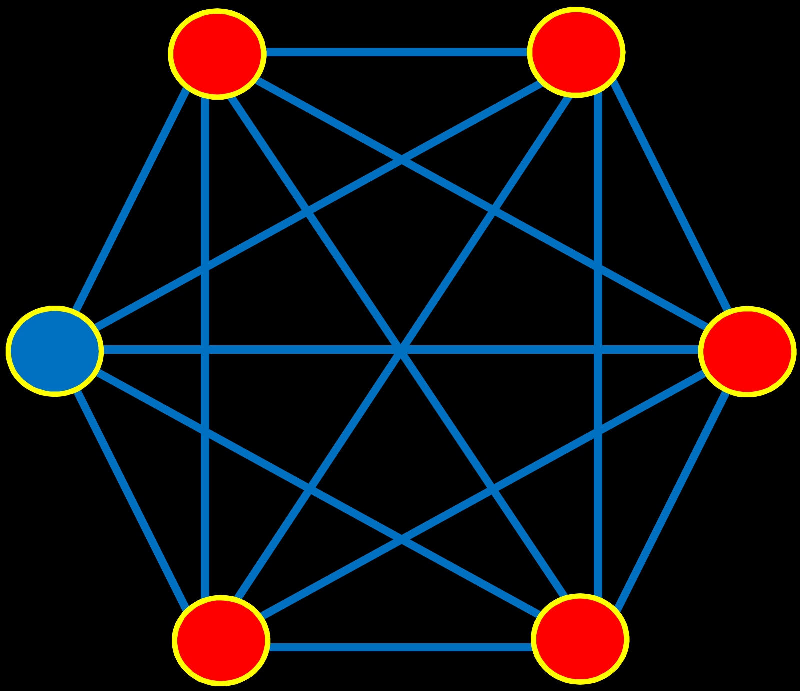
Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq res(G)$$

$=$

a

$\dim(K_n)$=n-1



Realizability ← [Chartrand et al.,2000]

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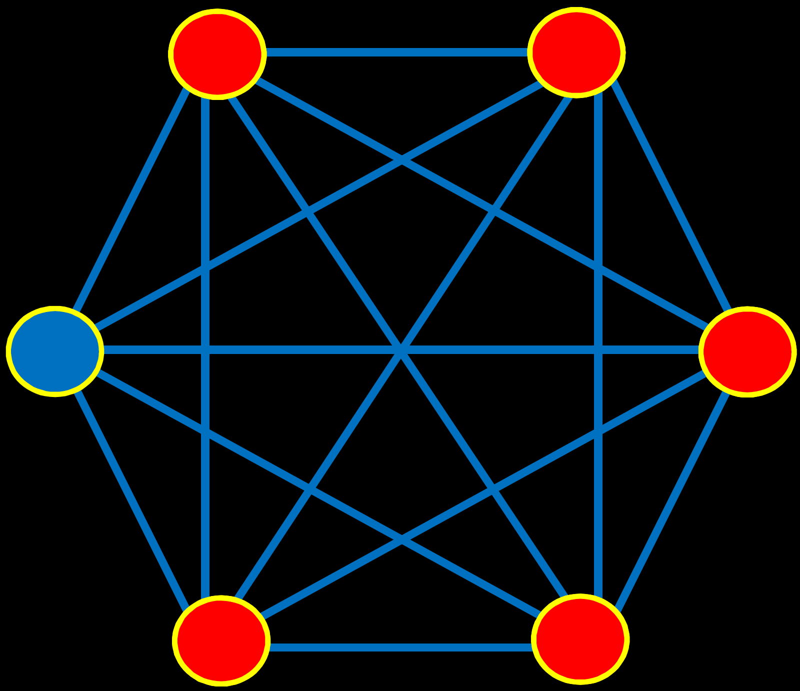
$\dim(K_n)$=n-1



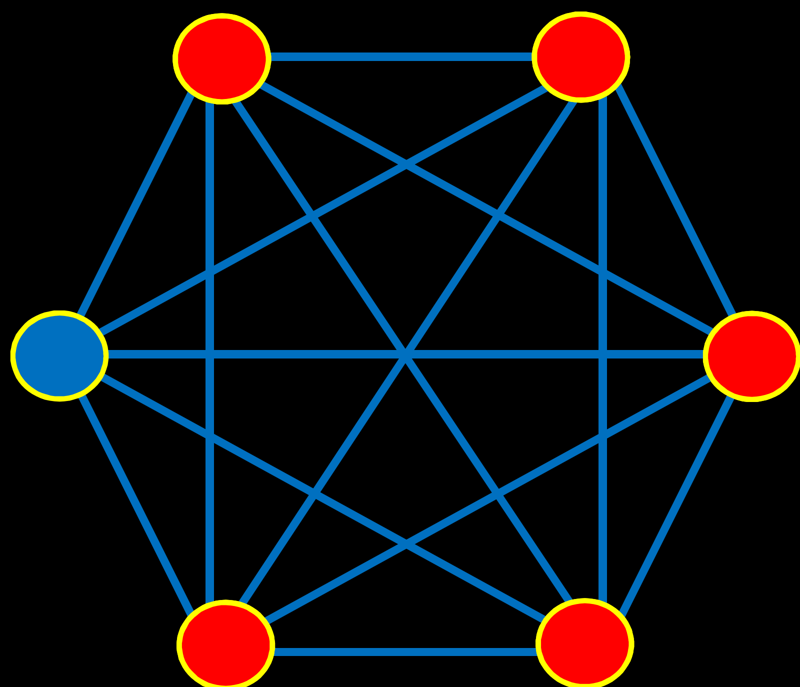
Realizability

[Chartrand et al.,2000]

$$\dim(G) \le \dim^{+}(G) \le \mathrm{res}(G)$$

$=$
c

$\dim(K_n)$=n-1

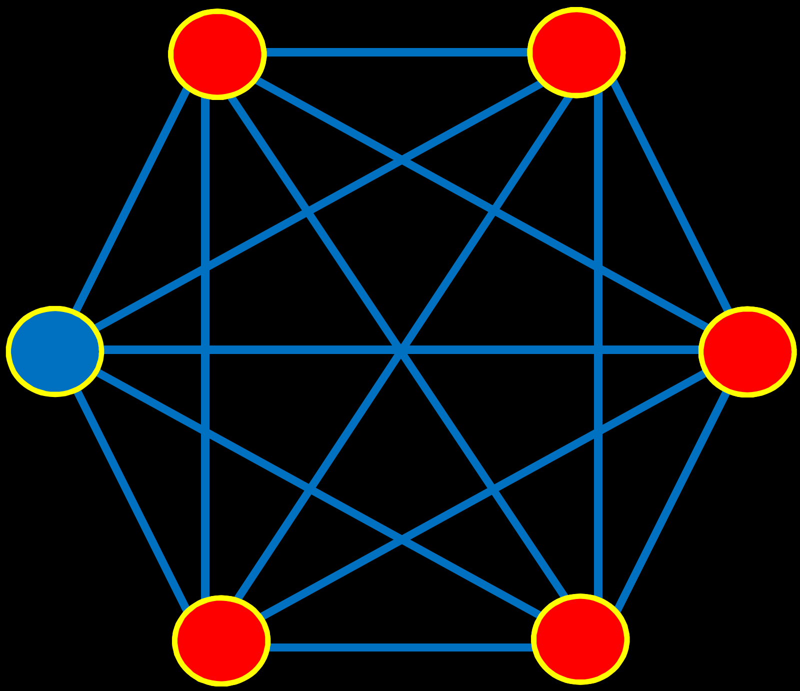


Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

$$= c$$

$\dim(K_n)$=n-1



$\mathrm{res}(K_n)$=n-1

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^{+}(G) \le \mathrm{res}(G)$$

$$\mathrm{res}(G) = c$$

Realizability ← [Chartrand et al.,2000]

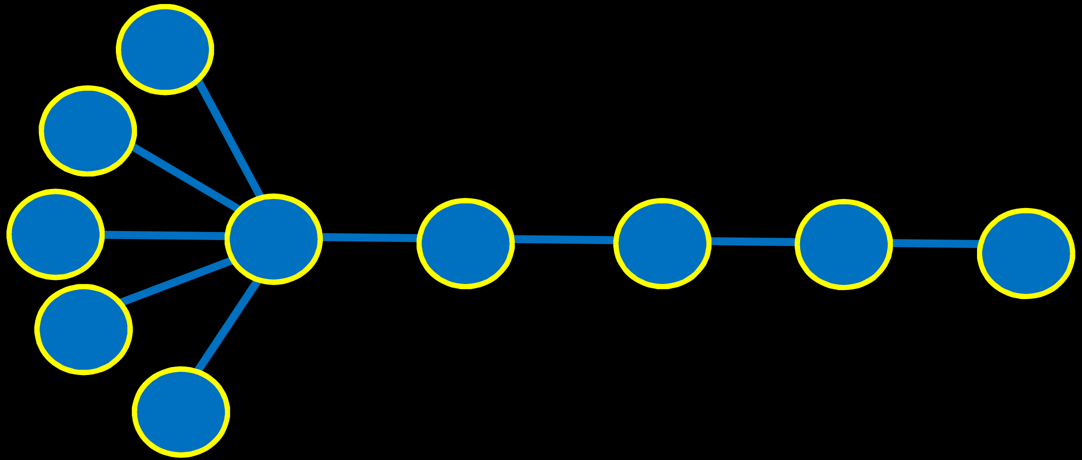
$$\dim(G) \leq \dim^+(G) \leq \mathrm{res}(G)$$

$$\dim(G) = a, \quad \mathrm{res}(G) = c$$

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^+(G) \le \mathrm{res}(G)$$

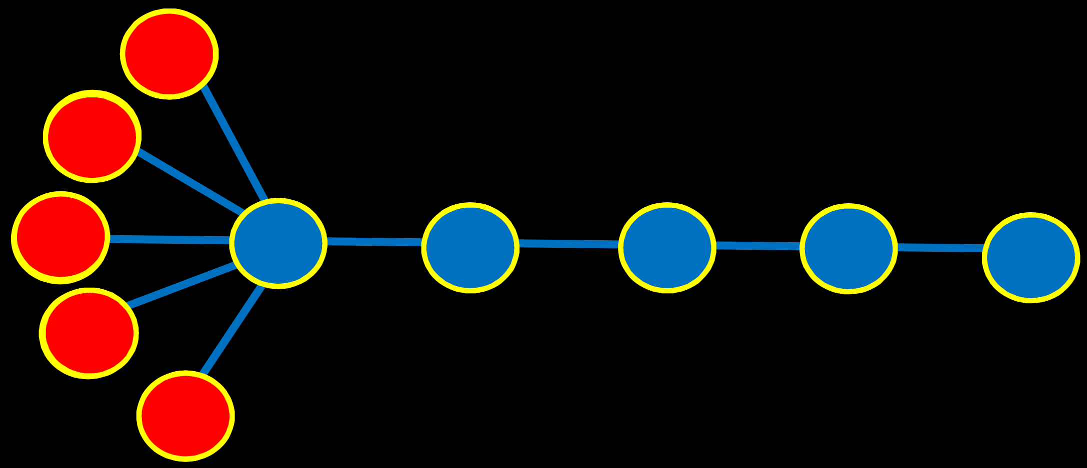
$\dim(G) = a$, $\mathrm{res}(G) = c$



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^+(G) \le \mathrm{res}(G)$$

$\dim(G) = a$, $\mathrm{res}(G) = c$



Realizability ⟵ [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

$$\dim(G) = a, \quad \mathrm{res}(G) = c$$

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^+(G) \leq \mathrm{res}(G)$$

$$\begin{matrix} \shortparallel & & \shortparallel \\ a & & b \end{matrix}$$

Realizability

[Chartrand et al.,2000]

$$\dim(G) \le \dim^+(G) \le \mathrm{res}(G)$$

$$\dim(G) = a, \quad \dim^+(G) = b$$

Conjecture: For every pair *a,b* of integers with $2 \le a \le b$, there exists a conected graph *G* such that $\dim(G)=a$ and $\dim^+(G)=b$.

Realizability

[Chartrand et al.,2000]

$$\dim(G) \leq \dim^+(G) \leq res(G)$$

$\dim(G) = a$, $\dim^+(G) = b$

Theorem: For every pair *a,b* of integers with $2 \leq a \leq b$, there exists a conected graph G such that $\dim(G)$=a and $\dim^+(G)$=b.

It is true!!! [Garijo,G.,Márquez,2011]

Realizability ← [Chartrand et al.,2000]

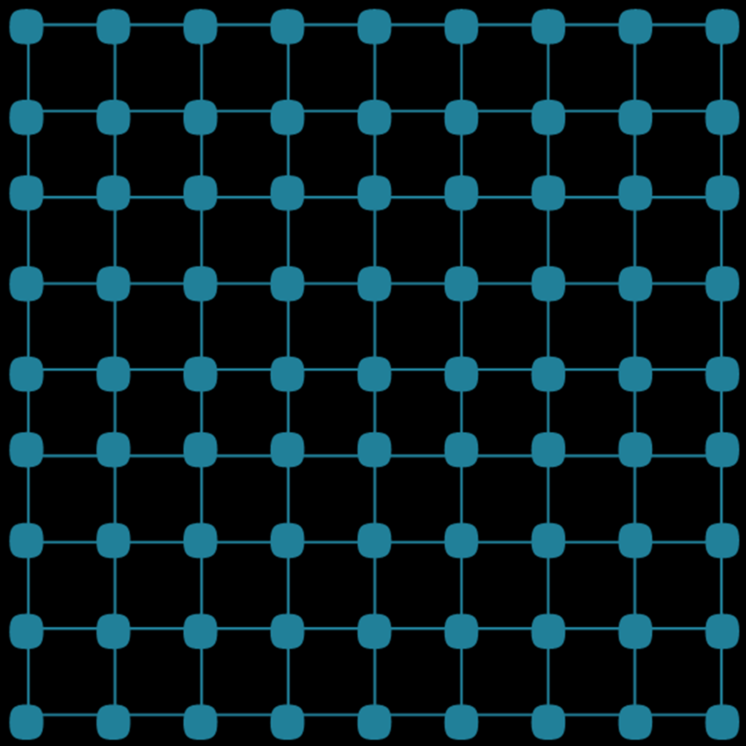
$$\dim(G) \;\le\; \dim^{+}(G) \;\le\; \mathrm{res}(G)$$

$$\begin{matrix} \| & & \| \\ a & & b \end{matrix}$$

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

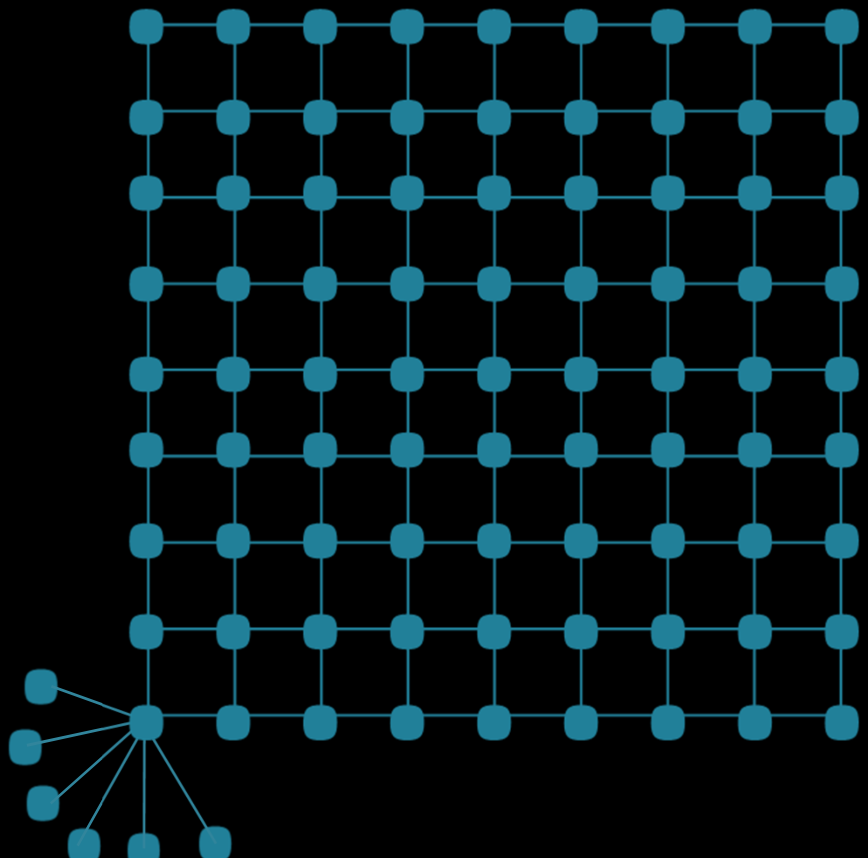
$\dim(G) = a$, $\dim^{+}(G) = b$



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^+(G) \le \mathrm{res}(G)$$

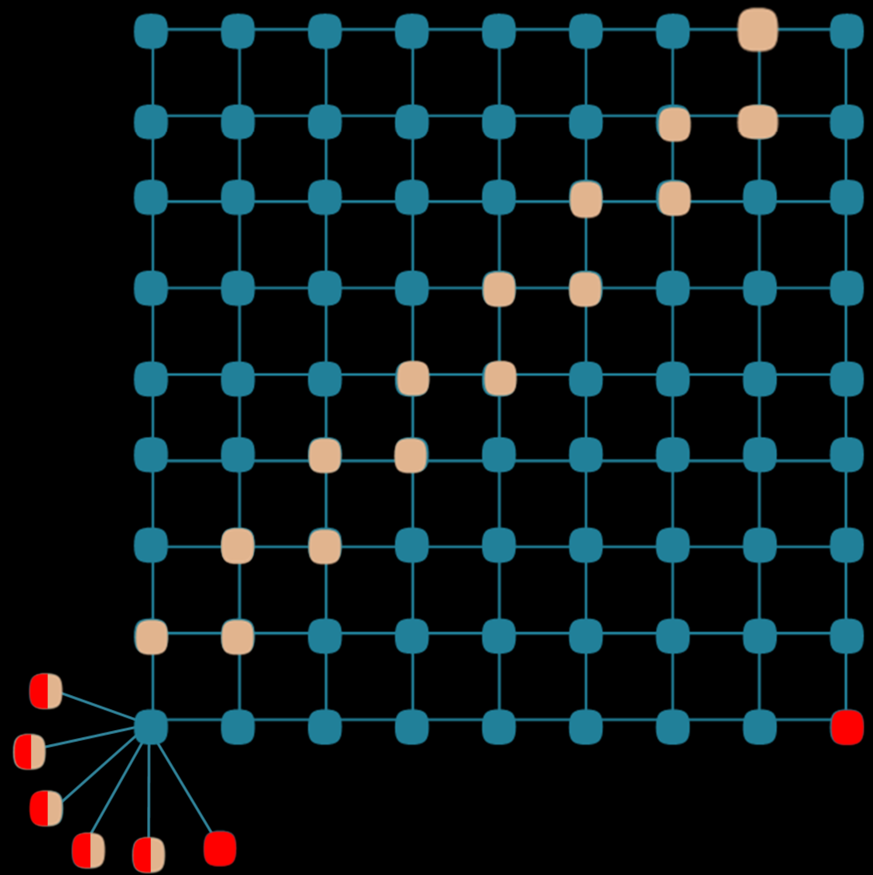
$$\dim(G) = a, \quad \dim^+(G) = b$$



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq res(G)$$

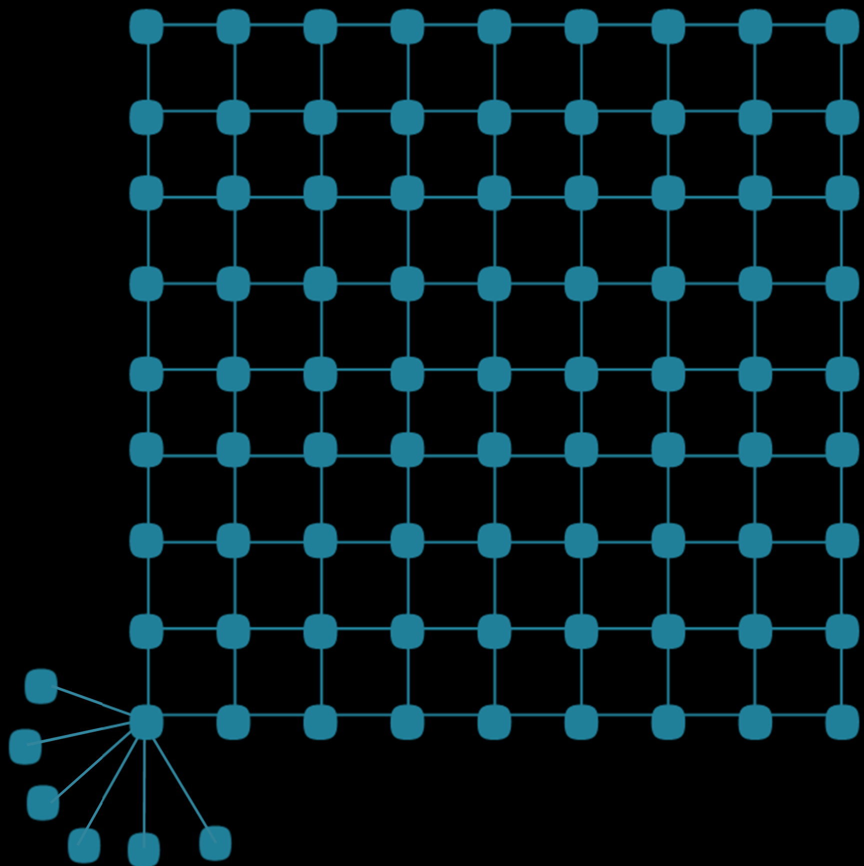
$\dim(G) = a$, $\dim^{+}(G) = b$



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \leq \dim^{+}(G) \leq \mathrm{res}(G)$$

$\dim(G) = a$, $\dim^{+}(G) = b$

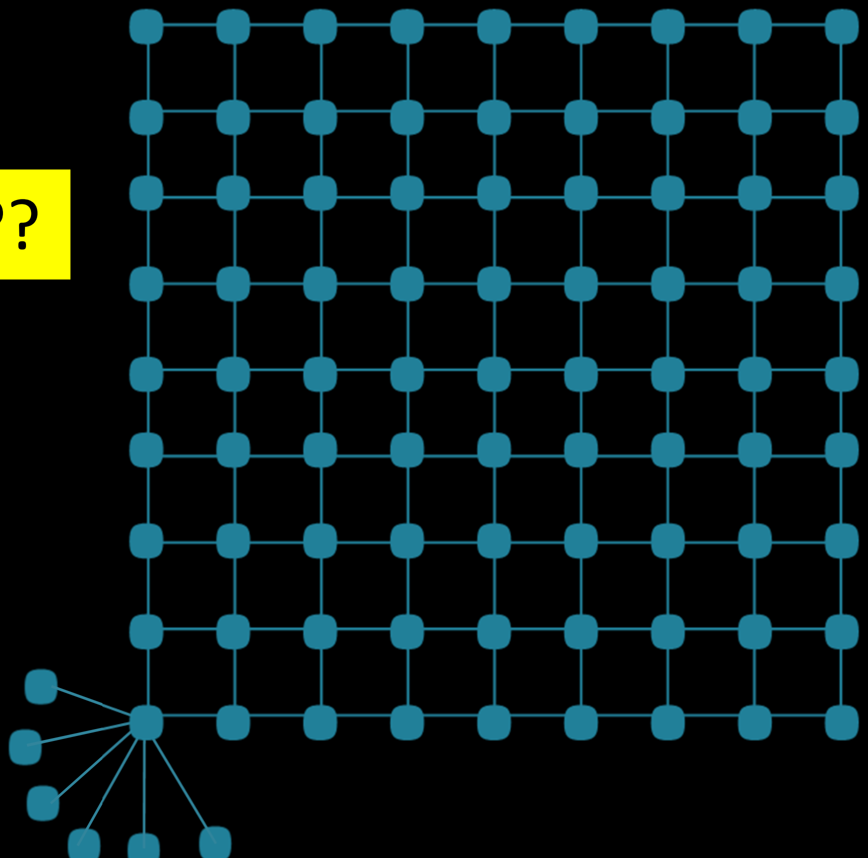


Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^{+}(G) \le res(G)$$

$\dim(G) = a$, $\dim^{+}(G) = b$

How many???



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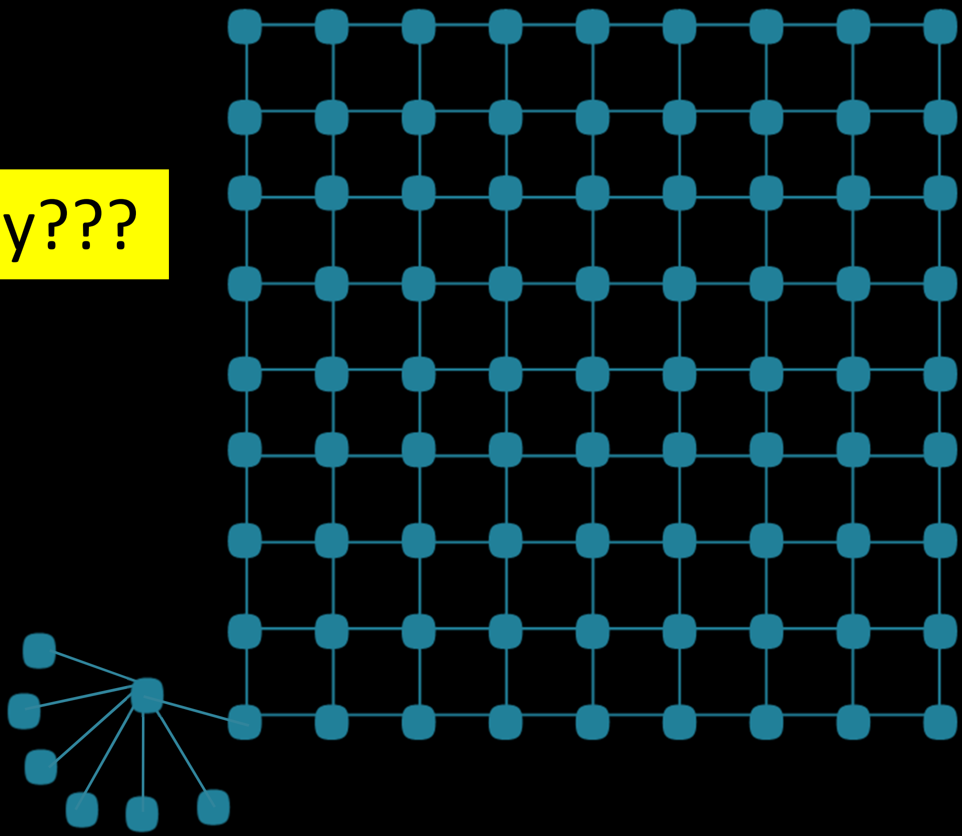
$\dim(G) = a$, $\dim^{+}(G) = b$

How many???



Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^{+}(G) \le \mathrm{res}(G)$$

$$\dim(G) = a, \quad \dim^{+}(G) = b$$

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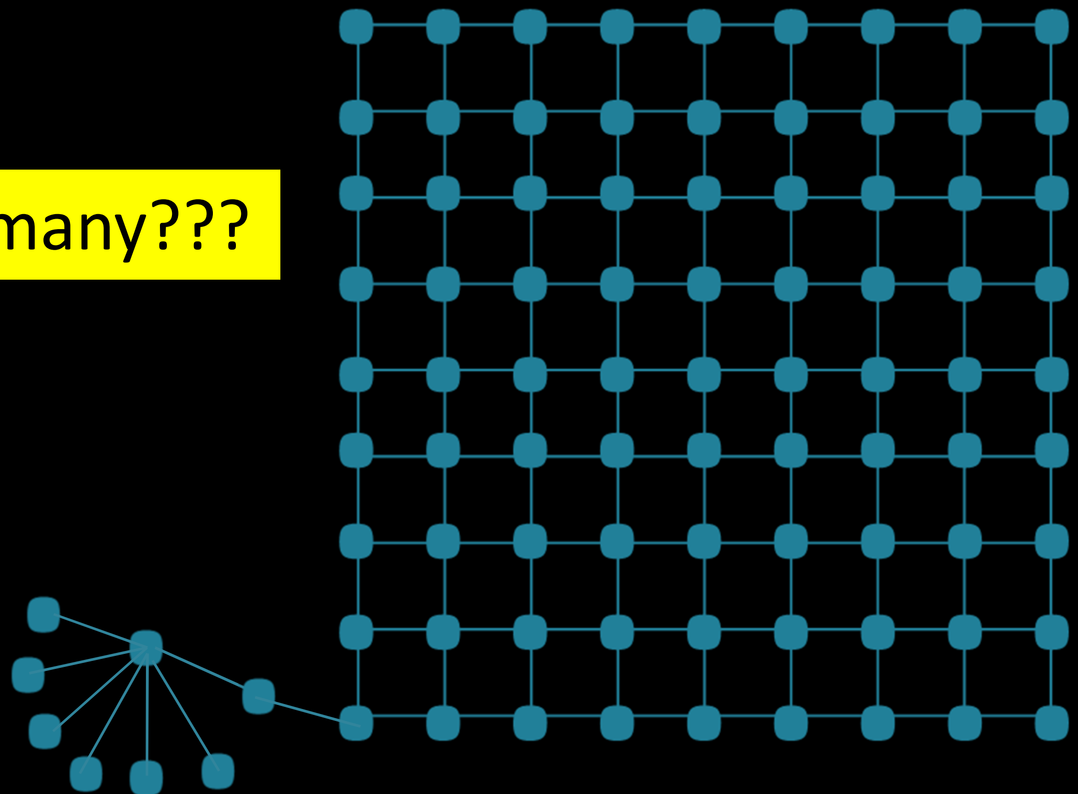

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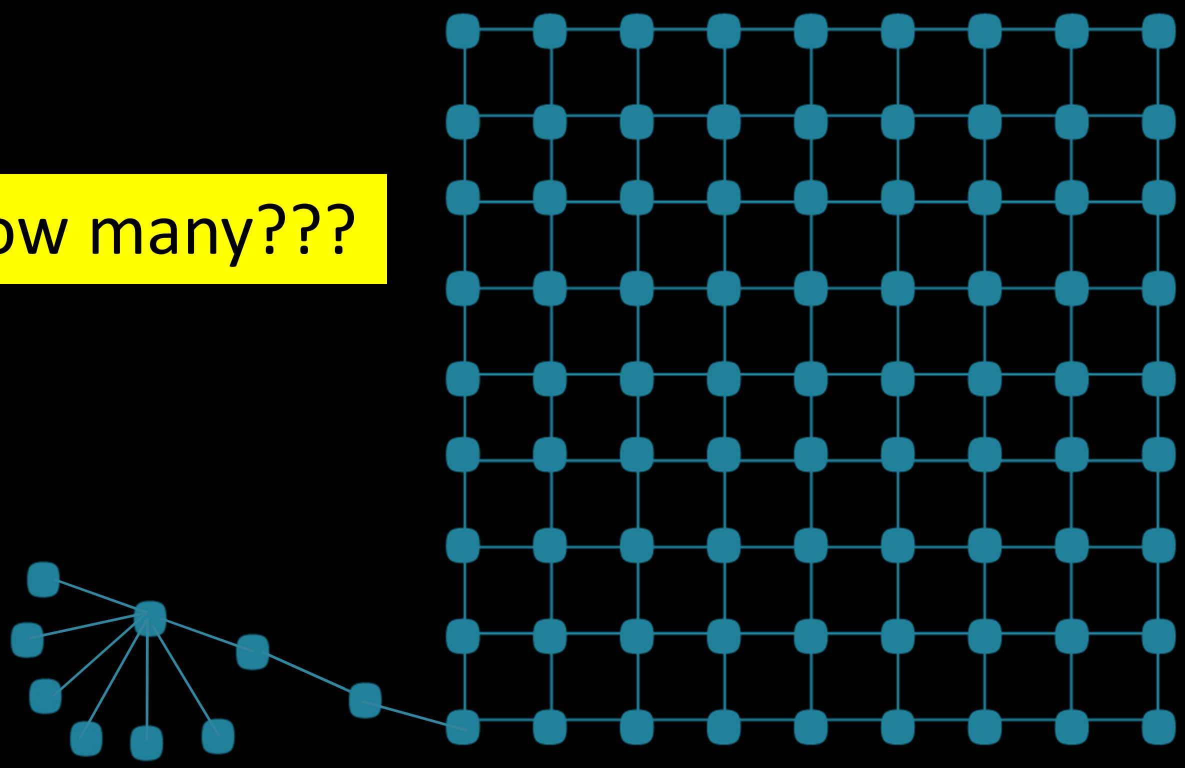

Realizability ← [Chartrand et al.,2000]

$$\dim(G) \le \dim^{+}(G) \le \mathrm{res}(G)$$

$$\begin{matrix} \| & & \| & & \| \\ a & & b & & c \end{matrix}$$

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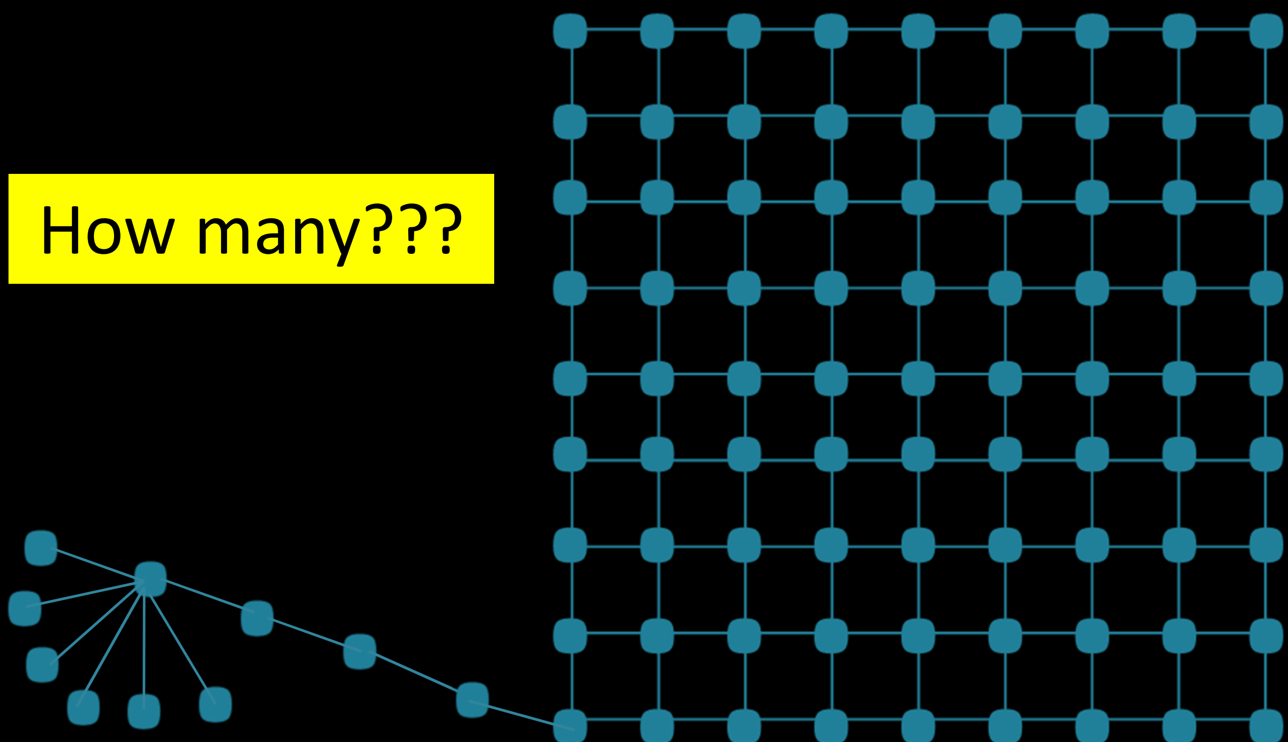

Realizability

[Chartrand et al.,2000]

$$\dim(G) \;\le\; \dim^+(G) \;\le\; \mathrm{res}(G)$$
$$\quad\;\; \| \qquad\qquad\quad \| \qquad\qquad\quad \|$$
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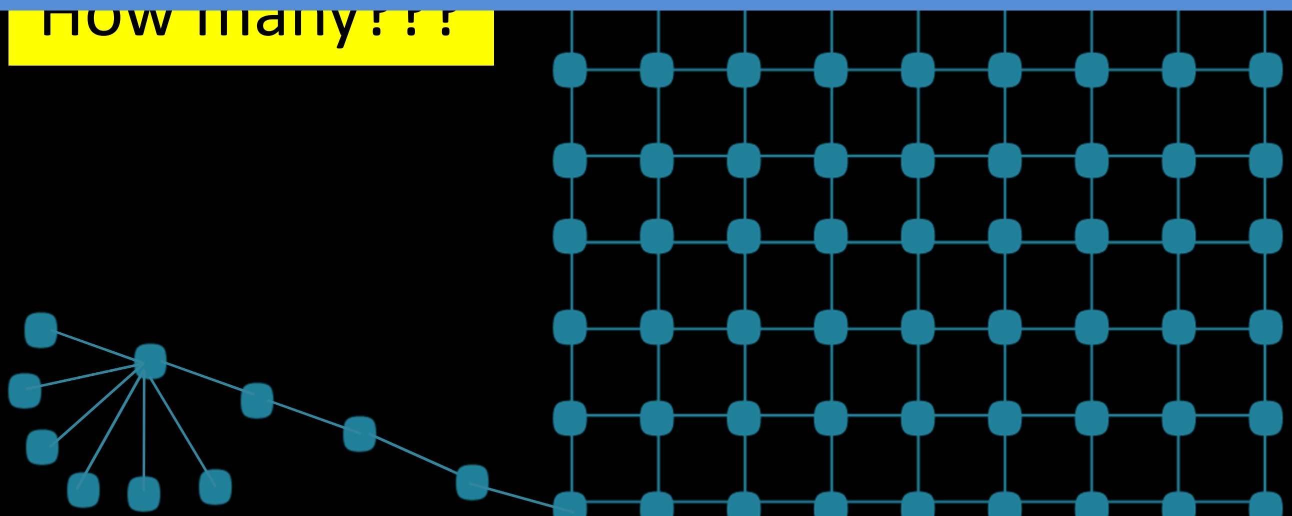
<u>Theorem</u>:[Garijo,G.,Márquez] Given c>3, the set of graphs with resolving number *c* is finite.

How many???



Realizability

[Chartrand et al.,2000]

$$\dim(G) \leq \dim^+(G) \leq \operatorname{res}(G)$$

$$a = \dim(G), \quad b = \dim^+(G), \quad c = \operatorname{res}(G)$$

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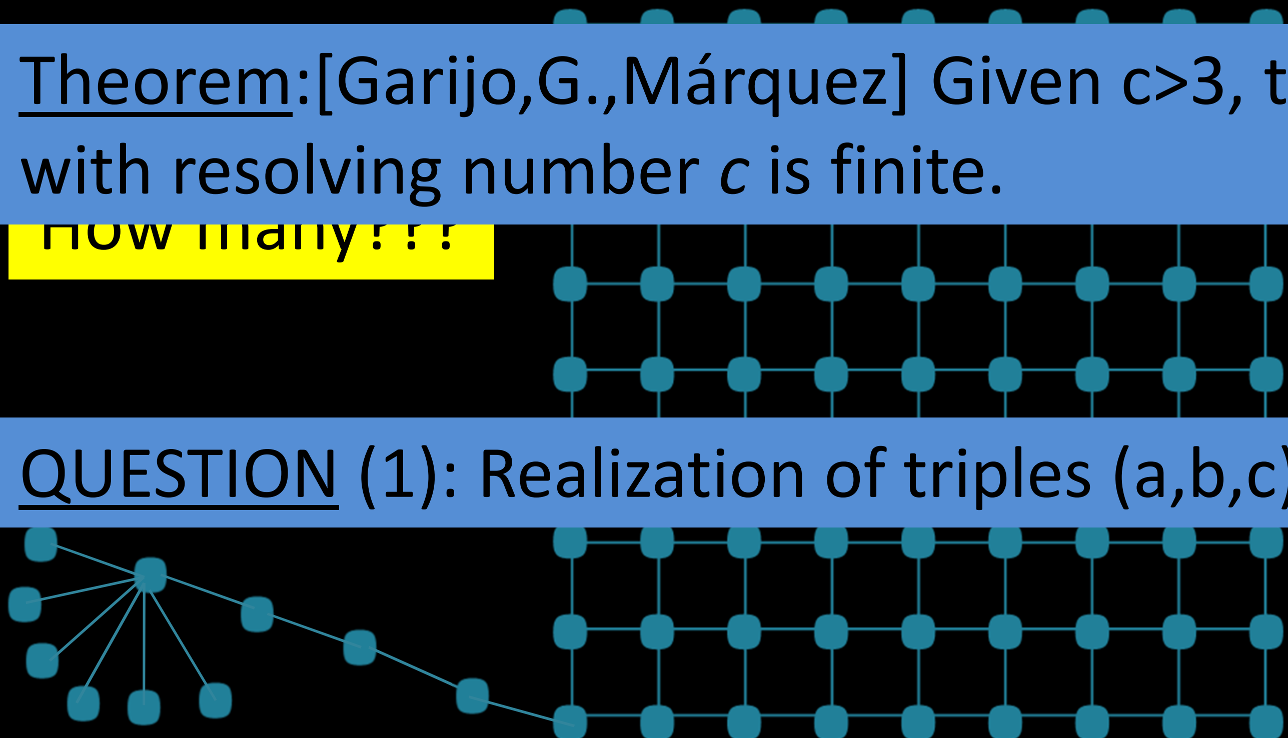
<u>QUESTION</u> (1): Realization of triples (a,b,c).



Realizability

[Chartrand et al.,2000]

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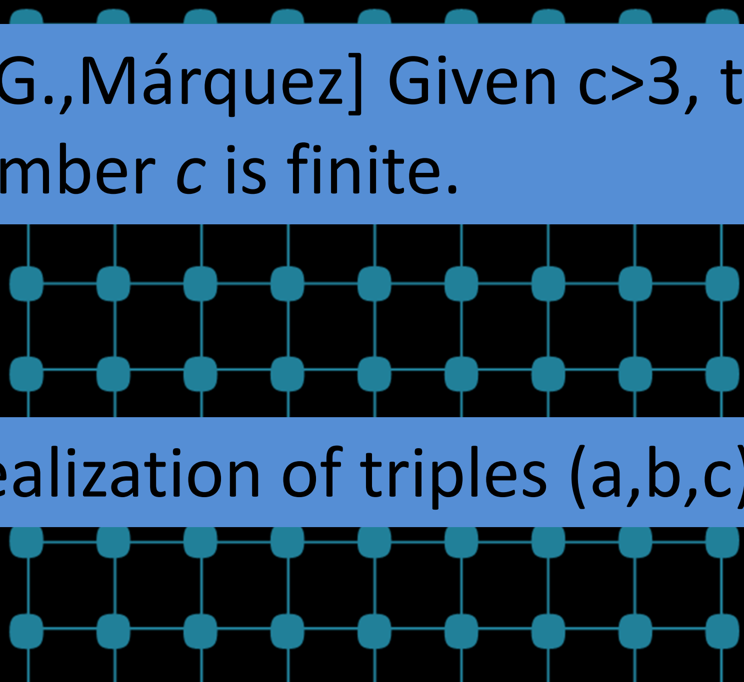
$$\begin{matrix} \shortparallel & \shortparallel & \shortparallel \\ a & b & c \end{matrix}$$



Theorem:[Garijo,G.,Márquez] Given c>3, the set of graphs with resolving number *c* is finite.

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QUESTION (1): Realization of triples (a,b,c).

QUESTION (2): RECONSTRUCTION!!!

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Reconstruction

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<u>Problem</u>: given $c > 0$, which are the graphs G such that res(G) = c?

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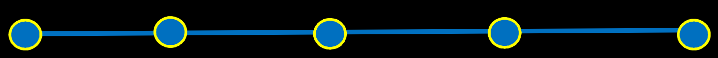

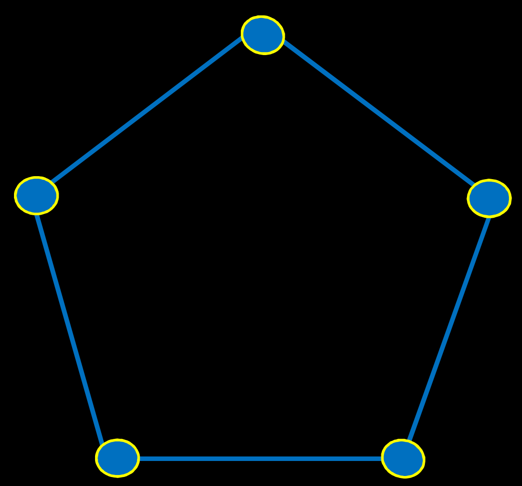

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[Garijo,G.,Márquez,2011] If G is neither a tree nor a cycle, then:

1. $g(G) \leq 2\, res(G) - 1$
2. $D(G) \leq 3\, res(G) - 5$
3. $n \leq 2 res(G)$ whenever G has diameter 2
4. $\Delta(G) \leq 2\, res(G)$

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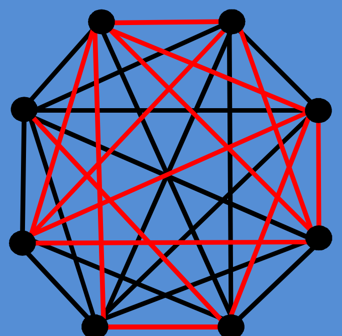

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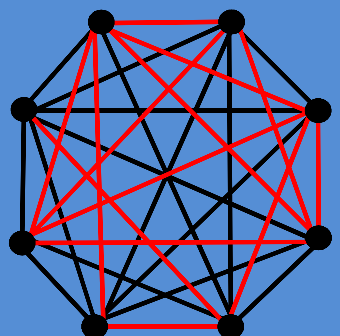

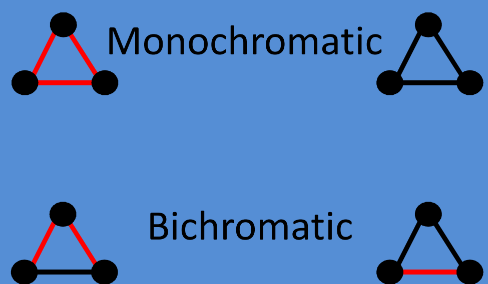

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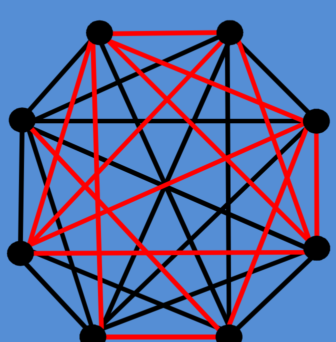

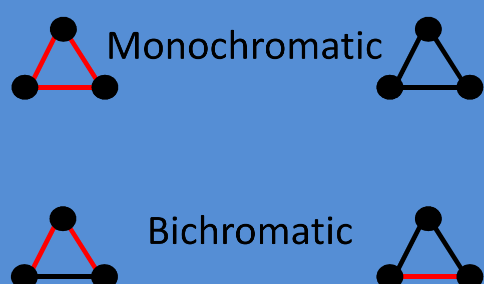

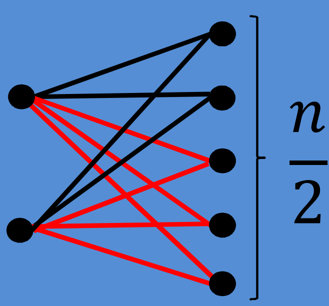

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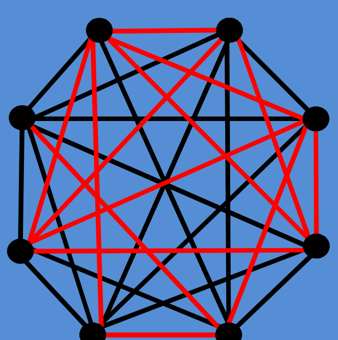

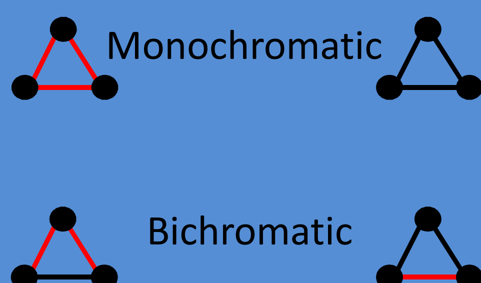

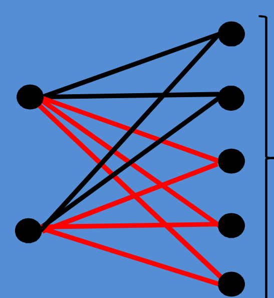

$\frac{n}{2}$ ⟹ $\mathrm{res}(G) \geq \frac{n}{2}$

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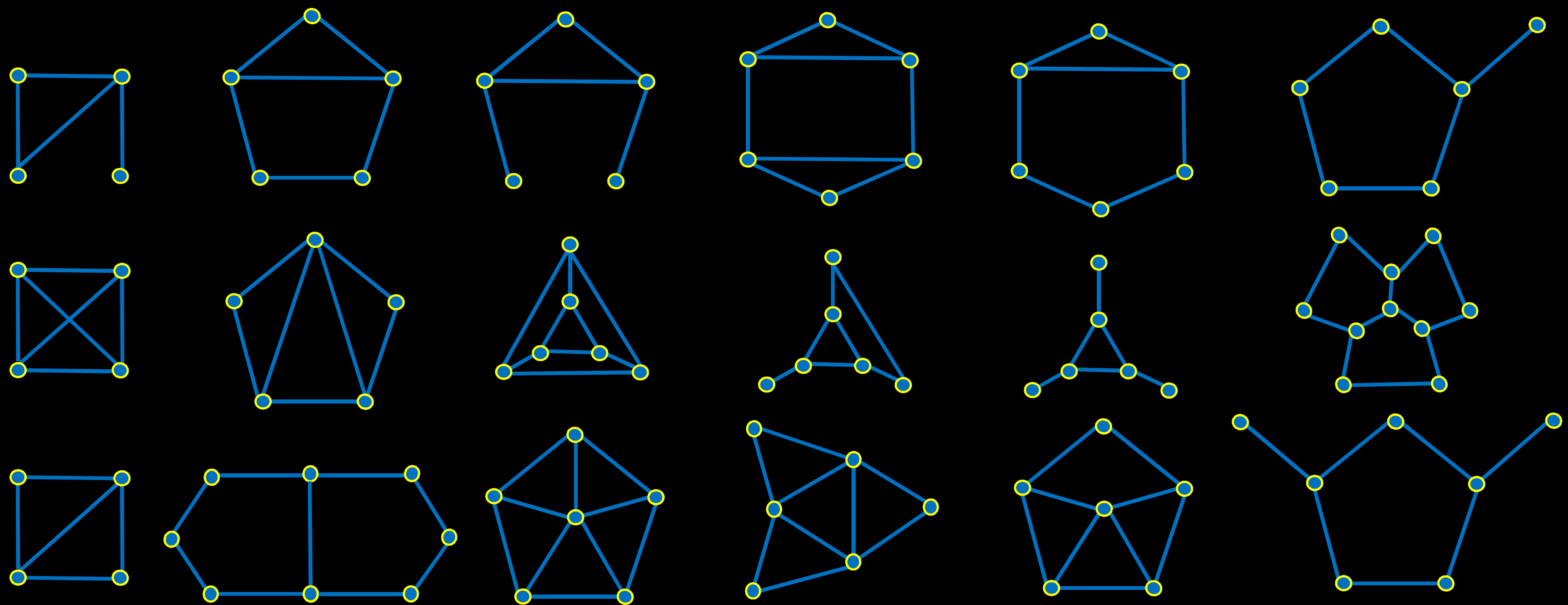

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Open problem: Reconstruction of trees.

Thanks!